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Specification and negotiation in incomplete contracts

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We investigate contractors' bargaining power and holdup on buyers in procurement auctions of incomplete contracts held by California Department of Transportation. Using a model where contractors bid competitively in response to a buyer's choice of initial contract design, we infer the contractors' costs and bargaining power from the bids and transfers negotiated after the auction. We find that the contract winners have substantial bargaining power in post-auction negotiation. The average holdup on the buyer is about 20% of project costs. Counterfactual cost-plus contracts would reduce the buyer's surplus in 72% of the projects, with an average reduction over $*382,000.*

1. Introduction

■ Government procurement contracts are often subject to negotiation and modification *after* their initial assignment through low-price auctions. The initial contract specification, or design, might be revised, and an additional transfer be negotiated between the buyer and the auction winner. Examples include the auctions of procurement contracts by California Department of Transportation (CalTrans) in Bajari, Houghton, and Tadelis (2014) and Texas Department of Transportation (DOT) in De Silva et al. (2016). Awareness of such incompleteness influences how the sellers (contractors) compete in the auction and interact with the buyer. It also affects the answers to policy questions such as comparing the buyer surplus under alternative forms of contracts.

We investigate empirical and theoretical questions related to the strategic incentives of the buyer and the contractors under incomplete contracts in highway procurement auctions held by CalTrans. The questions we study are partly motivated by several stylized facts in the data. First, a nonnegligible proportion of the contracts are not revised after the procurement auctions. Second,

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among the revised contracts, the negotiated transfers vary significantly, even conditional on the size of revision. Third, although the characteristics of the contracts (e.g., the job type and the capacity of contractors involved) have no obvious effect on the size of revision, they do have a significant impact on the negotiated transfers.

These patterns naturally lead to the following questions. What factors determine the revision of a contract and the transfer to the contractor in the subsequent negotiation? How do those factors depend on the bargaining power of the buyer and the features of the contract? Is it possible to use data on the bids and the negotiated transfers to infer the social surplus generated by the contract and the costs for contractors? How to quantify the holdup on the buyer due to uncertainty about the new design? How would a buyer's surplus change under an alternative (counterfactual) form of cost-plus contracts?

We use a new structural model to answer these questions. In our model, a buyer announces an initial contract specification based on a private signal that is correlated with a new feasible design to be drawn later. The contractors are notified of the initial specification, and then bid competitively in a low-price auction. Both the buyer and the contractors are aware that the new design will supplant the initial specification if it leads to an increase in net social surplus. If a contract is revised, the buyer then makes a transfer to the auction winner in addition to the initial auction payment. The size of this transfer is negotiated via Nash Bargaining. The auction price affects the bargaining outcome through disagreement values. The buyer and contractors take this into account in strategic decisions before and during the auction. The holdup on the buyer is defined as the negotiated share of net incremental surplus paid to the auction winner.

We recover structural elements of the model through sequential steps, using multiple sources of variation in the data. First, we exploit the consistency of equilibrium beliefs to identify the buyer's belief about the new design, using the joint distribution of initial and revised contract specification. Next, we apply an argument in Guerre, Perrigne, and Vuong (2000) to recover the contractors' adjusted costs, using the bid distribution under the initial specification. These costs include a downward adjustment made by the contractors after accounting for their holdup on the buyer. We then use the observed relation between the initial contract specifications, the auction prices, and the negotiated transfers to recover the marginal effect of contract design on social surplus by utilizing the buyer's first-order condition in equilibrium. This allows us to back out the costs for revising contracts and the buyer's bargaining power under flexible shape restrictions on the revision costs. With these model elements recovered, the contractor's holdup on the buyer is identified. It then follows that the bidding strategy of contractors and the distribution of their costs are also identified. Using these model primitives, we compare the buyer surplus in the data with that under counterfactual cost-plus contracts.

Applying this model to CalTrans highway procurement auctions, we find that the auction winners have significant bargaining power against the buyers, which depends on the intensity of competition as well as contractor characteristics such as the utilization rate (i.e., the ratio between a contractor's backlog and its capacity). Our estimates also indicate that there is an increasing return in the social surplus from highway construction, and that the net surplus from the contract revision is nonlinear in the size of revision. These results are consistent with the reduced-form patterns in the data, which motivate our structural approach.

The average markup in a contractor's bid is around 11%. Markups vary with contract characteristics such as the job type and the utilization rate of competitors, and decrease sharply with the intensity of competition in auctions. Besides, our estimates suggest that auction winners have high markups, possibly due to their cost advantage over competitors. In addition, we find that markups are overestimated by about 26% on average if the impact of uncertainty in contract revision on contractors' bids is not accounted for.

Our estimates suggest that incomplete contracts lead to sizable holdup on the buyer (on average, 20% of CalTrans' engineering estimate for project costs). The holdup as a percentage of the engineering estimate is higher for contracts involving major jobs or more bidders. We show that counterfactual cost-plus contracts would lower buyer surplus for 72% of the projects in the data, with an average reduction of $\$382,074$. This indicates that the buyer's gains in *ex ante* surplus under fixed-price contracts mostly outweigh the cost of holdup due to incomplete contracts.

Models of incomplete contracts have been used to study employment relation (Simon, 1951; Klein, Crawford, and Alchian, 1978), ownership and the property rights of firms (Williamson, 1985; Grossman, and Hart, 1986; Hart and Moore, 1990), and international trade (Spencer, 2005). Our work in this article is related to Tirole (2009), which models contractual incompleteness as a consequence of the buyer's optimal choice of cognitive effort. We extend Tirole (2009) to an environment where the initial contract price is determined via competitive bidding.

We contribute to a growing empirical literature on bargaining in various contexts. This includes the bilateral negotiation between hospitals and managed-care organizations in Gowrisankaran, Nevo, and Town (2014), the employer-insurer and hospital-insurer negotiation over premiums and reimbursements in Ho and Lee (2017), and the price negotiation between suppliers and buyers in the health industry in Grennan (2013).

Our work is closely related to the literature on post-auction bargaining (e.g., Elyakime et al., 1997; Larsen, 2014) and on incomplete contracts (e.g., Crocker and Reynolds, 1993; Bajari, McMillan, and Tadelis, 2009; Bajari, Houghton, and Tadelis, 2014; Lewis and Bajari, 2014; De Silva et al., 2016). Compared with these other articles, our structural model has distinctive new features about the rationale and the information structure of the incomplete contracts. (See Section 2 for details.) In addition, we address new empirical questions such as measuring the holdup on the buyer and quantifying the difference in buyer surplus under various contract formats. The identification of our model requires an original method that has not been used in other contexts.

Our article also contributes to a broader literature on the identification of structural models of auctions and contracts (e.g., D'Haultfoeuille and Février, 2007; Aryal, Perrigne, and Quang, 2012; Perrigne and Vuong, 2011; Perrigne and Vuong, 2012). These articles establish mappings between the unobserved agent types and the observed contract features. In comparison, we consider a model where the classical arguments based on the contractor incentives alone are insufficient for identifying the full model. Hence, we construct a new argument that capitalizes on the buyer's rationality and the Nash Bargaining interpretation of the negotiated transfers to identify the model elements.

Although we use the same source of data as Bajari, Houghton, and Tadelis (2014), our model differs qualitative from the latter. Bajari, Houghton, and Tadelis (2014) maintained that the buyer's choice of the initial contract specification is exogenous and that the contractors have perfect foresight about the negotiated transfers as well as the new feasible design, both of which are assumed exogenous.[1] Their model does not rationalize why the buyer and contractors would include an incompleteness pact that allows them to adopt a new design after the auction. Nor does it link the holdup on the buyer to *ex ante* uncertainty about the new design. In comparison, we relax these assumptions and model a game of sequential moves with private information that rationalizes incomplete contracts. We also answer new empirical and policy questions mentioned above.

In Section 2, we describe our model and define its equilibrium. In Section 3, we discuss the identification of model elements. Section 4 describes the data and institutional background of CalTrans highway procurement auctions, and summarizes stylized facts that motivate our structural model. Section 5 defines our estimation method. Section 6 reports estimates for structural parameters as well as contractor markups and holdup on the buyer. In Section 7, we compare the buyer surplus under counterfactual cost-plus contracts with that in the data. Section 8 concludes. Proofs are collected in the Appendix.

[1] See the second paragraph in Section II.A and equation (1) in Section II.B in Bajari, Houghton, and Tadelis (2014).

2. The model

■ Our model of procurement auctions with incomplete contracts accommodates heterogeneity on the contract level, which we suppress in the notation in order to simplify exposition. In what follows, we denote random variables by uppercase letters and their realized values by lowercase letters.

□ **Strategic decisions and contract incompletness.** A buyer announces an initial specification (design) $X \in \mathcal{X}$ for a procurement contract, where the support $\mathcal{X}$ is convex and compact in the real line $\mathbb{R}$. There are N sellers (contractors) competing for the contract. Once informed of X, each contractor i draws a private cost C_i for completing the contract with the design X, and quotes a price $P_i \in \mathbb{R}_+$. The distribution of C_i conditional on the initial design is stochastically increasing in X. The buyer awards the contract to the seller who quotes the lowest price. For any initial design X, the costs of contractors are drawn independently from a continuous distribution $F_{C|X}$ with support $\mathcal{C} \subset \mathbb{R}_+$, which may also depend on X.

The contract is incomplete in that the buyer and the auction winner agree that the initial design may be replaced by a new design $X^* \in \mathcal{X}$ following the auction. The modification takes place *after* a winner is chosen to execute the contract with the initial design. The new design X^* is unknown *ex ante* to the buyer and all contractors. Thus, it is considered stochastic when the buyer announces the initial design and the contractors quote their prices. The private cost C_i for implementing the contract conditional on the initial design X is independent from X^*.

The buyer and the auction winner observe the realization of X^* after the auction, and make a joint decision on whether to adopt the new design or not, based on the following rule. Let $\pi : \mathcal{X} \to \mathbb{R}$ be the social surplus collected by the buyer. (For example, suppose the procurement contract is about constructing a tollway. Then, $\pi(X)$ is the present value of the stream of revenues to be collected by the buyer from that tollway.) The incremental surplus under the new design is $\phi(X, X^*) \equiv \pi(X^*) - \pi(X)$. Let $a : \mathcal{X} \times \mathcal{X} \to \mathbb{R}$ be the incremental cost for changing the design from X to X^*. These include costs for additional construction or logistic tasks. We maintain that such incremental costs are nonseparable in X and X^*, so that the marginal costs in general depend on (X, X^*). Assume both π and a are bounded and continuously differentiable over their domains. Upon seeing the new design X^* after the auction, the buyer and the auction winner agree to adopt it if and only if it yields a positive net incremental surplus relative to the initial design X. That is, $s(X, X^*) \equiv \phi(X, X^*) - a(X, X^*) > 0$. Otherwise, the initial design X is implemented.

□ **Post-auction negotiation.** If X^* is adopted, the buyer and the winner negotiate transfers *in addition to* the payment determined in the auction. The contractor covers the incremental costs up front as they arise in construction. The incremental surplus is collected by the buyer. Both parties take these into account as they negotiate the transfers. We maintain that the post-auction transfer is determined via Nash Bargaining as follows. Let $y(X, X^*) \in \mathbb{R}$ denote the negotiated transfer from the buyer to the auction winner; and let $\gamma \in (0, 1)$ be a constant parameter reflecting the bargaining power of the winner.[2] To simplify notation, we suppress the argument (X, X^*) in the functions y, ϕ, a, s. The contract price from the auction enters disagreement values in Nash Bargaining. After the negotiated transfers are made, the auction winner obtains a share of the net incremental surplus that is proportional to its bargaining power. That is,

$$y - a = \gamma s \Leftrightarrow y = \gamma\phi + (1 - \gamma)a, \tag{1}$$

if a new design is adopted (see the Appendix for details in deriving the Nash Bargaining solution). On the other hand, there is no negotiated transfer, or $y = 0$, if no new design is adopted.

[2] The bargaining power depends on project and contractor characteristics in general. We focus on homogeneous auctions to simplify the exposition in the model and identification sections. Our results are generalizable conditional on observed auction and contractor heterogeneity. In the application, we estimate a model with heterogeneous auctions.

□ **The information structure.** The information structure of the model is as follows. Prior to announcing the initial design, the buyer observes a signal $\widetilde{X} \in \widetilde{\mathcal{X}}$, where the support $\widetilde{\mathcal{X}}$ is a compact interval in $\mathbb{R}$, and in general is not the same as $\mathcal{X}$. The private signal $\widetilde{X}$ is correlated with X^*. In what follows, we let $F_{R'|R}$ and $f_{R'|R}$ denote the conditional distribution and the conditional density of R' given R, respectively, and write $F_{R'|R=r}$ and $f_{R'|R=r}$ if there is need to be specific with the value conditioned on. The model elements $\gamma, \pi, a, F_{C|X}$ and $F_{X^*|\tilde{X}}$ are common knowledge among the buyer and contractors. Assume $F_{X^*|\tilde{X}=\tilde{x}}$ is absolutely continuous and increasing over $\widetilde{\mathcal{X}}$ for all $\tilde{x}$.

To reiterate, $\widetilde{X}$ and C_i are private information for the buyer and the contractor i, respectively. At the beginning of the auction, the buyer announces X to maximize its *ex ante* payoff, based on its signal $\widetilde{X}$, and taking into account the strategic incentives of contractors under incomplete contracts. A contractor is informed of X, draws its private cost C_i from $F_{C|X}$, and quotes a price to maximize its *ex ante* profit, which also takes into account the negotiated transfer *ex ante*. We maintain that the number of contractors in an auction N is common knowledge among the buyer and contractors in the bidding stage.

Compared with the existing literature, our model has several distinctive features motivated by the empirical questions we investigate. First, it endogenizes the buyer's choice of the initial contract specification. Accounting for strategic incentives in the choice of initial design as well as the bids is important for measuring the contractors' markup in the bids and the holdup on the buyer. Second, we rationalize contract revision by a new design which increases the net surplus, and model negotiated transfers as Nash Bargaining solutions. Contract characteristics affect the negotiated transfers through their impact on bargaining power. Third, we maintain a flexible information structure whereby neither the buyer nor the contractors have perfect foresight or rational expectation of the revised contract or transfers. In comparison, Bajari, Houghton, and Tadelis (2014) and De Silva et al. (2016) assumed that the contractors have rational expectation about the actual quantities to be used in the new design conditional on the contract being revised. Jung et al. (2018) assumed that contractors form expectations about the future adjustments on each item based on the historical probability of revision and negotiation.

□ **The equilibrium concept.** A contractor i's pure strategy is a mapping from his information (C_i, X) to the price he quotes; a buyer's pure strategy is a mapping from a signal $\widetilde{X}$ to an initial design X. Let $s_+ \equiv \max\{s, 0\}$ denote the *realized* net incremental surplus. In a symmetric pure-strategy Perfect Bayesian Equilibrium (*psPBE*), the buyer follows a pure strategy α^*, and each contractor follows a pure strategy β^* and holds a belief about the new design $\lambda^*(X^*|X)$: $\mathcal{X} \times \mathcal{X} \longrightarrow [0, 1]$ such that: (a) for all (x, c_i),

$$\beta^*(c_i, x) = \underset{b \in \mathbb{R}_+}{\operatorname{argmax}} \Pr\left(\min_{j \neq i} \beta^*(C_j, X) \geq b \middle| X = x\right)[b - c_i + \delta(x; \lambda^*)], \tag{2}$$

where $\delta(x; \lambda^*) \equiv E_{\lambda^*}[\gamma s_+(x, X^*)|X = x]$ is a contractor's expected share of the net incremental surplus according to his belief $\lambda^*(\cdot|x)$; (b) λ^* is consistent with $F_{X^*|\tilde{X}}$ and α^* for all x on the support of $\alpha^*(\widetilde{X})$; and (c) α^* is the buyer's best response when all contractors follow β^*:

$$\alpha^*(\tilde{x}) = \underset{x \in \mathcal{X}}{\operatorname{argmax}} \{\pi(x) - \varphi(x; \beta^*) + \mu(x, \tilde{x})\}, \tag{3}$$

where $\mu(x, \tilde{x}) \equiv E[(1 - \gamma)s_+(x, X^*)|\widetilde{X} = \tilde{x}]$ and $\varphi(x; \beta^*)$ is the buyer's expected payment in the auction with design x when all contractors follow the strategy β^*. (We provide the closed form for φ in the Appendix.) In addition, the contractor's belief off the equilibrium support of $\alpha^*(\widetilde{X})$ is such that any deviation from the equilibrium path is not payoff-improving for the buyer.

As (3) shows, the buyer's choice of the initial design X involves a trade-off. First, the initial design affects the expected payment in the auction, with a marginal effect determined by the dependence between the design X and the contractor's cost. Furthermore, the initial design affects the buyer's expected share of the net incremental surplus due to contract revision. However, the sign of this marginal effect is ambiguous because it depends on the form of the incremental surplus function and the joint distribution of the initial and the new design.

The term $\mu(x, \tilde{x})$ in (3) is the buyer's *ex ante* share of the net incremental surplus due to contract revision, and is assumed continuous in the buyer's private signal. The expectation in $\mu(x, \tilde{x})$ integrates out X^* with respect to its distribution conditional on $\widetilde{X} = \tilde{x}$, and does not depend on the contractor belief λ^*. We refer to $\delta(x; \lambda^*)$ as a contractor's *holdup* on the buyer under the initial design in equilibrium. In *symmetric monotone psPBE*, α^* is increasing over $\widetilde{\mathcal{X}}$, and β^* is increasing over $\mathcal{C}$ for any $x \in \mathcal{X}$. The consistency of contractor beliefs in (b) means $F_{X^*|X=x} = F_{X^*|\widetilde{X}=\alpha^{*-1}(x)}$ for all x on the equilibrium support of $\alpha^*(\widetilde{X})$.

We provide a heuristic overview of our argument for the existence of symmetric monotone psPBE, which takes several steps and is presented formally in the Appendix. First, for any given initial design X and belief λ, a contractor's optimization problem is similar to that in a standard lowest-price procurement auction, except that its private cost is adjusted downward into $C_i - \delta(x; \lambda)$ because of its holdup on the buyer. Using auction theory, we characterize the bidder strategy and the expected auction price in a bidding equilibrium as functions of X and λ. Next, note the first-order condition that characterizes the buyer's rationality takes the form of an ordinary differential equation (ODE). We argue that there exists a strictly monotone solution to this ODE under appropriate conditions based on the Picard's Existence Theorem. These conditions include the Lipschitz continuity of the functional form of ODE in the initial design, and the stochastic increasingness of $F_{C|X}$ in the initial design. Finally, we characterize contractor beliefs outside the equilibrium support of the initial contract design. Such beliefs are ultrapessimistic in that they assign no probability mass to new designs with positive net surplus. We show that such beliefs rationalize the buyer's choice of initial designs on the equilibrium path.

□ **Further remarks.** In this model, the buyer has a noisy signal $\widetilde{X}$ about the new design X^* before the auction, whereas the contractors have no private signals and update their beliefs about the new design given the buyer's choice of the initial design X. Such a specification of information structure is largely motivated by practical considerations. In most cases, the social surplus from public projects accrues directly to the buyer, who may well have some idea about which contract designs result in higher social surplus. This is in part reflected in the buyer's private signal about X^*. It is also worth noting that there is no explicit information asymmetry between the buyer and the contractors in equilibrium. This is because in a psPBE, the contractor's belief about X^* is consistent via Bayesian updating, conditional on any X on the equilibrium support of the initial design. Thus, in equilibrium, the contractors could correctly recover the buyer's signal $\widetilde{X}$ by inverting α^* on the equilibrium support of X. With $F_{X^*|\widetilde{X}}$ assumed common knowledge for the buyer and the contractors, this means a contractor is no less informed about the feasible new design X^* relative to the buyer on the equilibrium path.

Our model simplifies several aspects in the negotiation that follows the procurement auction. First, the Nash Bargaining solution used in the negotiation posits the net incremental surplus is known to both the buyer and the auction winner. Hence, our model consists of an incomplete information aspect (during the initial low-price procurement auction) and a complete information aspect (in the negotiation after the auction). This is a limitation in our approach of modelling, and it leads to a loss of generality in the information structure. An alternative setup of bargaining with incomplete information would be more robust. Nevertheless, as Myerson (1984) noted, the generalization of the Nash Bargaining solution with private information is complicated. The identification of the model elements in that case remains an open question.[3]

[3] In a related but different environment, Elyakime et al. (1997) takes a similar approach and uses complete-information Nash Bargaining as an approximation of the acutal information structure for the sake of model tractability. Larsen (2014) estimated a bargaining game with two-sided incomplete information that follows ascending auctions, using insights from the implications of a Bayesian Nash Equilibrium. The equilibrium characterization under the auction format he considers is qualitatively different; and his approach does not apply in our setting of lowest-price procurement auctions, given the data available.

We introduce this working assumption above as a first-order approximation of the actual information structure, in order to keep the identification and estimation of the model tractable. We believe that the complete-information Nash Bargaining solution provides a reasonable approximation of the buyer-seller negotiation in our specific application, because the new design X^* and the negotiation take place after the auction. At that point, both the auction winner and the buyer may well be informed about the actual costs of contract revision through implementation and monitoring. In contrast, a contractor's initial cost C_i is drawn before the auction and hence remains the contractor's private information in the bidding stage.

Another working assumption in our model is that the incremental costs for revising contracts depend on the features of the project (such as the job type) but not on the identity of the auction winner. Without this simplification, there would be an additional source of *ex ante* uncertainty regarding contractor costs, thus making the equilibrium characterization and the identification of model elements intractable. We believe this simplifying assumption is a reasonable first-order approximation of the relation between the contract features and the negotiated outcome in the data. Descriptive analyses of our data suggest that the variation in the negotiated transfers in the data are mostly explained by the observed heterogeneity on the contract level, rather than individual characteristics of the auction winner. Furthermore, despite the simplification, the negotiated transfer depends on contract heterogeneity through the net surplus due to contract revision and through the contractor's bargaining power in negotiation.

In our model, the specification of a contract is summarized by a scalar variable that could be interpreted as the buyer's engineer estimates for the total cost. This simplifies the practice of specifying contracts in terms of itemized quantities. Such a simplification allows us to construct a structural model with two desirable features. On the one hand, the dimension of unknown parameters is low enough to warrant the robust identification of model elements; on the other hand, the model is rich enough to capture the strategic interaction between the buyer and contractors given the uncertainty about the new design and given the order of sequential actions. An alternative model with multidimensional contracts would be more general and closer to the actual practice, but would increase the dimension of unknown parameters (which include the joint distribution of item-specific costs). The equilibrium characterization of that multidimensional model would be qualitatively different and more complicated; and its robust identification remains a challenging open question.

3. Identification

■ We consider model identification in an environment where the data report the auction payment V, the initial contract designs X, as well as the adopted new designs X^* and negotiated transfers Y if the new design is adopted. We explain how to use the joint distribution of these variables to recover the model parameters π, a, γ, $F_{C|X}$ and $F_{X^*,\tilde{X}}$. To do so, we use the implications of buyer and contractor rationality in equilibrium as well as Nash Bargaining. Our identification method applies conditional on contract heterogeneity observed in data and the number of auction participants N. In presenting our method in this section, we suppress both variables in the notation for simplicity.

□ **Buyer and contractor strategies.** The model is identified up to a monotone transformation of the buyer's signal $\widetilde{X}$ at best. (See the web appendix of this article for a formal statement and proof.) Thus, without loss of generality, we normalize the distribution of $\widetilde{X}$ to a standard uniform over [0,1]. Let $D = 1$ if the new design X^* is adopted with a negotiated transfer, and 0 otherwise. This dummy variable is reported in the data.

The buyer's strategy $\alpha^*(\cdot)$ is identified because its monotonicity implies that $\alpha^*(\tau) = x_\tau$ for all $\tau \in [0, 1]$, where x_τ is the τ-th quantile of the initial designs reported in the data. Besides, the monotonicity of α^* implies

$$F_{X^*|s(X,X^*)>0,\widetilde{X}=\tau}(x^*) \equiv \Pr\{X^* \leq x^*|\widetilde{X} = \tau, s(X, X^*) > 0\} = \Pr\{X^* \leq x^*|X = x_\tau, D = 1\} \quad (4)$$

for all x^* and $\tau \in (0, 1)$. Hence, the distribution of the new design conditional on the contract revision and the buyer's signal is identified.

□ **Adjusted costs for contractors.** Contractors bid strategically in the procurement auction, knowing that additional gains are possible due to contract revision. Thus, we can use a standard argument in Guerre, Perrigne, and Vuong (2000) to recover the (endogenous) adjusted costs for contractors in the auction based on the initial design X . Such costs include a downward adjustment from actual costs C_i, which take into account the contractors' holdup on the buyer. That is, the adjusted cost for a contractor i under an initial design x is

$$\widetilde{C}_i \equiv C_i - \delta(x; \lambda^*), \tag{5}$$

where $\delta(x; \lambda^*)$ is the *ex ante* share of net surplus due to contract revision in equilibrium.

To do so, first note that for a given pair of the initial design x and equilibrium belief λ^*, the contractor's holdup on the buyer $\delta(x; \lambda^*)$ is constant. Thus, we can define a *hypothetical* low-price procurement auction without incompleteness, where contractors' costs are i.i.d. draws from the distribution of adjusted costs $\widetilde{C}_i$ given x and λ^*. Let $\tilde{\beta}(\cdot, x)$ denote the contractor strategy in symmetric monotone psPBE in such a procurement auction with no incompleteness. That is,

$$\tilde{\beta}(\widetilde{c}_i, x) = \underset{b \in \mathbb{R}_+}{\operatorname{argmax}} \Pr\left\{\min_{j \neq i} \tilde{\beta}(\widetilde{c}_j, X) \geq b \middle| X = x\right\}\left(b - \tilde{c}_i\right) \text{ for all } x. \tag{6}$$

The solution to (6) is related to the equilibrium strategy β^* in the original game in (2) through the equality $\tilde{\beta}(\tilde{c}_i, x) = \beta^*(\tilde{c}_i + \delta(x; \lambda^*), x)$ for all x. Therefore, we can use a standard argument from Guerre, Perrigne, and Vuong (2000) to invert $\tilde{\beta}$ and recover the contractors' adjusted costs $\tilde{c}_i$ from the distribution of quoted prices in the auction. (See Lemma A.2 in the Appendix.)

□ **Social surplus.** We recover the social surplus π using the buyer's rationality in equilibrium, and the link between negotiated transfers and γ, a, π in Nash Bargaining. This approach is robust to the contractor's belief off the equilibrium support of $\alpha^*(\widetilde{\mathcal{X}})$. We maintain that for all $\tilde{x} \in \widetilde{\mathcal{X}}$, the buyer's solution to (3) admits a unique solution in the interior of $\mathcal{X}$.[4]

In a symmetric monotone psPBE, a buyer with private signal $\tilde{x}$ chooses an initial design x to maximize *ex ante* payoff $\pi(x) + \mu(x, \tilde{x}) - \varphi(x; \beta^*)$, where $\varphi(x; \beta^*)$ is the buyer's expected payment under the design x when contractors follow the strategy β^*. The first-order condition for an interior solution is:

$$\varphi'(\alpha^*(\tilde{x}); \beta^*) = \pi'(\alpha^*(\tilde{x})) + \frac{\partial}{\partial x}\left[\int_{\{s(x,t)>0\}} (1-\gamma)s(x,t)dF_{X^*|\tilde{X}=\tilde{x}}(t)\right]_{x=\alpha^*(\tilde{x})}. \tag{7}$$

That is, the optimal choice of initial designs must strike a balance between a buyer's marginal cost in expected auction payment and its marginal benefit in social surplus under the initial design plus its *ex ante* share of net surplus from revision.

Recall that the negotiated transfer in Nash Bargaining is $y = \gamma\phi + (1-\gamma)a$. Substituting this equality into the first-order condition (7) and applying the Leibniz rule, we show that the marginal effect of initial designs on the social surplus is:

$$\pi'(x) = [1 - p^*(x)]^{-1}\left(\varphi'(x; \beta^*) + \int_{\{s(x,t)>0\}} y_1(x,t)dF_{X^*|X=x}(t)\right) \tag{8}$$

for all x on the equilibrium support of X, where $p^*(x) \equiv \Pr\{D = 1|X = x\}$ is the probability that the contract is revised to the new design, and y_1 denotes the partial derivative of y with respect to its first argument. (See the Appendix for a formal statement of conditions and a proof of this result.) The right-hand side of (8) consists of identifiable quantities only. Both $y(x, t)$

[4] If the solution to (3) is on the boundary of $\mathcal{X}$ for some $\tilde{x} \in \widetilde{\mathcal{X}}$, then our results hold with $\mathcal{X}_e$ defined as $\{x \in \mathcal{X}_I : x = \alpha^*(\tilde{x}) \text{ for some } \tilde{x} \in [0, 1]\}$ where $\mathcal{X}_I$ is the interior of $\mathcal{X}$.

and $\varphi(x;\beta^*)$ are directly identified in the data for all x on the equilibrium support of X and new design t such that $s(x,t)>0$; and the distribution of X^* conditional on $X=x$ is also identified over the set of new designs t with $s(x,t)>0$. Thus, π' is identified over the equilibrium support of X.

At best, the social surplus π is identified up to location normalization. This is because the equilibrium strategies and the negotiated transfers depend only on the derivative $\pi'(\cdot)$ and the difference in social surplus between the initial and new designs. Without loss of generality, set $\pi(\underline{x})=\pi_0$ for some constant π_0, with $\underline{x}$ being the infimum of the equilibrium support of X. Then, we can recover the surplus function as $\pi(x)=\pi_0+\int_{\underline{x}}^{x}\pi'(z)dz$ for all x on the equilibrium support.

□ **Bargaining power and costs for contract revision.** Recall that the negotiated transfer y is related to the bargaining power γ, and the costs for contract revision a as follows:

$$y(x,x^*)=\gamma[\pi(x^*)-\pi(x)]+(1-\gamma)a(x,x^*). \tag{9}$$

With the (difference in) social surplus already identified, we use the joint variation in y,x,x^* to back out the remaining parameters γ and a. To do so, we maintain the following condition on revision costs over the equilibrium support of X.

(A1) *There exist $x,\xi,x^*,\xi^*\in\mathcal{X}$ such that $s(x,x^*)>0$, $s(\xi,\xi^*)>0$, $a(x,x^*)=a(\xi,\xi^*)$, and $\pi(x^*)-\pi(x)\neq\pi(\xi^*)-\pi(\xi)$.*

Under this condition, there exist two pairs of initial and new designs that lead to the same costs for revisions but different incremental surplus. To locate such pairs, it is not necessary to know the functional forms of a and π; shape restrictions on these functions are sufficient. For example, suppose a is a k-th order polynomial of the difference x^*-x, whereas π is a k'-th order polynomial in x with $k,k'\geq 2$. Then, (A1) holds with any two pairs on the equilibrium support of designs such that $x^*-x=\xi^*-\xi$ and $x\neq\xi$.

For (x,x^*) and (ξ,ξ^*) that satisfy (A1), $y(x,x^*)-y(\xi,\xi^*)=\gamma[\pi(x^*)-\pi(x)-(\pi(\xi^*)-\pi(\xi))]$, where the differences in social surplus on the right-hand side is already identified in the previous step. Thus, the bargaining power parameter γ is identified. With γ,π recovered, the revision cost a is also identified from (9) for all x and x^* on the equilibrium support of X with $s(x,x^*)>0$.

□ **Holdup on the buyer.** To quantify the contractor's holdup on the buyer and the distribution of costs for contractors, we need to extend the identification of π over the set of designs

$$\mathcal{X}_s\equiv\mathcal{X}_e\cup\{x^*:\exists x\in\mathcal{X}_e \text{ s.t. } s(x,x^*)>0\},$$

where $\mathcal{X}_e\equiv\{x:\alpha^*(\tilde{x})=x \text{ for some } \tilde{x}\in[0,1]\}$ denotes the equilibrium support of the initial design $\alpha^*(\widetilde{\mathcal{X}})$. We also need to recover the revision costs a over the set of pairs $\{(x,x^*):x\in\mathcal{X}_e$ and $s(x,x^*)>0\}$. To do so, we use the following condition:

(A2) *There exists a real-valued, differentiable function $\tilde{a}$ such that $a(x,x^*)=\tilde{a}(x^*-x)$ for all $x,x^*\in\mathcal{X}$.*

Under this condition, $a_1(x,x^*)+a_2(x,x^*)=0$, where a_j is the partial derivative of a with respect to its j-th argument. This provides a useful link between the marginal effect of designs on observed transfers and that on the social surplus. That is,

$$\frac{\partial}{\partial t}y(t,x^*)|_{t=x}+\frac{\partial}{\partial t}y(x,t)|_{t=x^*}=\gamma\pi'(x^*)-\gamma\pi'(x)$$

for all (x,x^*) such that $x\in\mathcal{X}_e$ and $s(x,x^*)>0$. This equality allows us to recover the marginal effect $\pi'(x^*)$ outside the equilibrium support of initial design (i.e., at $x^*\in\mathcal{X}_s\backslash\mathcal{X}_e$). Thus, we can recover the cost of contract revisions and the contractor's holdup on the buyer, using this and the knowledge of other elements identified earlier, such as the equilibrium belief and bargaining power. Furthermore, with knowledge of the contractor holdup, we can recover the distribution of actual costs from that of the adjusted costs identified earlier. (See corollary A.1 in the Appendix for details.)

□ **Discussions.**

Alternative identifying conditions. There are alternative restrictions on a and π that are sufficient for identifying the bargaining power. For example, suppose that for some x_0 and x_0^*, the level of the adjustment cost is known: $a(x_0, x_0^*) = a_0$ where $s(x_0, x_0^*) > 0$. Let y_0 and ϕ_0 denote the observed transfer $y(x_0, x_0^*)$ and incremental surplus $\phi(x_0, x_0^*)$, respectively, where $\phi(x_0, x_0^*) \equiv \pi(x_0^*) - \pi(x_0)$ is already identified. Then, knowledge of (x_0, x_0^*, a_0) allows us to recover $\gamma = (y_0 - a_0)/(\phi_0 - a_0)$.

Another alternative condition is when the incremental cost is homogeneous of degree one, whereas the incremental surplus has nonconstant (diminishing or increasing) returns to scale. In this case, consider any two pairs with positive net incremental surplus (x_1, x_1^*) and $(x_2, x_2^*) \equiv (tx_1, tx_1^*)$ with a known constant $t > 0$. Let y_k, ϕ_k, a_k be shorthand for the functions y, ϕ, a evaluated at (x_k, x_k^*) for $k = 1, 2$. Because the incremental cost a is homogeneous of degree one, we have $a_2 = ta_1$ and

$$(y_2 - \gamma\phi_2)/(y_1 - \gamma\phi_1) = t,$$

which implies γ is identified as $(y_2 - ty_1)/(\phi_2 - t\phi_1)$. (The nonconstant returns to scale in the incremental surplus ϕ ensures the denominator is nonzero.)

Finally, if the model assumes a parametric form of π and a, then we do not need the exclusion and shape restrictions in (A1)–(A2) to point identify the bargaining power and recover π and a off the equilibrium support.

Exogenous participation. So far, we have maintained that the number of bidders N is known to the contractors as well as the buyer during the auctions. Athey and Haile (2007) argued that in some procurement auctions, the contractors may in fact know which of their competitors have the capability to compete for a given contract or which firms have been invited to bid. Bajari, Houghton, and Tadelis (2014) maintained this assumption in their analysis of the highway procurement auctions by CalTrans.

In other contexts, the actual number of auction participants N is not public information to the parties involved in the auction. Nevertheless, our identification strategy remains valid in such cases as long as the variation in N is exogenous. (Athey and Haile, 2007 provides an example of how such exogenous variation arises in a model where bidders' entry decisions are related to costly signal acquisition.) With the actual distribution of N being common knowledge among the buyer and the contractors, the existence of symmetric monotone psPBE $\{\alpha^*, \beta^*\}$ follows from an argument similar to the proof in the Appendix. The only necessary change is that the *ex ante* return for the contractors and the *ex ante* payment by the buyer need to integrate out the number of bidders using the commonly known distribution of N.

As for identification of the model when N is not known to the bidders, the results on identifying the distribution of the new design conditional on contract revision and the buyer's signal is built on the monotonicity of the buyer's strategy. Thus, the results remain valid under any symmetric monotone psPBE in a model with bidders' uncertainty about participation. Also, the result about recovering the adjusted costs of contractors holds when the contractors and the buyer are uncertain about the participation, provided the data report the prices quoted by all contractors in an auction. In this case, Athey and Haile (2007) showed the markup in the inverse bidding strategy needs to incorporate the uncertainty about participation. This is done by integrating out N using the actual distribution of the number of bidders, which is common knowledge among all parties and is directly recoverable from the data.[5] The identification of π, a and γ then follows from an argument similar to that presented above.

It remains an open question how to identify a model where decisions to participate in auctions are selective in the sense that the distribution of costs for active participants differs from the unconditional cost distribution in the population (e.g., if entry is based on a preliminary signal that is correlated with private costs to be drawn in the bidding stage). We leave this topic for future research.

4. CalTrans auctions: background and data

□ **Data.** The California Department of Transportation (CalTrans) is a government department in the state of California that is responsible for the planning, construction, and maintenance of public transportation facilities such as highways, bridges, and rails. It awards highway construction projects to contractors through lowest-price procurement auctions. Before each auction, CalTrans announces the initial design of the contract in the form of an engineer estimate X for total project costs. These engineer estimates are reported as the dot product of the quantity for each category of inputs and their per-unit prices. Once informed of engineer estimates, the contractors draw private costs for completing the project and quote their prices. The contractor who quotes the lowest price is awarded the contract. In most cases, CalTrans and the auction winner end up with an agreement to adopt a new specification X^* after the auction, and additional transfers in the form of adjustment, deduction, or payment for extra work are made through negotiation. We measure the change in the specification as $X^* - X$.

Some institutional facts indicate that CalTrans chooses an initial specification to maximize its *ex ante* payoff. For example, CalTrans' Highway Design Manual (2015) states that it is necessary to take into account uncertainty about the costs and welfare in the design (specification) to be implemented. In practice, engineers who prepare the estimates do anticipate project uncertainty, such as changes proposed by the winning contractor (Project Risk Management Handbook: A Scalable Approach, CalTrans, 2012) and stochastic quotes from contractors (CalTrans Estimating Practices).

We use the same source of data as Bajari, Houghton, and Tadelis (2014). The data include 5908 bids submitted by contractors in 1306 procurement auctions by CalTrans between 1999 and 2008. Over 90% of the contracts in the data receive two to nine quotes from contractors. For each contract, the data report the initial specification (engineer estimates), the actual specification adopted (calculated using the Blue Book prices published in the Contract Cost Data Book (CCDB), and the actual quantities of items used for the project), and the negotiated transfer after the auction.[6] The data record zero transfer if no new specification is adopted after the auction. The data also report the bids submitted by all contractors in each auction, and cost-related characteristics for the contractors. These include the distance between the contractor's location and the worksite for the project (*dis*), a dummy variable that equals one if the contractor is a "fringe" competitor (*fri*), which is defined as a contractor that has won less than 1% of the value of contracts awarded in the data, and the contractor's utilization rate, defined as the ratio of its backlog over its capacity (*uti*).

In addition, we classify the contracts into two types based on the project description. Type-one contracts ($job = 1$) involve major construction or rebuilding (e.g., replace bridge or widen highway, etc.). Type-zero contracts ($job = 0$) require only relatively minor or decorative tasks (e.g., realign curves, install traffic signals, or other accessories, etc.).

[5] Furthermore, Song (2006) showed that identification of inverse bidding strategy in first-price procurement auctions, where neither the bidders nor the researcher observe the set of actual participants, is also possible, even if only the winning bid and the second-lowest bid are reported in the data.

[6] Following Bajari, Houghton, and Tadelis (2014), we calculate the total negotiated transfers after the auctions by adding up transfers under three categories in the data: "adjustment," "deduction," and "extra work."

TABLE 1 Summary Statistics[a]

Variable	Mean	Standard Deviation	5-th Pctile	Median	95-th Pctile
Contract (project)-level					
Number of bidders	4.521	2.267	2	4	9
Engineering estimate (million dollars)	2.813	6.681	.227	.947	10.212
Actual project size (million dollars)	2.667	5.978	.256	1.044	9.811
Job	0.379	.485	0	0	1
Winning bid (million dollars)	2.701	6.604	.195	.885	9.423
Transfer (million dollars)	.258	1.1	0	.039	1.09
Winner fringe	.484	.5	0	0	1
Winner distance (miles)	98.174	145.743	5	46.47	376
Winner utilization	.125	.217	0	.005	.637
Avg. fringe[b]	.564	.293	0	.6	1
Avg. utilization[b]	.121	.127	0	.084	.377
Avg. distance[b]	108.342	99.301	17.785	82.052	279
Contractor (bidder)-level					
Bids (million dollars)	3.055	17.185	.202	.912	10
Fringe	.620	.485	0	1	1
Distance (miles)	103.574	144.633	7.2	55.555	374
Utilization	.112	.225	0	0	.7
Min. dis. among rivals (miles)	43.346	57.868	2.88	25.4	149
Min. uti. among rivals	.017	.075	0	0	.11

[a]There are 1306 auctions and 5908 bids in the sample.
[b]The average is taken for all bidders within an auction.

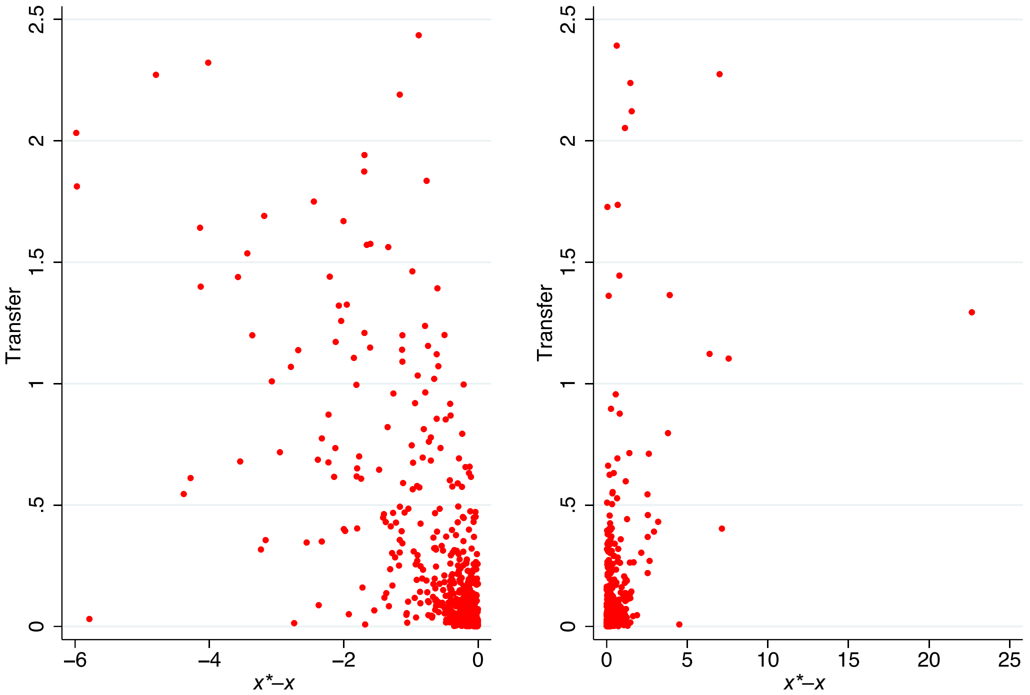
Table 1 presents the summary statistics for our data. The average transfer due to contract revisions after the auction is about \$258, 000 per contract, which is about 9.67% of the average size of the final specification implemented after the auction. Among all contracts, 94.10% (1229 cases) reported nonzero transfers negotiated after the adoption of a new design following the auction, whereas 5.90% (77 cases) report zero transfer. Type-one (major construction) jobs account for 37.9% of all contracts with an average transfer of \$280, 775. Type-zero contracts have an average transfer of \$243, 657. For each auction, we use the proportion of fringe contractors competing in the auction (*afri*) and the average utilization rate and distance across contractors (*auti* and *adis*) as contract-level characteristics. For each contractor in an auction, we also record the minimum distance to the job site (*rdis*) and the minimum utilization rate among its competitors (*ruti*). Table 1 also reports the summary statistics of these contract- and contractor-level variables.

□ **Nonstructural evidence and motivating facts.** We start with some descriptive analyses of how the transfers negotiated after the auction are related to contract specification and characteristics. Figure 1 is a scatter plot of negotiated transfers (Y) against the size of contract revisions ($X^* - X$) in the data. As expected, the size of contract revision has a substantial impact on the distribution of negotiated transfers. More formally, pairwise Kolmogorov-Smirnov tests for the equality between the distributions of negotiated transfers conditional on the low, medium, and high terciles of $(x^* - x)$ all reject the null at 1% level.

Figure 2 further illustrates the dependence of transfers on the size of revision, conditioning on contract characteristics such as the proportion of fringe competitors (*afri*), and the average utilization rate among competitors (*auti*), where both *afri* and *auti* are discretized into intervals defined by their lower, middle, and upper terciles. The scatter plots in Figure 2 reveal several distinctive patterns that motivate our structural model in Section 2. First, the negotiated transfers vary significantly across contracts with different characteristics, even after controlling for the size of contract revision. We compare the two distributions of transfers in each column plotted in the figure and find that in each of the four cases, the p-value from a Kolmogorov-Smirnov test

FIGURE 1

SCATTER PLOTS OF TRANSFER VERSUS SPECIFICATION DIFFERENCE
[Color figure can be viewed at wileyonlinelibrary.com]



Note: Both transfer and specification change $x^{*}-x$ are in million dollars. The left and right subplots are corresponding to positive and negative $x^{*}-x$, respectively.

is less than 0.05. These suggest that the negotiated transfers vary significantly across contracts with different characteristics (such as *afri* and *auti*), even after controlling for the size of contract revision.

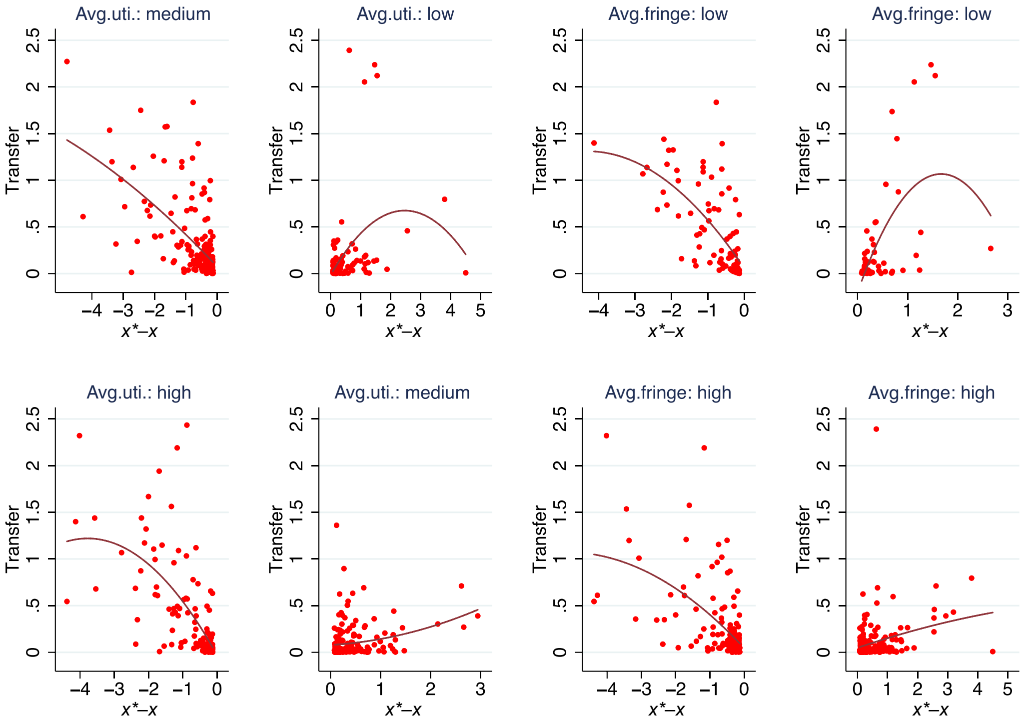
Second, there is evidence that the size of contract revision affects the transfers nonlinearly and through its interaction with contract characteristics. This is also evident from Figure 2, which shows the impact of *afri* and *auti* on transfers differ across various sizes of contract revisions. For instance, the subplots in the third and fourth column show that for lower and upper terciles of *afri*, the size of revision affects the transfers similarly for negative revisions, whereas such effect differs for positive revisions.

Table 2 further quantifies the effect of various factors on the transfers by regressing the latter on the contract characteristics and the size of contract revision $(x^{*}-x)$. The quadratic term of the size of contract revision and its interaction with contract characteristics are significant across all three model specifications. The type of work (*job*) has a significant positive effect on the observed transfer, and the effect becomes less pronounced as the size of the revision increases. The number of bidders, the proportion of fringe bidders in an auction, and the average utilization rate of participants (*nbid*, *afri*, *auti*) all affect the transfer through their interaction with the initial and the new specification. These results indicate that the size of specification change and contract characteristics affect the transfers significantly and nonlinearly. In addition, the impacts of the characteristics on the transfer depend on the size of specification change.

Third, there is little correlation between the contract characteristics and the size of specification change per se. In Table 3, we regress the size of specification changes on the

FIGURE 2

DEPENDENCE OF TRANSFER ON SPECIFICATION CHANGE AND CONTRACT CHARACTERISTICS [Color figure can be viewed at wileyonlinelibrary.com]



Note: In each subplot, the curve depicts the predicted transfers from a linear regression on $(x^* - x)$ and $(x^* - x)^2$. Subplots with *avg.uti.* : *low*, *medium*, and *high* are for subsets of data in which $(x^* - x)$ fall in the first, second, and third intervals defined by terciles of $(x^* - x)$. Similar definition applies for subplots for *avg.fringe*.

contract characteristics *nbid*, *job*, *afri*, *auti*, and *adis*. We find that these characteristics are jointly insignificant at the 5% level across all three specifications. Combining the results from Tables 2 and 3, we conclude that contract characteristics affect the negotiated transfers through some channel *other than* a direct impact on the size of contract revision. This further supports the setup of our model in Section 2, where contract characteristics affect the transfers through the contractor's bargaining power γ, but not via any direct impact on the size of specification change.

The model in Section 2 is motivated in part by these stylized facts in the CalTrans highway procurement data. It allows us to disentangle the roles of various factors determining the contract revision, the negotiated transfers, as well as the auction payment.

5. Econometric implementation

■ We estimate a parametric model which accommodates the heterogeneity in the contracts and sellers in the data. Our results in Section 3 are limited to the benchmark case with no unobservable structural errors that are known to the buyer and sellers. Nonparametric identification of an econometric model which incorporates these structural errors is an open question. In this section, we adopt a parametric approach by exploiting the restrictions on the functional forms of the model elements in order to recover the structural parameters.

TABLE 2 Regression Results of Transfer[a]

	(1)	(2)	(3)
Spec. change ($x^* - x$)	0.0681	0.122*	−0.0436
	(0.0662)	(0.0698)	(0.0805)
Avg. fringe	−0.211**	−0.206**	−0.104
	(0.0989)	(0.101)	(0.102)
Number of bidders	−0.0135	−0.0158	−0.0150
	(0.0130)	(0.0130)	(0.0129)
Job	0.0387	0.0295	0.00889
	(0.0549)	(0.0550)	(0.0547)
Spec. change ∗ avg. fringe	0.655***	0.648***	0.790***
	(0.0959)	(0.0960)	(0.103)
Spec. change ∗ number of bidders	−0.143***	−0.144***	−0.144***
	(0.0158)	(0.0158)	(0.0158)
Spec. change ∗ job	−0.0541	−0.0826*	−0.132***
	(0.0456)	(0.0471)	(0.0482)
Spec. change ∗ spec. change	0.00236***	0.00279***	0.00225***
	(0.000646)	(0.000671)	(0.000682)
Avg. utilization		−0.238	−0.240
		(0.217)	(0.215)
Spec. change ∗ avg. utilization		−0.419**	−0.305*
		(0.173)	(0.174)
Avg. distance			0.00103***
			(0.000290)
Spec. change ∗ avg. distance			0.000456***
			(0.000132)
Constant	0.355***	0.389***	0.219**
	(0.0714)	(0.0819)	(0.0916)
N	1224	1224	1224
R^2	0.334	0.338	0.351
Adj. R^2	0.330	0.332	0.344

[a]Standard errors in parentheses. $*p < 0.10$, $**p < 0.05$, $***p < 0.01$.
We use only observations of contracts that are modified with negotiated transfers.

TABLE 3 Regression Results of Specification Change[a]

	(1)	(2)	(3)
Number of bidders	−0.0479*	−0.0485*	−0.0485*
	(0.0281)	(0.0281)	(0.0281)
Avg. fringe	0.519**	0.503**	0.498**
	(0.215)	(0.219)	(0.223)
Job	0.0835	0.0815	0.0818
	(0.120)	(0.120)	(0.120)
Avg. utilization		−0.186	−0.185
		(0.468)	(0.469)
Avg. distance			−0.0000769
			(0.000634)
Constant	−0.264*	−0.228	−0.218
	(0.155)	(0.179)	(0.200)
R^2	0.006	0.006	0.006
Adj. R^2	0.003	0.002	0.002
p-value (joint significance)	0.08	0.14	0.22

[a]Standard errors in parentheses. $*p < 0.10$, $**p < 0.05$, $***p < 0.01$. The regression uses the observations with contract revisions and observed transfers.

□ **Parametric specifications.** For simplicity, we drop the indices of a contract j and a contractor i in the notation. The bargaining power of a contractor in the negotiation after the auction is:

$$\gamma(w;\varrho) = \exp(w'\varrho)/[1+\exp(w'\varrho)], \tag{10}$$

where $w \equiv [job, nbid, afri, auti, adis]$ is the vector of contract characteristics, with *nbid* being the number of contractors who participate in the auction. The vector w consists of contract/auction characteristics known to CalTrans and contractors at the time of their decisions.

The specification in (10) is consistent with the reduced-form evidence in Section 4 that the contract/auction characteristics (such as job type and the intensity of competition) affect transfers through some other channel than the size of contract revision. Under this specification, the bargaining power as perceived by CalTrans and contractors in the auctions does not depend on *ex post* individual characteristics of the auction winner, whose identity is unknown before the auction is concluded. We run a regression of negotiated transfers on the winners' characteristics (fringe, utilization, and distance), the specification change $x^* - x$, and the contract characteristics *job*, *nbid*, *afri*, *auti*, and *adis*. The results suggest that the auction winner's characteristics do not affect the transfer significantly across different model specifications.

The transfer Y due to Nash Bargaining is

$$Y = D\left[\gamma\phi + (1-\gamma)a + \varepsilon\right], \tag{11}$$

where

$$D = 1\{\phi - a + \eta > 0\} \tag{12}$$

is the dummy for contract revision, and (ε, η) are bivariate normal with zero mean, standard deviation $(\sigma, 1)$, and a correlation coefficient ρ. The change in social surplus due to contract revision is $\pi(x^*, job;\theta) - \pi(x, job;\theta)$, where

$$\pi(x, job;\theta) = \theta_1 x + \theta_2(x \times job) + \theta_3 x^2; \tag{13}$$

and the costs for revising the contract is

$$a(x, x^*, job;\theta) \equiv \theta_0 + \theta_4(x^* - x) + \theta_5(x^* - x) \times job + \theta_6(x^* - x)^2, \tag{14}$$

where θ_0 is the fixed cost. The errors (ε, η) capture measurement noises, or any idiosyncratic factors that affect the negotiated transfer or the joint decision to revise the contract, but that are orthogonal to the initial and new specification (e.g., additional compensation for delay in payment). Under (13), the social surplus depends on the job type and specification, but not the characteristics of the auction winner. The costs for revising the contract $a(\cdot)$ include the adaptation costs in Bajari, Houghton, and Tadelis (2014).

The specifications in (13) and (14) are supported by the reduced-form patterns in Table 2 (as discussed in Section 4). In addition to the sources of variation we use for nonparametric identification in Section 3, parametric restrictions in this section also provide further identifying power to recover model elements. We discuss the identification of the parametric model in the web appendix.

□ **Two-step estimation of structural parameters.** We estimate $(\theta, \varrho, \rho, \sigma)$ via two steps. In the first step, we estimate some components in θ by applying the probit procedure to (12) with a vector of explanatory variables $[x^* - x,\ (x^* - x) \times job,\ x^{*2} - x^2,\ (x^* - x)^2]$. This returns estimates for $(\hat{\theta}_1 - \hat{\theta}_4, \hat{\theta}_2 - \hat{\theta}_5, \hat{\theta}_3, \hat{\theta}_6) \equiv \hat{\theta}_{-1,2}$. The second step is to estimate the remaining parameters $\tau \equiv (\theta_1, \theta_2, \varrho, \rho, \sigma)$ using an extremum estimator:

$$\hat{\tau} = \operatorname*{argmax}_{\tau} \left\{\mathcal{L}_J(\tau) - \mathcal{M}_J(\tau)\right\}, \tag{15}$$

where $\mathcal{L}_J$ is the log-likelihood for the transfers under contract revision, and $\mathcal{M}_J$ is based on a set of moments derived from the first-order condition due to CalTrans' optimization in equilibrium, as shown in (7).[7] That is,

$$\mathcal{L}_J(\tau) \equiv J^{-1} \sum_j \log \Phi \left(\frac{\hat{I}_{s,j} + \frac{\rho}{\sigma}\left(y_j - \hat{I}_{o,j}\right)}{\sqrt{1-\rho^2}} \right) + \log \Phi' \left(\frac{y_j - \hat{I}_{o,j}}{\sigma} \right) - \log \sigma - \log \Phi(\hat{I}_{s,j}),$$

where j is an index for contracts, Φ and Φ' denote standard normal cumulative distribution function (cdf) and probability distribution function (pdf), and

$$\hat{I}_{o,j} \equiv \gamma_j \hat{\phi}_j + (1 - \gamma_j)\hat{a}_j \text{ and } \hat{I}_{s,j} \equiv \hat{\phi}_j - \hat{a}_j,$$

with $\gamma_j \equiv \gamma(w_j; \varrho)$, $\hat{\phi}_j \equiv \pi(x_j^*, w_j; \tau, \hat{\theta}_{-1,2}) - \pi(x_j, w_j; \tau, \hat{\theta}_{-1,2})$ and $\hat{a}_j \equiv a(x_j, x_j^*, w_j; \tau, \hat{\theta}_{-1,2})$; and

$$\mathcal{M}_J \equiv J^{-1} \sum_j \left[\hat{\varphi}_1(x_j, w_j) - \hat{\pi}_1(x_j, job_j; \tau) - \hat{\mu}_1(x_j, \tilde{x}_j, job_j; \tau)\right]^2,$$

where $\hat{\varphi}_1, \hat{\pi}_1, \hat{\mu}_1$ are estimates for the marginal effect of x on $\varphi(x, w)$, $\pi(x, job)$, and $\mu(x, \tilde{x}, job)$ (which is defined as $E[(1 - \gamma(W))s_+(x, X^*, job)|\tilde{X} = \tilde{x}, job]$); and $\tilde{x}_j \equiv r$ if x_j is the r -th quantile of the initial specification in the sample. Note that here, $\mu(\cdot)$ is defined as the buyer's *ex ante* share of the net surplus conditional only on the job type, rather than the full information $(x, \tilde{x}, w)$ available to the buyer prior to the realization of new design. To calculate $\hat{\varphi}_1$, first estimate a regression model of auction payment on x, w, and their interaction terms. (Note this *ex ante* payment does not depend on x^*, which is unknown in the auction stage.) Then, calculate $\hat{\varphi}_1$ by plugging in ordinary least squares (OLS) estimates. To calculate $\hat{\mu}_1$, note that under our specification,

$$\mu_1(x, \tilde{x}, job; \tau) = E[(1 - \gamma(W; \varrho)|job] \times E\left[\Phi(s(X^*, x, job)) \frac{\partial s(X^*, x, job)}{\partial x} \middle| \tilde{x}, job \right], \quad (16)$$

where the first expectation is with respect to the identity of the auction winner, and the second with respect to X^* conditional on $\tilde{x}$. For each trial value τ, construct $\hat{\mu}_1(x_j, \tilde{x}_j, job_j; \tau)$ by plugging $\hat{\theta}_{-1,2}$ into s and $\partial s/\partial x$ and taking the conditional sample average to estimate the first term in (16), and then estimating the second term using simulated observations of x^* drawn from the estimated distribution of X^* given $\widetilde{X} = \tilde{x}_j$. Recall that we normalize the distribution of $\widetilde{X}$ to $U(0, 1)$. The monotone strategy in equilibrium implies that $F_{X^*|\tilde{X}=\alpha^{*-1}(x)}$ equals $F_{X^*|X=x}$. Conditional on X, the distribution of X^* is approximately normal among revised contracts in the data. Hence, we parametrize this conditional density of X^* as $N((1 + v)X, \tilde{\sigma}^2 X^2)$, which implies $(X^* - X)/X \sim N(v, \tilde{\sigma}^2)$ in auctions with negotiated transfers. Our specification here is motivated by some reduced-form evidence. By regressing x^* and $x^* - x$ on (x, w), we find that the coefficients for contract characteristics in w are statistically insignificant. We use a maximum likelihood to estimate ν and $\tilde{\sigma}$. The second expectation is then calculated for each τ using simulated draws of x^* based on these estimates.

This two-step estimator is consistent for $(\theta, \varrho, \rho, \sigma)$. First, $\theta_{-1,2}$ is consistently estimated via probit in the first step by a standard argument. Furthermore, under our parametrization, the true τ is a unique maximizer of the probability limit of $\mathcal{L}_J$. (See the web appendix for details.) Next, note $\mathcal{M}_J$ is a sample analog for

$$E\left\{\left[\varphi_1(X, W) - \pi_1(X, job; \tau) - \mu_1(X, \tilde{X}, job; \tau)\right]^2\right\}, \quad (17)$$

[7] An alternative to our estimation method would be to derive the first-order conditions of the log-likelihood for the negotiated transfers, and stack them with the first-order condition from the buyer's optimization problem in a Generalized Method of Moments (GMM) estimator. This approach is algebraically demanding because the score function of the log-likelihood of negotiated transfers is cumbersome (mostly due to our parametrization of the bargaining power).

which is minimized to zero at the true parameters under the first-order condition of CalTrans' optimization in equilibrium, as shown in (7). In the equation above, the *ex ante* payment $\varphi(x, w)$ depends on the contract characteristics w and is directly identifiable in data. Buyers' optimization in equilibrium implies that the true parameter τ is a minimizer of (17). Hence, the true parameter τ is a unique maximizer of the probability limit of $\mathcal{L}_J - \mathcal{M}_J$ in the second step.

Because the estimand in (15) is a smooth function of the sample analogs, the two-step extremum estimator is asymptotically normal under regularity conditions. (If we were to follow the asymptotic plug-in approach for inference, then the asymptotic variance of $\hat{\tau}$ would need to include a term that accounts for the first-stage estimation error from the probit procedure.) In practice, we use the bootstrap procedure to construct the standard error. To implement the maximization routine in the second step, we pick an initial value for τ by estimating a two-stage Maximum Likelihood Estimation (MLE) that maximizes $\mathcal{L}_J$ alone (which produces a consistent estimator for τ).

□ **Estimation of contractor costs and holdup.** Recall that $\delta(x; \lambda^*) = E_{\lambda^*}[\gamma s_+(x, X^*)|X = x]$ and $y - a = \gamma s$ whenever $s > 0$. Let $q(x, job)$ denote the probability that a contract is revised with a negotiated transfer ($D = 1$) conditional on information available to contractors in the auction. Let $f_{P|Z}(\cdot|z_{j,k})$ and $F_{P|Z}(\cdot|z_{j,k})$ denote the density and the distribution of bids conditional on bidder k's characteristics in project j. From (5) and (6), we obtain the cost of contractor i in project j, $c_{j,i}$ in equilibrium,

$$c_{j,i} = p_{j,i} - \left(\sum_{k \neq i} \frac{f_{P|Z}(p_{j,i}|z_{j,k})}{1 - F_{P|Z}(p_{j,i}|z_{j,k})} \right)^{-1} + E\left[Y - a(x_j, X^*, job_j)|x_j, z_{j,i}, D_j = 1\right] q(x_j, job_j), \tag{18}$$

where $z_{j,i} \equiv [fri_{j,i}, uti_{j,i}, dis_{j,i}, job_j, ruti_{j,i}, rdis_{j,i}]$ and $ruti_{j,i}, rdis_{j,i}$ are minimum distance and utilization rate among the competitors against i. Note that the first two terms on the right-hand side of (18) constitute the inverse bidding strategy in a standard low-price procurement auction with asymmetric sellers (see, e.g., Bajari, Houghton, and Tadelis, 2014). More specifically, the first two terms in (18) identify the adjusted costs $\widetilde{C}_i$ as defined in (5), which accounts for the seller's expected share of net surplus from contract revision; the third term in (18) identifies the downward adjustment $\delta(\cdot)$ in (5), using the Nash Bargaining solution characterized in (1). Let $\Delta_{j,i}$ denote the conditional expectation of $Y - a$ in (18). It conditions only on x_j and $z_{j,i}$ because the contractor's *ex ante* share of the net surplus depends on contractor characteristics known prior to the auction.

In (18), we adopt a logit specification for the probability of contractual incompleteness $q(x_j, job_j; \vartheta)$. This probability differs from (12) in that the former specifies the *ex ante* probability of contract revisions, which does not condition on *ex post* information such as X^*. Denote the maximum likelihood estimator in this step by $\hat{\vartheta}$.

We model contractors' bidding strategies (normalized by engineer estimates) via the following regression:

$$\frac{p_{j,i}}{x_j} = g(z_{j,i}; \nu) + e_{j,i}, \tag{19}$$

where $g(\cdot)$ is linear in ν, and $e_{j,i}$ is independent from $z_{j,i}$. As in Bajari, Houghton, and Tadelis (2014), the specification conditions on the number of contractors in an auction (which is suppressed in notation), and allows for heterogeneity in the structural error via the contract size x_j.[8] We adopt a pooled-OLS to estimate the parameter in $g(.; \nu)$ conditional on the number of

[8] Note that the inverse bidding strategy for contractors is identified nonparametrically due to (18). Hence, in principle, the reduced-form linear specification in (19) is not necessary for estimating bidding strategies in large samples. We implement this regression form to estimate bidding strategies mostly due to data constraints.

contractors in the auction. Let $\hat{\nu}$ and $\hat{e}_{j,i}$ be the estimated parameters and the residuals, respectively. We then use these estimates to estimate $F_{P|Z}$ and $f_{P|Z}$. To see how, note that for a given number of contractors, $F_{P|Z}$ equals:

$$F_{P|Z}(p_{j,i}|z_{j,i}) = \Pr\{g(z_{j,i};\nu) + e_{j,i} \leq p_{j,i}/x_j\} = F_e(p_{j,i}/x_j - g(z_{j,i};\nu)),$$

where $F_e(\cdot)$ is the distribution of $e_{j,i}$ for contracts with the same number of bidders as contract j (we suppress n in $F_{P|Z}$ and F_e to simplify notation). The conditional density of bids is:

$$f_{P|Z}(p_{j,i}|z_{j,i}) \equiv \tfrac{\partial}{\partial p} F_e(p_{j,i}/x_j - g(z_{j,i};\nu)) = f_e(p_{j,i}/x_j - g(z_{j,i};\nu))/x_j.$$

To estimate $F_{P|Z}$ and $f_{P|Z}$, we first obtain the empirical distribution $\hat{F}_e$ and kernel density $\hat{f}_e$ of $\hat{e}_{j,i}$, then plug $\hat{F}_e$, $\hat{f}_e$, and $\hat{\nu}$ into the two equations above. Let $\hat{F}_{P|Z}$ and $\hat{f}_{P|Z}$ denote these estimates.

The expectation of $Y - a$ is estimated as

$$\widehat{\Delta}_{j,i} = \widehat{E}\left(Y_j|x_j, z_{j,i}, d_j = 1\right) - S^{-1}\sum\nolimits_{s=1}^{S} a(x_j, x^*_{j,s}, job_j;\hat{\theta}), \tag{20}$$

where the first term (the conditional expected transfer) in $\widehat{\Delta}_{j,i}$ is estimated using a single-index specification and Ichimura's semiparametric least squares in Ichimura (1993); the second term in $\widehat{\Delta}_{j,i}$ is a simulation-based estimate for the *ex ante* revision costs a conditional on contractual incompleteness, initial specification, and contract characteristics. Specifically, $x^*_{j,s}$ are independent draws from the estimated density of x^*, given x_j and $d_j = 1$.

We replace $f_{P|Z}$, $F_{P|Z}$, $\Delta_{j,i}$, and $q(x_j, job_j)$ by their estimates $\hat{f}_{P|Z}$, $\widehat{F}_{P|Z}$, $\widehat{\Delta}_{j,i}$, and $\hat{q}_j \equiv q(x_j, job_j;\hat{\vartheta})$ to obtain the estimate of costs for bidder i in contract j:

$$\hat{c}_{j,i} = p_{j,i} - \left(\sum\nolimits_{k=1,k\neq i}^{n_j} \frac{\hat{f}_{P|Z}(p_{j,i}|z_{j,k})}{1 - \widehat{F}_{P|Z}(p_{j,i}|z_{j,k})}\right)^{-1} + \widehat{\Delta}_{j,i}\hat{q}_j. \tag{21}$$

A contractor's holdup on the buyer is estimated as $\widehat{\Delta}_{j,i}\hat{q}_j$. A contractor's markup is defined as the difference between its *ex ante* payoff, estimated by $p_{j,i} + \widehat{\Delta}_{j,i}\hat{q}_j$, and its initial costs in the auction $\hat{c}_{j,i}$. We estimate the markup by the second term in (21).

6. Results

■ We start with a descriptive analysis of how the buyer's initial payment in the procurement auction is related to the contract characteristics. As explained in the next paragraph, we find that the initial contract design has a positive nonlinear effect on auction payment, especially via interaction with contract characteristics. This corroborates several key aspects in our model: that the contractor costs are stochastically increasing in the contract design, that the contractors adopt monotone strategies, and that the contract characteristics affect the auction payment through bargaining power and *ex ante* holdup on the buyer.

Table 4 reports estimates from regressing the auction payment on contract characteristics. In each specification, the initial design (engineer estimate) has a significant positive marginal effect on the auction payment. There is also evidence in the third specifications that the effect diminishes as the engineer estimates increase. Another pattern consistent across all specifications is that the contract/auction-level characteristics, which include the job type, the proportion of fringe competitors (*afri*) as well as the average distance and utilization rate of competitors (*auti*) and (*adis*), are all significant via their interaction with engineer estimates. This is consistent with the notion that auction characteristics affect contractors' adjusted costs through the bargaining power of contractors and *ex ante* holdup on the buyer. Also, it is worth mentioning that a higher average utilization rate among the contractors tends to lower the auction payment. This indicates that there is an economy of scale in the costs for contractors that work on multiple contracts simultaneously.

TABLE 4 Regression Results of Expected Payment[a]

	(1)	(2)	(3)
Engineering estimate	1.105***	1.080***	1.029***
	(0.0160)	(0.0165)	(0.0170)
Engineering estimate ∗ engineering estimate	0.0000175	0.000107	−0.000228*
	(0.000133)	(0.000133)	(0.000135)
Avg. fringe	−0.0465	−0.0574	−0.108
	(0.117)	(0.118)	(0.115)
Number of bidders	0.0151	0.0126	0.0121
	(0.0146)	(0.0145)	(0.0140)
Job	−0.137**	−0.145**	−0.121**
	(0.0620)	(0.0614)	(0.0596)
Engineering estimate ∗ avg. fringe	0.128***	0.120***	0.150***
	(0.0198)	(0.0197)	(0.0194)
Engineering estimate ∗ number of bidders	−0.0490***	−0.0484***	−0.0487***
	(0.00314)	(0.00312)	(0.00302)
Engineering estimate ∗ job	0.0622***	0.0648***	0.0539***
	(0.00842)	(0.00835)	(0.00818)
Avg. utilization		−0.631**	−0.613**
		(0.246)	(0.239)
Engineering estimate ∗ avg. utilization		0.150***	0.142***
		(0.0283)	(0.0274)
Avg. distance			−0.0000780
			(0.000302)
Engineering estimate ∗ avg. distance			0.000299***
			(0.0000347)
Constant	−0.0460	0.0583	0.104
	(0.0872)	(0.0972)	(0.104)
N	1306	1306	1306
R^2	0.977	0.978	0.979
Adj. R^2	0.977	0.978	0.979

[a]The units of payment and engineering estimate are million dollars. Standard errors in parentheses. *$p < 0.10$, **$p < 0.05$, ***$p < 0.01$.

Our estimates in Table 5 indicate that the contractor tends to retain a substantial share of the *ex ante* net surplus due to contract revision, and that on average, the bargaining power depends on the intensity of competition as well as the utilization rates of alternative contractors.

The upper panel in Table 5 reports the average bargaining power of the contractor in a negotiation following contract revisions, that is, the mean of $\gamma(w_j; \hat{\varrho})$ across contracts, conditional on the job type ($job = 1$ for major work and 0 otherwise). In both cases, the contractor has significant bargaining power against CalTrans. For example, in each of the first two specifications, the contractor's bargaining power is significantly higher than 50%, regardless of the job type. In the third and most comprehensive specification, the average, bargaining power is 68.9% for major jobs and 79.3% for minor jobs.

The lower panel in the table reports the estimates for the mean marginal effect of contract and contractor characteristics on bargaining power. According to the first specification, on average, the bargaining power for a contractor is about 6% higher if it had to defeat an additional competitor in the auction. This might be because a contractor's competitive advantage (such as cost or logistic efficiency) leads to more leverage in the negotiation with CalTrans. The average utilization rate is important in explaining the bargaining power in the third specification. Our estimates show that if the average utilization rate in an auction is increased by 10%, then the bargaining power of the contractor increases by 4.4%. This conforms with the intuition that a contractor has larger bargaining power against CalTrans when its competitors are, on average, more occupied or committed to other projects.

TABLE 5 Estimates of Average Marginal Effects on Bargaining Power[a]

		(1)	(2)	(3)
Avg. bargaining power	Job=1	0.911^{***}	0.887^{***}	0.689^{**}
		(0.185)	(0.192)	(0.323)
	Job= 0	0.903^{***}	0.953^{***}	0.793^{**}
		(0.214)	(0.254)	(0.323)
Avg. marginal effects	Number of bidders	0.060^{**}	0.074	0.051
		(0.032)	(0.047)	(0.071)
	Avg. fringe	−0.077	−0.086	−0.281
		(0.178)	(0.238)	(0.471)
	Avg. utilization		−0.178	0.440^{**}
			(0.214)	(0.189)
	Avg. distance			0.001
				(0.002)

[a]Standard errors in parentheses are calculated using 200 bootstrap samples $*p < 0.10$, $**p < 0.05$, $***p < 0.01$. Specifications (1), (2), and (3) are for $w \equiv [job, nbid, afri]$, $w \equiv [job, nbid, afri, auti]$, and $w \equiv [job, nbid, afri, auti, adis]$, respectively.

TABLE 6 Estimates of Surplus and Cost Functions[a]

		(1)	(2)	(3)
Surplus function $\pi(\cdot)$	X	-0.239^{**}	-0.233^{**}	-0.262^{***}
		(0.083)	(0.082)	(0.083)
	$X * job$	−0.118	−0.134	−0.115
		(0.088)	(0.085)	(0.114)
	X^2	0.006^{***}	0.006^{***}	0.006^{***}
		(0.002)	(0.002)	(0.002)
Cost function $a(\cdot)$	$(X^* - X)$	0.031	0.037	0.008
		(0.048)	(0.051)	(0.061)
	$(X^* - X) * job$	−0.104	-0.120^{*}	−0.101
		(0.065)	(0.071)	(0.083)
	$(X^* - X) * (X^* - X)$	-0.010^{**}	-0.010^{**}	-0.010^{**}
		(0.005)	(0.005)	(0.004)
	Constant	1.566^{***}	1.566^{***}	1.566^{***}
		(0.057)	(0.057)	(0.057)

[a]Standard errors in parentheses are calculated using 200 bootstrap samples $*p < 0.10$, $**p < 0.05$, $***p < 0.01$. Columns (1), (2), and (3) report estimates based on the first-stage estimates from specifications (1), (2), and (3) in Table 4, respectively.

Our estimates in Table 6 indicate that there is an increasing return in the social surplus for highway construction, and that the net incremental surplus from revisions is nonlinear in the size of revisions.

Table 6 reports parameter estimates in the social surplus and incremental costs for contract revision. The coefficient for the squared engineer estimates is significantly positive, suggesting some economy of scale in social surplus as the contract size increases. We reject the null hypothesis that s is linear in $(x^* - x)$ (coefficients for x^2 in π and coefficients for $(x^* - x)^2$ in a are jointly zero) with a p-value less than 0.001 in the most comprehensive specification. In addition, a Wald test for the joint significance of x and x^* in s also yields a p -value less than 0.001.

To quantify the effect of job types on the negotiated transfer, we use the estimates in Table 6 to calculate the average difference between the truncated mean of transfer (conditional on contract revision) for major contracts ($job = 1$) and minor contracts, respectively. This measures the *ex ante* marginal effect of the job type on the truncated means, after integrating out other contract characteristics and the initial and final specification. For the most general specification, this estimated difference is \$52, 000 with a bootstrap standard error of \$20, 000. The tests under the other two nested specifications also report statistical significance of *job* at the 1% level.

TABLE 7 Logit Regression of Probability of Incompleteness[a]

	(1)	(2)	(3)
Constant	0.997***	1.179***	1.161***
	(0.249)	(0.273)	(0.275)
Engineering estimate	2.149***	1.798***	1.846***
	(0.381)	(0.410)	(0.417)
Job	0.226	−0.403	−0.391
	(0.256)	(0.512)	(0.512)
Engineering estimate ∗ job		1.309	1.287
		(0.968)	(0.967)
Engineering estimate ∗ engineering estimate	−0.0198***		−0.0202***
	(0.00529)		(0.00675)
Observations	1306	1306	1306
Log-likelihood	−246.72	−245.81	−245.70
Pseudo R^2	0.157	0.160	0.161

[a]Standard errors in parentheses. *$p < 0.10$, **$p < 0.05$, ***$p < 0.01$.

TABLE 8 Estimates of Markup Ratio (unit: %)[a]

		10-th Pctile.	25-th Pctile.	Median	75-th Pctile.	90-th Pctile.
Job	1	.017	.031	.058	.131	.236
	0	.020	.036	.065	.129	.244
Winners	Yes	.038	.063	.122	.221	.397
	No	.017	.028	.054	.105	.190
Fringe	Yes	.017	.028	.054	.112	.210
	No	.024	.045	.078	.154	.291
Utilization[b]	0	.017	.028	.054	.109	.205
	(0,0.3)	.025	.047	.082	.161	.289
	[0.3,1]	.021	.040	.067	.147	.292
Distance[b]	[0.1,11.9]	.022	.037	.065	.142	.278
	[12,247.9]	.018	.033	.062	.128	.233
	> 247.9	.020	.036	.063	.132	.284
Number of bidders[c]	2	.153	.181	.232	.343	.523
	$3 \leq n \leq 5$	.038	.051	.078	.137	.251
	$6 \leq n \leq 8$	.017	.022	.035	.073	.152
	$8 < n$	.010	.014	.019	.039	.081
Overall		.019	.034	.062	.129	.241

[a]The markup ratio is defined as a contractor's markup over her initial cost $\hat{c}_{j,i}$.
[b]The intervals of distance are defined by their lower, middle, and upper terciles. For all the positive utilization rates, we divide them into (0, 0.3) and [0.3, 1] such that the observations in the two intervals equal.

Table 7 estimates a logit model for contract revision $q(x_j, job_j; \vartheta)$. Our estimates illustrate that the engineer estimates are a significant determinant for contractual revision, whereas job types are not. This conforms with the observation in Tirole (2009) that the holdup problem occurs under the incomplete contracts where the probability of incompleteness is determined by the endogenous choice of the buyer.

Table 8 reports the estimates for the ratio between markups and contractor costs. We find that the markups are substantial (with a sample average of 10.6%), and are slightly lower for contracts with substantial work ($job = 1$). Our estimates also reveal how markups depend on other features of the auctions and contractors.

The markup ratios are lower in auctions involving more competitors, which is consistent with our assumption that contractors are aware of the number of competitors in auctions. Besides, the auction winners tend to bid with higher markups than the others. One possible interpretation is that the cost for the auction winner is lower than the other contractors, thus allowing

TABLE 9 Estimates of Markup Ratios Assuming Away Incompleteness (unit: %)[a]

		10-th Pctile.	25-th Pctile.	Median	75-th Pctile.	90-th Pctile.
Job	1	.022	.038	.071	.162	.301
	0	.024	.045	.077	.161	.306
Winning bidders	Yes	.050	.082	.149	.277	.512
	No	.022	.036	.064	.129	.240
Fringe	Yes	.022	.036	.066	.137	.271
	No	.029	.054	.092	.189	.359
Utilization[b]	0	.020	.036	.065	.134	.262
	(0,0.3)	.032	.058	.098	.194	.365
	[0.3,1]	.026	.042	.083	.175	.363
Distance[b]	[0.1,11.9]	.025	.046	.074	.168	.334
	[12,247.9]	.023	.042	.076	.161	.299
	> 247.9	.024	.042	.073	.156	.326
Number of bidders[c]	2	.188	.199	.269	.406	.670
	$3 \leq n \leq 5$	.047	.061	.094	.170	.311
	$6 \leq n \leq 8$	.022	.027	.045	.095	.211
	$8 < n$	.014	.017	.027	.058	.106
Overall		.023	.043	.075	.161	.305

[a]The markup ratio assuming away incompleteness is defined as a contractor's markup over her initial cost $\hat{c}_{j,i}$.
[b]The intervals of distance are defined by their lower, middle, and upper terciles. For all the positive utilization rates, we divide them into (0, 0.3) and [0.3, 1] such that the intervals have equal number of observations.

it to quote competitive prices even with a high markup. This might also explain why our estimates for markup ratios are slightly higher for major competitors in the industry (nonfringe contractors).

About half of the contractors competing in the auctions are not committed to other projects simultaneously (with zero utilization rates). The markup ratios for these contractors are on average lower than those with positive utilization rates. Our estimates suggest that the contractors with moderate or high utilization rates have greater markup ratios than those with lower utilization rates. On the other hand, the distance between the contractor's location and the job sites has no substantial effect on the estimated markups.

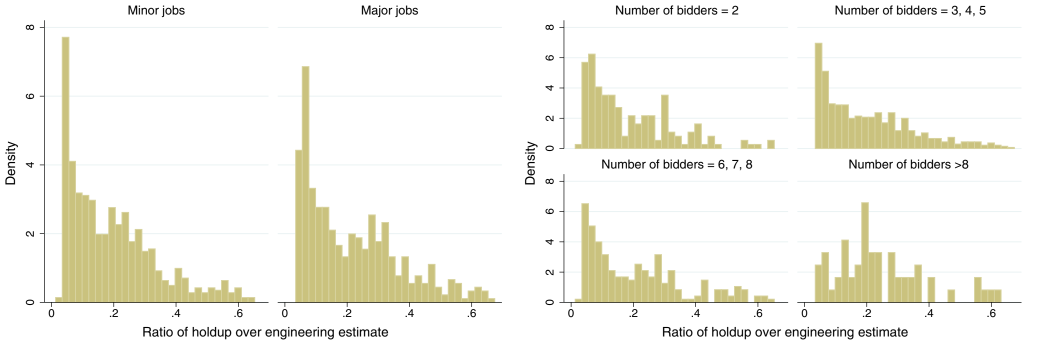
We now quantify the overestimation of the markups if the effect of holdups from incomplete contracts are ignored in estimation, so that the bids are interpreted as generated from equilibrium in standard lowest-price procurement auctions. Table 9 reports estimates for markup ratios under these assumptions. The average markup ratio is estimated to be 13.3%, which is 25.5% higher than the estimates in Table 8. We present the estimates at different quantiles in Table 9. The markups are overestimated up to 26.4% (at the 25th quantiles of overall estimates) if contract incompleteness is not accounted for.

We use the parameter estimates above to calculate the ratio between the contractor's holdup on CalTrans and the engineer estimates based on the initial contract design in each auction. The average holdup ratio is 20.4% across contracts with revisions in the data. Figure 3 illustrates histograms of these estimated ratios conditional on the type of contracts (major or minor) and contractor participation. The histograms show that these ratios tend to be higher for contracts involving major jobs. The average ratio is 21.6% for major projects and 19.5% for minor projects. The difference is statistically significant at 1% level based on a two-sample Kolmogorov-Smirnov test. The estimates also indicate that the ratios are higher for contracts involving more bidders.

To see how contract characteristics impact the holdups, we regress the estimated holdup ratios on *auti*, *afri*, *nbid*, and *job* conditioning on bidder participation and report their marginal effects on holdups in Table 10. Across all specifications, the proportion of fringe competitors in an auction (*afri*) has a significantly positive effect on the holdup ratio. For example, the holdup ratio is about 13.4%–14.4% higher in the auctions that only involve fringe contractors than those only involving nonfringe contractors when the number of bidders $n \in \{3, 4, 5\}$.

FIGURE 3

HISTOGRAM OF THE HOLDUP RATIO
[Color figure can be viewed at wileyonlinelibrary.com]



(a) The ratio: by job type (b) The ratio: by number of bidders

Note: The holdup ratio is defined as the ratio of holdup over the engineering estimate in each contract.

The impact of the job type of contracts (*job*) on holdup depends on the intensity of competition. For auctions with moderate competition ($3 \leq n \leq 8$), which accounts for more than 76% of contracts in the data, the job type has a significantly positive impact on holdup ratios. This is consistent with our structural estimates above. Table 7 suggests that the contract type has no significant effect on the *ex ante* probability for contract revision; Table 6 indicates that the net surplus from revisions is higher for contracts that involve major jobs (the average marginal effect of job type on revision surplus is statistically positive at 5% level). Taken together, those estimates explain why the holdup ratios are higher in contracts that require more substantial work.

7. Cost-plus versus fixed-price contracts

■ In practice, a common alternative to the "fixed-price" contract analyzed above is a "cost-plus" contract.[9] Cost-plus contracts do not specify any fixed payment, but reimburse contractors for actual costs incurred plus a profit margin negotiated *ex ante* (either in a lump-sum or *pro-rata* form). Cost-plus contracts are popular in the US defense industry, where the total value of cost-plus contracts is $78 billion for the fiscal year 2007.[10]

Using estimates from Section 6, we calculate the surplus for buyers under cost-plus contracts with a negotiated lump-sum profit margin, and compare it with those estimated for fixed-price contracts in the data.

□ **Basic setup.** We consider a counterfactual context where CalTrans and the contractors have common knowledge about *ex ante* uncertainty of new design X^*. Let j denote the index for projects reported in the data. A contractor i competes for a project j by quoting a lump-sum profit margin in a cost-plus contract $\zeta_{j,i}$; CalTrans awards the contract to the seller with the highest *ex ante* surplus for the government. Note that for the government, its expected cost for the project varies with the identity of the contractor it selects.

[9] The term "fixed-price" refers to the fact that, prior to the negotiation of transfers due to contract revisions, the payment to the contractor determined in the auction is fixed.

[10] Source: "Defense Industrial Initiatives Current Issues: Cost-Plus Contracts," by Joachim Hofbauer and Greg Sanders, Center for Strategic and International Studies, 2008.

TABLE 10 Estimates of Average Marginal Effects on the Holdup Ratio[a]

	$n=2$			$3 \le n \le 5$			$6 \le n \le 8$			$n > 8$		
Variable	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)
Avg. fringe	0.062^{***}	0.053^{***}	$.057^{***}$	0.144^{***}	0.135^{***}	0.134^{***}	0.135^{***}	0.129^{***}	0.0821^{***}	0.359^{***}	0.417^{***}	0.383^{***}
	(0.0001)	(0.011)	(0.0007)	(0.001)	(0.001)	(0.001)	(0.001)	(0.0007)	(0.0006)	(0.009)	(0.0007)	(0.014)
Job	-0.010^{***}	-0.011^{***}	-0.010^{***}	0.033^{***}	0.031^{***}	0.033^{***}	0.032^{***}	0.031^{***}	0.033^{***}	-0.010^{***}	-0.015	0.011
	(0.0001)	(0.0004)	(0.002)	(0.006)	(0.005)	(0.006)	(0.0004)	(0.001)	(0.0006)	(0.0003)	(0.011)	(0.013)
Avg. utilization		-0.126^{***}	-0.124^{***}		-0.095^{***}	-0.097^{***}		-0.045^{***}	-0.020^{***}		-0.251^{***}	-0.183^{***}
		(0.0008)	(0.001)		(0.001)	(0.001)		(0.005)	(0.002)		(0.065)	(0.074)
Avg. distance			$-1.13\text{e-}4^{***}$			-0.045^{***}			$-3.39\text{e-}4^{***}$			4.39e-4
			(1.10e-5)			(0.005)			(2.37e-6)			(7.00e-6)

[a]The holdup ratio is defined as the ratio of holdup over the engineering estimate in each contract. Standard errors in parentheses. $^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$. Specification (1): ratio=$\beta_0 + \beta_1$avg. fringe + $\beta_2$job + $\beta_3$avg. fringe × job + ϵ. Specification (2): ratio=$\beta_0 + \beta_1$avg. fringe + $\beta_2$avg. utilization + $\beta_3$job + $\beta_4$avg. fringe × job + $\beta_5$avg. utilization × job + ϵ. Specification (3): ratio=$\beta_0 + \beta_1$avg. fringe + $\beta_2$avg. utilization + $\beta_3$avg. distance + $\beta_4$job + $\beta_5$avg. fringe × job + $\beta_6$avg. utilization × job + $\beta_7$avg. distance × job + ϵ.

For each project j and contractor/seller i, let $\tilde{z}_{j,i} \equiv (job_j, uti_{j,i}, fri_{j,i}, dis_{j,i})$ denote the project and contractor characteristics in $z_{j,i}$ that affect private costs. The buyer's *ex ante* surplus from awarding the contract to i is

$$\int \left[\pi(t, job_j) - E(C^*_{j,i} | X^*_j = t, \tilde{z}_{j,i})\right] dF_{X^*_j|\tilde{z}_{j,i}}(t) - \zeta_{j,i}, \tag{22}$$

where π is the social surplus, $F_{X^*_j|\tilde{z}_{j,i}}$ the *ex ante* distribution of the new design conditional on $\tilde{z}_{j,i}$, and $C^*_{j,i}$ the costs for the new design X^*_j. The buyer chooses a contractor to maximize *ex ante* surplus in (22). Contractors compete by quoting profit margins in cost-plus contracts. A contractor i's profit from quoting $\zeta_{j,i}$ is:

$$\begin{aligned}
&\zeta_{j,i} \times 1\{E[\pi(X^*_j, job_j) - C^*_{j,i}|\tilde{z}_{j,i}] - \zeta_{j,i} \geq E[\pi(X^*_j, job_j) - C^*_{j,i'}|\tilde{z}_{j,i'}] - \zeta_{j,i'} \forall i' \neq i\} \\
&\quad = \zeta_{j,i} \times 1\{\zeta_{j,i} \leq \zeta_{j,i'} + E(C^*_{j,i'}|\tilde{z}_{j,i'}) - E(C^*_{j,i}|\tilde{z}_{j,i}) \forall i' \neq i\},
\end{aligned}$$

where $1\{\cdot\}$ is the indicator function. The distribution of X^*_j and $C^*_{j,i}$ are common knowledge among contractors. This is a simultaneous game of complete information between contractors. We calculate the buyer surplus under a Nash Equilibrium in which the winner is $i = \text{argmin}_k E(C^*_{j,k}|\tilde{z}_{j,k})$ with a quote $\zeta_{j,i} \equiv E(C^*_{j,i'}|\tilde{z}_{j,i'}) - E(C^*_{j,i}|\tilde{z}_{j,i})$, where $i' = \text{argmin}_{k \neq i} E(C^*_{j,k}|\tilde{z}_{j,k})$.[11]

Under cost-plus contracts, the surplus for the buyer is

$$E[\pi(X^*_j, job_j) - C^*_{j,i} - \zeta_{j,i}|w_j], \tag{23}$$

where i denotes the winner who has the lowest costs for design X^**ex ante*, and w_j consists of contract characteristics $(auti_j, afri_j, adis_j, nbid_j, job_j)$.

Buyer surplus under fixed-price contracts is

$$E[\pi(X_j, job_j) - \varphi(X_j, W_j; \lambda^*) + \mu(X_j, \tilde{X}_j, W_j)|w_j], \tag{24}$$

where μ is the buyer's expected share of net surplus due to the new design after conceding the holdup to the contractor, and φ is the expected auction price under initial specification X_j and contract characteristics. For a meaningful comparison, we maintain in our analysis that the distribution of X^*_j given w_j in counterfactual cost-plus contracts is the same as that in the sample.

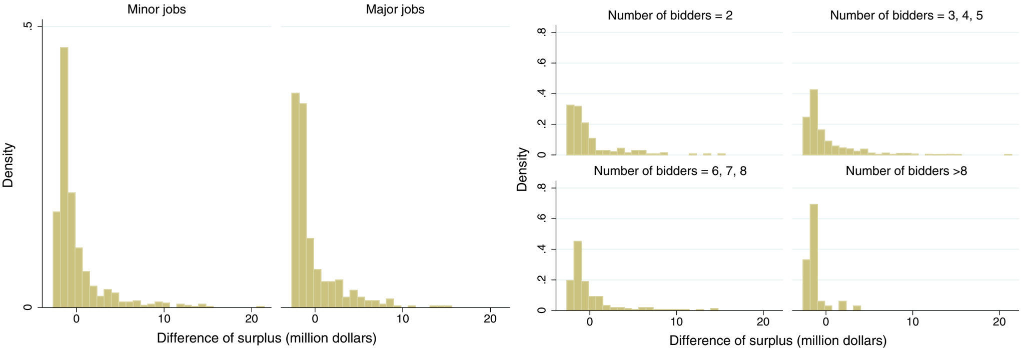
The ranking of buyer surplus across these two types of contracts is ambiguous because of the trade-offs involved. First, under cost-plus contracts, the auction winner is the one with the lowest *ex ante* costs under the design X^*, whereas under a fixed-price contract, the auction winner is the one with the lowest *ex post* costs under the initial design X, *after adjusting for the holdup*. Second, under cost-plus contracts, the buyer collects the expected *gross* surplus $E[\pi(X^*_j, job_j)|w_j]$; whereas under fixed-price contracts, the buyer concedes a proportion of the net surplus to the contractor, thus collecting an expected gross surplus of $E[\pi(\alpha^*(\tilde{X}_j), job_j) + \mu(\alpha^*(\tilde{X}_j), \tilde{X}_j, W_j)|w_j]$. Finally, the buyer's expected payment under cost-plus contracts is $E[C^*_{j,i} + \zeta_{j,i}|w_j]$, or *ex ante* second-lowest costs under the new design. In contrast, the buyer's expected payment under fixed-price contracts is $E[\varphi(X_j, W_j; \lambda^*)|w_j]$, where φ consists of the lowest costs under X_j and a markup that is adjusted based on contractor expected share of net surplus. All in all, measuring the difference in buyer surplus under these contracts is an open empirical question because of these trade-offs.

□ **Comparison of buyer surplus.** Our results of counterfactual analysis show that the buyer surplus under a cost-plus contract is lower than that under the fixed-price contract for 71.7% of the projects reported in the data. The average reduction in buyer surplus under the cost-plus contracts is \$382, 074, which is 13.6% of average engineer estimates. This implies that, for most

[11] We assume a random tie-breaking if there are multiple contractors who have the same, lowest expected costs. Such an *ad hoc* tie-breaking rule has no impact on the characterization of equilibrium and therefore subsequent counterfactual analysis.

FIGURE 4

HISTOGRAM OF THE SURPLUS DIFFERENCE
[Color figure can be viewed at wileyonlinelibrary.com]



(a) The surplus difference: by job type (b) The surplus difference: by number of bidders

Note: The surplus difference is defined as the surplus under cost-plus contract subtracts that of the fixed-price contract.

of the projects in the data, the buyer's gains in the expected surplus under a fixed-price contract are greater than the increase in its expected payment.

Figure 4 reports the histograms of estimated differences in buyer surplus (that under cost-plus minus that under fixed-price) in the auctions conditional on the job type and on the number of bidders. Although the job types do not seem to have any significant impact on the difference in buyer surplus, a greater number of bidders tends to make the distribution of the difference slightly positively skewed. To understand what is driving this pattern, recall the number of bidders affects a buyer's expected payment under both forms of contracts through contractor strategies and the lowest expected costs. It also affects buyers' *ex ante* surplus under the fixed-price contract through the holdup. The figure indicates that these trade-offs eventually work in favor of the buyer under fixed-price contracts in most cases.

We regress the estimated differences in buyer surplus on contract characteristics, that is, $(auti, afri, job, adis)$, and report their average marginal effects in Table 11. First, across all specification and level of bidder participation, the type of jobs have significantly negative impact on the difference in buyer surplus. That is, from a buyer's perspective, a fixed-price contract is *ex ante* more desirable relative to a cost-plus contract when the jobs involve major tasks ($job = 1$). For example, under the third specification, under moderate competition ($n \in \{3, 4, 5\}$), the difference between buyer surplus is reduced by \$237, 000 (about 8.4% of average engineer estimates) when the contract involves major jobs.

Second, cost-plus contracts compare more favorably with fixed-price contracts when there is a higher proportion of fringe competitors in the auction. With moderate bidder participation ($n \in \{3, 4, 5\}$), the difference in surplus would increase by \$549, 000 if the competition is between fringe contractors alone. This effect is explained by the difference in contractor costs and bargaining power between fringe and nonfringe competitors.

Third, the average marginal effect of utilization rates on the difference reported in Table 11 is positive, implying that the advantage of fixed-price contracts on buyer's surplus over cost-plus contracts diminishes if contractors are more occupied. This can be explained by utilization's positive impact on contractors' bargaining power presented in Table 5: higher bargaining power of the contractor due to higher average utilization rates benefits contractors in post-auction bargaining but hurts the buyer. In addition, more occupied contractors may have less incentives to exert efforts and lower costs in fixed-price contracts, and this negatively affects the advantage of fixed-price over cost-plus contracts.

TABLE 11 **Estimates of Average Marginal Effects on Surplus Difference (cost-plus minus fixed-price, in million dollars)[a]**

	$n = 2$			$3 \leq n \leq 5$			$6 \leq n \leq 8$			$n > 8$		
Variable	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)
Engineering estimate	1.029***	1.028***	1.036***	0.980***	0.979***	0.976***	0.878***	0.876***	0.868***	0.745***	0.741***	0.745***
	(0.007)	(0.007)	(0.006)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.017)	(0.019)	(0.015)
Avg. fringe	0.919***	0.943***	0.871***	0.454***	0.533***	0.549***	0.397***	0.484***	0.050***	0.451	0.072	0.028
	(0.518)	(0.062)	(0.046)	(0.013)	(0.014)	(0.015)	(0.015)	(0.017)	(0.003)	(0.146)	(0.102)	(0.093)
Job	−0.061***	−0.059***	−0.079***	−0.234***	−0.228***	−0.237***	−0.240***	-0.247***	−0.234***	−0.333***	−0.407***	−0.376***
	(0.017)	(0.016)	(0.013)	(0.011)	(0.011)	(0.009)	(0.008)	(0.010)	(0.009)	(0.038)	(0.004)	(0.030)
Avg. utilization		0.147***	0.102**		0.720***	0.727***		0.124***	0.137***		1.776	2.001***
		(0.049)	(0.043)		(0.011)	(0.009)		(0.030)	(0.001)		(0.157)	(0.078)
Avg. distance			2.78e-4***			5.01e-4***			−4.00e-4***			−3.36e-4***
			(6.75e-5)			(3.986e-5)			(9.06e-5)			(6.76e-5)

[a]Standard errors in parentheses. *$p < 0.10$, **$p < 0.05$, ***$p < 0.01$. Specification (1): diff=$\beta_0 + \beta_1$engineering estimate + $\beta_2$avg. fringe + $\beta_3$job + $\beta_4$avg. fringe × engineering estimate + $\beta_5$job × engineering estimate + $\beta_6$engineering estimate × engineering estimate + ϵ. Specification (2): diff=$\beta_0 + \beta_1$engineering estimate + $\beta_2$avg. fringe + $\beta_3$avg. utilization + $\beta_4$job + $\beta_5$avg. fringe × engineering estimate + $\beta_6$avg. utilization × engineering estimate + $\beta_7$job × engineering estimate + $\beta_8$engineering estimate × engineering estimate + ϵ. Specification (3): diff=β_0 + $\beta_1$engineering estimate + $\beta_2$avg. fringe + $\beta_3$avg. utilization + $\beta_4$avg. distance + $\beta_5$job + $\beta_6$avg. fringe × eng. + $\beta_7$avg. utilization × engineering estimate + $\beta_8$avg. distance × engineering estimate + $\beta_9$job × engineering estimate + β_{10}engineering estimate × engineering estimate + ϵ.

Our analysis of buyer surplus provides a benchmark comparison between fixed-price and cost-plus contracts in that it abstracts away from any disincentive for cost-saving under cost-plus contracts. If the contractor costs under the cost-plus format is stochastically higher than fixed-price contracts, the difference in buyer surplus would be even lower than what we estimate. Hence, our benchmark comparison can be interpreted as an upper bound on the difference in buyer surplus if there is cost-saving under fixed-price contracts. Our estimates show this upper bound is negative for most cases in the data.

8. Concluding remarks

■ This article studies procurement auctions with incomplete contracts. We introduce a model that endogenizes a buyer's initial specification of the contract, and maintain a flexible information structure. The model rationalizes the decision to revise a contract and the transfers negotiated via Nash Bargaining between the buyer and the auction winner. We show that the model components are nonparametrically identified from the contract prices, the bids, and negotiated transfers, and use the model to analyze CalTrans auctions of highway procurement contracts. Our estimates shed light on how contractors respond strategically to contractual incompleteness, and what determines the size of the holdup in highway procurement projects. We find that fixed-price contracts mostly yield higher surplus for the buyer over cost-plus contracts when there is uncertainty about the final design. A direction for future research is to use the method we propose to evaluate the impact of incomplete contracts on the efficiency of mechanisms, for example, whether revenue equivalence still holds once incompleteness occurs. Although our method is introduced in the context of procurement auctions, it might be extended to other formats of precontractual competition.

Appendix

This appendix contains proofs omitted in the body of the paper.

□ **Nash Bargaining in postauction negotiation.** Let v denote the auction price and c denote the auction winner's *ex post* cost for implementing the contract under the initial design x. The disagreement value (a.k.a., reservation value) for the auction winner is

$$d_c \equiv v - c,$$

whereas the disagreement value for the buyer is

$$d_b \equiv \pi - v,$$

with π being the social surplus under the design x. With a new feasible design x^*, the *ex post* total net social surplus to be shared among the buyer and the auction winner is

$$u_0 \equiv \pi^* - a - c,$$

where π^* is the social surplus under x^* and a is the *incremental* costs for delivering the contract under x^* (in addition to the cost c for delivering the contract under x). With γ denoting the bargaining power of the contractor, the Nash Bargaining solution is characterized by

$$\max_{u_c, u_b} (u_c - d_c)^{\gamma} (u_b - d_b)^{1-\gamma} \text{ s.t. } u_b + u_c \le u_0.$$

By a standard argument,

$$u_c \equiv \gamma(u_0 - d_b) + (1 - \gamma)d_c = \gamma s + v - c$$

and

$$u_b \equiv u_0 - u_c = (1 - \gamma)s + \pi - v,$$

where $s \equiv \pi^* - \pi - a$ is the net incremental surplus. As stated in the text, we maintain that the contractor covers the incremental costs a as they arise in construction, whereas the incremental surplus $\pi^* - \pi$ is eventually accrued to the buyer. Then, the negotiated transfer y needs to satisfy:

$$d_c + y - a = u_c \text{ and } d_p + \pi^* - \pi - y = u_b,$$

which is equivalent to

$$y - a = \gamma s \text{ and } \pi^* - \pi - y = (1-\gamma)s.$$

This proves that the negotiated transfer is as characterized in Section 2.

□ **Existence of symmetric monotone psPBE.** Our first step is to link the contractor beliefs to their bidding strategies in the auction stage. Denote a generic contractor belief about the new design conditional on the initial announcement by $\lambda : \mathcal{X} \times \mathcal{X} \to [0,1]$. Let $\delta(x;\lambda) \equiv E_\lambda[\gamma s_+(x, X^*)|X = x]$, and the notation $E_\lambda(.)$ is a reminder that X^* is integrated out with respect to the belief $\lambda(X^*|X)$. Let $G(\cdot \mid x;\lambda)$ denote the distribution of contractors' adjusted costs $\widetilde{C}_i(\lambda) \equiv C_i - \delta(x;\lambda)$ conditional on $X = x$ and λ. That is,

$$G(t \mid x;\lambda) \equiv \Pr\{C_i - \delta(x;\lambda) \le t \mid X = x\}.$$

A standard argument such as that in Krishna (2009) show that a symmetric monotone pure-strategy Bayesian Nash Equilibrium exist in a procurement auction where contractors' costs are drawn from $G(\cdot \mid X;\lambda)$. More specifically, let $\widetilde{G} \equiv 1 - G$ so that $\widetilde{G}(c \mid x;\lambda)^{N-1} \equiv \Pr\{\min_{j\neq i} \widetilde{C}_j(\lambda) \ge c \mid x;\lambda\}$ and $1 - \widetilde{G}(c \mid x;\lambda)^{N-1} \equiv \Pr\{\min_{j\neq i} \widetilde{C}_j \le c \mid x;\lambda\}$. For all $i = 1, 2, \cdots, N$, $c \in \mathcal{C}$, and $x \in \mathcal{X}$,

$$\beta_i(c, x;\lambda) = \frac{\int_c^{\bar{c}} s d[1 - \widetilde{G}(s \mid x;\lambda)^{N-1}]}{\widetilde{G}(c \mid x;\lambda)^{N-1}}, \tag{A1}$$

where $\bar{c}$ denotes the supremum of $\mathcal{C}$.

Next, we link contractor beliefs to the buyer's expected payment and the buyer's choice of initial design. It follows from (A1) that the buyer's expected payment when contractors hold belief λ is

$$\begin{aligned}\varphi_\lambda(x) \equiv \varphi(x;\beta(.,x;\lambda)) &= N\int_0^{\bar{c}} \beta_i(c,x;\lambda) \Pr\left\{\min_{j\neq i} \widetilde{C}_j \ge c \,\middle|\, X = x;\lambda\right\} dG(c \mid x;\lambda)\\ &= N\int_0^{\bar{c}} sd\left[1 - \widetilde{G}(s \mid x;\lambda)^{N-1}\right] - N\int_0^{\bar{c}} s\widetilde{G}(s \mid x;\lambda) d\left[1 - \widetilde{G}(s \mid x;\lambda)^{N-1}\right],\end{aligned}$$

where the first term is $NE[\min_{j\neq i} \widetilde{C}_j(\lambda)|X = x;\lambda]$; and the second term is

$$\begin{aligned}&N\int_0^{\bar{c}} s\widetilde{G}(s \mid x;\lambda)(N-1)\widetilde{G}(s \mid x;\lambda)^{N-2} g(s \mid x;\lambda) ds\\ &= (N-1)\int_0^{\bar{c}} sd\left[1 - \widetilde{G}^N(s \mid x;\lambda)\right] = (N-1)E\left[\min_i \widetilde{C}_i(\lambda)\,\middle|\, X = x;\lambda\right].\end{aligned}$$

Because $\widetilde{C}_i(\lambda) \equiv C_i - \delta(x;\lambda)$, we can write the right-hand side of (A2) as $\sigma(x) - \delta(x;\lambda)$, where

$$\sigma(x) \equiv NE[C^{(1:N-1)}|X = x] - (N-1)E[C^{(1:N)}|X = x],$$

with $C^{(m:n)}$ being the m-th smallest out of n independent draws from $F_{C|X}$. Given the contractor belief λ, the buyer's optimization problem is:

$$\alpha(\tilde{x};\lambda) \equiv \operatorname*{argmax}_{x\in\mathcal{X}} \{\pi(x) - \sigma(x) + \mu(x,\tilde{x}) + \delta(x;\lambda)\},$$

where π, σ, and μ are belief-free in that they depend only on model primitives that are common knowledge to both the buyer and contractors. We maintain that $E(\min_{i=1,\ldots,N} C_i|X = x)$ is differentiable in x over $\mathcal{X}$ for all N.

Suppose $\alpha^* : \mathcal{X} \to \mathcal{X}$ is the buyer's strategy in a strictly monotone pure-strategy PBE, and is differentiable. By the restriction of consistent beliefs on the equilibrium path of initial announced design, $\lambda^*(.|x) = F_{X^*|\tilde{X}=\alpha^{*-1}(x)}(.)$ for all x on the support of $\alpha^*(\tilde{X})$. For $\alpha^*(.)$ to be the buyer's equilibrium strategy, it must be the case that for all $\tilde{x} \in \mathcal{X}$,

$$\pi'(\alpha^*(\tilde{x})) + \mu_1(\alpha^*(\tilde{x}),\tilde{x}) = \sigma'(\alpha^*(\tilde{x})) - \delta'(\alpha^*(\tilde{x});\lambda^*), \tag{A2}$$

where $\mu_1(z,\tilde{x}) \equiv \partial\mu(x,\tilde{x})/\partial x|_{x=z}$. Let

$$\Lambda(x,\tilde{x}) \equiv E[s_+(x,X^*) \mid \widetilde{X} = \tilde{x}] = \int_{\omega(x)} [\pi(x^*) - \pi(x) - a(x,x^*)]\, dF(x^*|\tilde{x}),$$

where $\omega(x) \equiv \{x^* : s(x,x^*) > 0\}$, as in the text. Note that the functional form of Λ depends on the common knowledge primitive $F_{X^*|\widetilde{X}=\tilde{x}}$ but not the belief of contractors. By construction, $\mu(x,\tilde{x}) = (1-\gamma)\Lambda(x,\tilde{x})$ and $\delta(x;\lambda^*) = \gamma\Lambda(x,\alpha^{*-1}(x))$. Thus,

$$\delta'(\alpha^*(\tilde{x});\lambda^*) = \gamma\Lambda_1(\alpha^*(\tilde{x}),\tilde{x}) + \frac{\gamma\Lambda_2(\alpha^*(\tilde{x}),\tilde{x})}{\alpha^{*\prime}(\tilde{x})}, \tag{A3}$$

where Λ_1 (and Λ_2) denote partial derivatives of Λ with respect to its first (and second) argument, respectively. Note that the function form of σ and Λ are determined by model primitives $F_{C|X}$ and $F_{X^*|\tilde{X}}$, respectively, and do not depend on the belief of contractors. Substitute (A3) into (A2) to get

$$\alpha^{*\prime}(\tilde{x}) = \Psi(\alpha^*(\tilde{x}), \tilde{x}),$$

where $\Psi : \mathcal{X} \times \widetilde{\mathcal{X}} \to \mathbb{R}$ is defined as

$$\Psi(x, \tilde{x}) \equiv \frac{\gamma \Lambda_2(x, \tilde{x})}{\sigma'(x) - \pi'(x) - \Lambda_1(x, \tilde{x})}. \tag{A4}$$

We say a contractor's conditional belief λ is *ultra-pessimistic at* $x \in \mathcal{X}$ if it assigns no probability mass to new designs to the event that the new designs yield positive net surplus. That is,

$$\int 1\{x^* \in \omega(x)\} d\lambda(x^*|x) = 0,$$

where $\omega(x) \equiv \{x^* : s(x, x^*) > 0\}$, as in the text. In other words, a contractor with an ultra-pessimistic belief at x thinks $\Pr\{s(X, X^*) \leq 0 | X = x\} = 1$, which implies $\delta(x; \lambda) = 0$ for all $x \in \mathcal{X}$. Therefore, a buyer's objective function when contractors hold "ultra-pessimistic" beliefs is:

$$H^o(x, \tilde{x}) \equiv \pi(x) + (1 - \gamma)\Lambda(x, \tilde{x}) - \sigma(x),$$

which is the buyer's objective function when contractors hold an "ultra-pessimistic" belief that $\Pr\{s(X, X^*) \leq 0 \mid X = x\} = 1$ for all $x \in \mathcal{X}$. This is because $\delta(x; \lambda) = 0$ if the belief $\lambda(.|x)$ is ultra-pessimistic at x.

Assumption A.1. (Smoothness and Concavity) π, σ, and Λ are differentiable and bounded over their domains. For each $\tilde{x} \in \widetilde{\mathcal{X}}$, H^o is strictly concave in x over $\mathcal{X}$.

It follows from Assumption A.1 and the Theorem of Maximum that

$$\mathcal{X}_o \equiv \left\{ x : x = \operatorname*{argmax}_{z \in \mathcal{X}} H^o(z, \tilde{x}) \text{ for some } \tilde{x} \in \mathcal{X} \right\}$$

is convex and compact in $\mathcal{X}$. Let x_h^o (and x_l^o) denote the supremum (and infimum) of $\mathcal{X}_o$; x_h (and x_l) denote the supremum (and infimum) of $\mathcal{X}$; and $\tilde{x}_h$ (and $\tilde{x}_l$) denote the supremum (and infimum) of $\widetilde{\mathcal{X}}$.

Assumption A.2. (a) Ψ is continuous over $\mathcal{X} \times \widetilde{\mathcal{X}}$; and there exists $L \in \mathbb{R}_{++}$ such that for all $(\tilde{x}, x, x')$ in $\widetilde{\mathcal{X}} \times \mathcal{X} \times \mathcal{X}$, $|\Psi(x', \tilde{x}) - \Psi(x, \tilde{x})| \leq L|x' - x|$. (b) $M(\tilde{x}_h - \tilde{x}_l) \leq x_h - x_l^o$, where $M \equiv \sup_{(x,\tilde{x})\mathcal{X}\times\widetilde{\mathcal{X}}} \Psi(x, \tilde{x}) < \infty$.

Define the following ordinary differential equation with an initial condition:

$$\alpha'(\tilde{x}) = \Psi(\alpha(\tilde{x}), \tilde{x}) \text{ and } \alpha(\tilde{x}_l) = x_l^o. \tag{A5}$$

It follows from Assumption A.2 and Picard's Local Existence Theorem that there exists a solution to (A5) over $\widetilde{\mathcal{X}} \equiv [\tilde{x}_l, \tilde{x}_h]$. The range for the solution is a subset of $[x_l^o, x_h]$ under Assumption A.2.

Assumption A.3. (Monotonicity) (a) For any pair $x' > x$ on $\mathcal{X}$, $F_{C|X=x'}$ first-order stochastically dominates $F_{C|X=x}$. (b) For each $x \in \mathcal{X}$, $\Lambda(x, \tilde{x})$ is increasing in $\tilde{x}$. (c) For each $\tilde{x} \in \widetilde{\mathcal{X}}$, $\pi(x) + \Lambda(x, \tilde{x})$ is nonincreasing in x.

Lemma A.1. Under Assumptions A.1, A.2, and A.3, there exists a strictly monotone solution $\alpha^* : \widetilde{\mathcal{X}} \to \mathcal{X}$ to (A5).

Proof of Lemma A.1. We first show that if the solution exists, it must be strictly monotone over its domain. By construction,

$$\sigma(x) \equiv N \int_0^{\bar{c}} [1 - F_{C|X=x}(s)]^{N-1} ds - (N-1) \int_0^{\bar{c}} [1 - F_{C|X=x}(s)]^N ds.$$

Therefore,

$$\sigma'(x) = -N(N-1) \int_0^{\bar{c}} \frac{\partial F_{C|X}(s|x)}{\partial x} [1 - F_{C|X}(s)]^{N-2} F_{C|X}(s) ds > 0,$$

where the last line is due to Assumption A.3-(a) (that $F_{C|X}$ is stochastically increasing in x). Together with Assumption A.3-(c), this implies the denominator in the definition of Ψ in (A4) is positive for all $x, \tilde{x}$. Assumption A.3-(b) then implies the numerator in (A4) is also positive. Thus, the solution to (A5), if it exists, must be increasing over $\widetilde{\mathcal{X}}$. By the Picard's Existence Theorem, there exists a solution α^* to (A5) over $\widetilde{\mathcal{X}} \equiv [\tilde{x}_l, \tilde{x}_h]$ under Assumption A.2. □

Assumption A.4. (Support Conditions) (a) For any $(x, \tilde{x}) \in \mathcal{X} \times \tilde{\mathcal{X}}$, the integral $\int 1\{x^* \in \omega(x)\} dF_{X^*|\tilde{X}=\tilde{x}}(x^*) > 0$. (b) For some α^* that solves (A5), $\mathcal{X}_o \subseteq \mathcal{X}_e \equiv \{x : x = \alpha^*(\tilde{x}) \text{ for some } \tilde{x} \in \tilde{\mathcal{X}}\}$.

We now define a symmetric monotone pure-strategy Perfect Bayesian Equilibrium as follows:

$$\alpha^* \text{ solves (A5)}; \quad \beta^* \text{ is defined in (A1) with beliefs } \lambda^*;$$
$$\lambda^*(.|x) = F_{X^*|\tilde{X}=\alpha^{*-1}(x)}(.) \text{ for } x \in \mathcal{X}_e; \text{ and } \lambda^*(.|x) \text{ is ultra-pessimistic for } x \notin \mathcal{X}_e. \tag{A6}$$

Proposition A.1. Under Assumptions A.1–A.4, the profile of strategies and beliefs in (A6) exists, and is a symmetric monotone pure-strategy Perfect Bayesian Equilibrium.

Proof of Proposition A.1. Existence of the profile $(\alpha^*, \beta^*, \lambda^*)$ in (A6) follows from Lemma A.1. It only remains to show that such a profile $(\alpha^*, \beta^*, \lambda^*)$ indeed forms a symmetric monotone psPBE. That β^* is the contractors' best response given their beliefs λ^* is shown earlier in the text of this Appendix. That α^* is a solution to (A5) ensures that the buyer cannot make profitable (local) deviation over the equilibrium path of X. Also by definition, λ^* is consistent on the equilibrium support of $\alpha^*(\tilde{X})$.

It remains to check that the off-equilibrium beliefs in (A6) guarantee that for any $\tilde{x}$, there is no profitable deviation of the buyer from $\alpha^*(\tilde{x})$ to some $x' \notin \mathcal{X}_e$. To see this, first note that under Assumption A.4, $x' \notin \mathcal{X}_e$ implies $x' \notin \mathcal{X}_o$. Thus, there exists $x^o \in \mathcal{X}_o$ so that $H^o(x^o, \tilde{x}) \geq H^o(x', \tilde{x})$. By construction,

$$H^o(z, \tilde{x}) \leq \pi(z) + \mu(z, \tilde{x}) - \sigma(z) + \delta(z; \lambda^*),$$

where the inequality holds with equality for $z \notin \mathcal{X}_e$ (because $\delta(z; \lambda^*) = 0$ for such z due to ultra-pessimistic beliefs), and the inequality holds strictly for $z \in \mathcal{X}_e$ due to Assumption B4-(a). Because $x^o \in \mathcal{X}_o$ implies $x^o \in \mathcal{X}_e$ under Assumption A.4-(b),

$$\begin{aligned}
&\pi(x^o) + \mu(x^o, \tilde{x}) - \sigma(x^o) + \delta(x^o; \lambda^*) \\
&> H^o(x^o, \tilde{x}) \geq H^o(x', \tilde{x}) \\
&= \pi(x') + \mu(x', \tilde{x}) - \sigma(x') + \delta(x'; \lambda^*).
\end{aligned}$$

This implies x' could not be the buyer's optimal choice of initial design given the belief λ^*. □

Discussion of Assumption A.4-(b). We conclude this part of the Appendix with primitive conditions that are sufficient for the second support condition in Assumption A.4. By construction and the monotonicity of the solution to (A5), $\alpha^*(\tilde{x}_l) = x_l^o = \inf \mathcal{X}_e = \inf \mathcal{X}_o$, and

$$x_h^e - x_l^e = x_h^e - x_l^o = \int_{\tilde{x}_l}^{\tilde{x}_h} \Psi(\alpha^*(\tilde{x}), \tilde{x}) d\tilde{x}.$$

Define $\alpha_o(\tilde{x}) \equiv \operatorname{argmax}_{x \in \mathcal{X}} H^o(x, \tilde{x})$ for all $\tilde{x} \in \tilde{\mathcal{X}}$. Suppose the solution is in the interior of $\mathcal{X}$ for all $\tilde{x}$. By the Implicit Function Theorem,

$$\frac{d}{d\tilde{x}} \alpha_o(\tilde{x}) = -\frac{(1-\gamma)\Lambda_{12}(\alpha_o(\tilde{x}), \tilde{x})}{H_{11}^o(\alpha_o(\tilde{x}), \tilde{x})}$$

for all $\tilde{x}$ such that $\operatorname{argmax}_{x \in \mathcal{X}} H^o(x, \tilde{x})$ is in the interior of $\mathcal{X}$, where H_{11}^o denotes the second-order derivative with respect to its first argument, and Λ_{12} denotes the cross-derivatives. Assume Λ_{12} is positive over its domain. Then, $x_h^o - x_l^o$ is bounded above by $\int_{\tilde{x}_l}^{\tilde{x}_h} \alpha_o'(t) dt$. Therefore, for Assumption A.4-(b) to hold, it suffices to have

$$-\frac{(1-\gamma)\Lambda_{12}(x, \tilde{x})}{H_{11}^o(x, \tilde{x})} \leq \frac{(1-\gamma)\Lambda_2(x, \tilde{x})}{\sigma'(x) - \pi'(x) - \Lambda_1(x, \tilde{x})}$$

for all $(x, \tilde{x}) \in \mathcal{X} \times \tilde{\mathcal{X}}$. For example, the inequality in the display above holds when π and a are close to being linear in x. In this case, Λ_{12} and therefore the left-hand side will be positive and close to zero, whereas the right-hand side is positive and not close to zero in general.[12]

[12] To see this, note that by construction,

$$\Lambda_1(x, \tilde{x}) \equiv \frac{\partial}{\partial x} \int_{\omega(x)} [\pi(x^*) - \pi(x) - a(x, x^*)] \, dF(x^*|\tilde{x}) = \int_{\omega(x)} \left[-\pi'(x) - a_1(x, x^*)\right] dF(x^*|\tilde{x}).$$

□ **Proofs in Section 3.** Let $\mathcal{X}_e \equiv \{x : x = \alpha^*(\tilde{x}) \text{ for some } \tilde{x} \in [0,1]\}$ denote the equilibrium support of the initial design, which is convex and compact under assumptions of our model by the Theorem of Maximum.

Lemma A.2. (a) The inverse of a contractor's equilibrium bidding strategy in (2) *is*

$$\beta^{*-1}(b,x) = \tilde{\beta}^{-1}(b,x) + \delta(x;\lambda^*) \tag{A7}$$

for all x and b on the support of bids in symmetric monotone psPBE. (b) For all x on the equilibrium support of initial designs, $\tilde{\beta}^{-1}(.,x)$ is identified from the distribution of the auction price (the winning bid) V.

Proof of Lemma A.2. By changing variables between contractor costs C_i and quoted prices in the procurement auctions P_i, we can write the inverse of contractors' equilibrium bidding strategy as:

$$\tilde{\beta}^{-1}(p,x) = p - \frac{1}{N-1}\frac{1-F_{P|X=x}(p)}{f_{P|X=x}(p)} \text{ for all } x \in \mathcal{X}_e, \tag{A8}$$

where the right-hand side is identified from the distribution of quoted prices conditional on the initial design.

In fact, $\beta^{-1}(.,x)$ is identified for all $x \in \mathcal{X}_e$, even if the data only report the contract prices determined in the procurement auctions. By definition, the contract price V is the minimum of prices quoted by participating contractors $(P_i : i = 1,2,\ldots,N)$. Because contractor costs are i.i.d. conditional on X, their bids in a symmetric monotone psPBE are i.i.d. given X. Thus,

$$1 - F_{V|X} = \left(1 - F_{P|X}\right)^N \Rightarrow F_{P|X} = 1 - \left(1 - F_{V|X}\right)^{\frac{1}{N}}.$$

Substituting this into (A8) and using $f_{V|X=x}(b) = \frac{\partial}{\partial t}F_{V|X=x}(t)|_{t=b}$, we get

$$\tilde{\beta}^{-1}(p,x) = p - \frac{N}{N-1}\frac{1-F_{V|X=x}(p)}{f_{V|X=x}(p)} \text{ for all } x \in \mathcal{X}_e,$$

where the right-hand side is identified using the distribution of auction prices conditional on each $x \in \mathcal{X}_e$. □

For each $x \in \mathcal{X}$, define $\omega(x) \equiv \{t \in \mathcal{X} : s(x,t) > 0\}$. That is, $\omega(x)$ is the set of new designs that could be adopted to replace the initial design x.

Assumption A.5. For each $x \in \mathcal{X}$, $\omega(x)$ is a nondegenerate interval $(\omega_l(x), \omega_h(x))$. Both ω_l and ω_h are differentiable in x over $\mathcal{X}$.[13]

Under this assumption, $\mu(x,\tilde{x})$ is continuous in both arguments and is differentiable in x for each $\tilde{x}$. To see how this condition can be satisfied, suppose for any x, the net incremental surplus $s(x,x^*)$ is monotone (or concave or convex) in x^*. The Implicit Function Theorem then implies that this condition holds if s is continuously differentiable in both of its arguments.

Theorem A.1. Under Assumption A.5, the marginal effect of contract design on social surplus $\pi'(x)$ is identified for all $x \in \mathcal{X}_e$.

Proof of Theorem A.1. Rewrite (7) as:

$$\begin{aligned}&\varphi'(\alpha^*(\tilde{x});\beta^*) - \pi'(\alpha^*(\tilde{x}))\\ &= -\frac{\partial}{\partial x}\left[(1-\gamma)\pi(x)p(x,\tilde{x})\right]_{x=\alpha^*(\tilde{x})} + \frac{\partial}{\partial x}\left[\int_{\omega(x)}(1-\gamma)[\pi(t)-a(x,t)]dF_{X^*|\tilde{x}}(t)\right]_{x=\alpha^*(\tilde{x})}\end{aligned} \tag{A9}$$

for any $\tilde{x}$ with an interior solution, where $p(x,\tilde{x}) \equiv \Pr(s(X,X^*) > 0|X = x, \widetilde{X} = \tilde{x})$.

Recall for any $x \in \mathcal{X}_e$ and $t \in \omega(x)$, $y(x,t) = (1-\gamma)a(x,t) + \gamma[\pi(t) - \pi(x)]$. Hence, the second term on the right-hand side is

$$-\frac{\partial}{\partial x}\left[\pi(x)p(x,\tilde{x})\right]_{x=\alpha^*(\tilde{x})} + \frac{\partial}{\partial x}\left[\int_{\omega(x)}\{\pi(t) - y(x,t)\}\,dF_{X^*|\tilde{x}}(t)\right]_{x=\alpha^*(\tilde{x})}, \tag{A10}$$

[13] Our main identification result in Theorem A.1 holds when $\omega(x) \subseteq \mathcal{X}$ is nonconvex, that is, partitioned into disjoint intervals such as $(-\infty,\omega_1(x))$, $(\omega_2(x),\omega_3(x))$, $(\omega_4(x),+\infty)$.

and (A9), under Assumption C1, can be written as

$$\varphi'(\alpha^*(\tilde{x});\beta^*) - \pi'(\alpha^*(\tilde{x}))$$
$$= -\pi'(\alpha^*(\tilde{x}))p(\alpha^*(\tilde{x}),\tilde{x}) - \pi(\alpha^*(\tilde{x}))p_1(\alpha^*(\tilde{x}),\tilde{x}) + \frac{\partial}{\partial x}\left[\int_{\omega_l(x)}^{\omega_h(x)} \{\pi(t) - y(x,t)\}\, dF_{X^*|\tilde{x}}(t)\right]_{x=\alpha^*(\tilde{x})} \quad \text{(A11)}$$

for all $\tilde{x}$, where $p_1(z,\tilde{x}) \equiv \frac{\partial}{\partial x} p(x,\tilde{x})|_{x=z}$.

In what follows, let z be shorthand for $\alpha^*(\tilde{x})$ at each $\tilde{x}$. Then, by Assumption C1,

$$p_1(z,\tilde{x}) = f_{X^*|\tilde{x}}(\omega_h(z))\omega_h'(z) - f_{X^*|\tilde{x}}(\omega_l(z))\omega_l'(z).$$

Applying the Leibniz rule to the last term on the right-hand side of (A11), we can write (A11) as:

$$\begin{aligned}\varphi'(z;\beta^*) = {} & [1 - p(z,\tilde{x})]\pi'(z) + \pi(z)[f_{X^*|\tilde{x}}(\omega_l(z))\omega_l'(z) - f_{X^*|\tilde{x}}(\omega_h(z))\omega_h'(z)] \\ & + [\pi(\omega_h(z)) - y(z,\omega_h(z))]\, f_{X^*|\tilde{x}}(\omega_h(z))\omega_h'(z) - [\pi(\omega_l(z)) - y(z,\omega_l(z))]\, f_{X^*|\tilde{x}}(\omega_l(z))\omega_l'(z) \\ & - \int_{\omega_l(z)}^{\omega_h(z)} y_1(z,t) f_{X^*|\tilde{x}}(t)dt \end{aligned} \quad \text{(A12)}$$

for any $z \in \mathcal{X}_e$. Next, note that for all (x, x^*),

$$\begin{aligned}\pi(x^*) - y(x,x^*) &= \pi(x^*) - \gamma[\pi(x^*) - \pi(x)] - (1-\gamma)a(x,x^*) \\ &= \pi(x) + (1-\gamma)[\pi(x^*) - \pi(x) - a(x,x^*)] \\ &= \pi(x) + (1-\gamma)s(x,x^*).\end{aligned}$$

Because $s(x,x^*) = 0$ at $x^* = \omega_l(x)$ or $\omega_h(x)$, we have $\pi(x^*) - y(z,x^*) = \pi(z) + (1-\gamma)\cdot 0$ for all $z \in \mathcal{X}_e$ and $x^* = \omega_l(z)$ or $\omega_h(z)$. Substitute these into (A12) and cancel out several terms. Then, for all $\tilde{x} \in \widetilde{\mathcal{X}}$,

$$\pi'(\alpha^*(\tilde{x})) = [1 - p(\alpha^*(\tilde{x}),\tilde{x})]^{-1}\left(\varphi'(\alpha^*(\tilde{x});\beta^*) + \int_{\omega_l(\alpha^*(\tilde{x}))}^{\omega_h(\alpha^*(\tilde{x}))} y_1(\alpha^*(\tilde{x}),t) f_{X^*|\tilde{X}=\tilde{x}}(t)dt\right). \quad \text{(A13)}$$

The right-hand side of (A13) consists of quantities that are directly identifiable in the data. First, because the distribution of $\tilde{X}$ is normalized to a standard uniform distribution, the monotonicity of the buyer strategy implies that $\alpha^*(\tilde{x})$ equals the $(100 \times \tilde{x})$-th percentile of the initial design announced in the data. For any $\tilde{x} \in \widetilde{\mathcal{X}}$ and $t \in \omega(\alpha^*(\tilde{x})) \equiv (\omega_l(\alpha^*(\tilde{x})), \omega_h(\alpha^*(\tilde{x})))$,

$$\begin{aligned} f_{X^*|\tilde{X}=\tilde{x}}(t) &= f_{X^*|X=\alpha^*(\tilde{x})}(t) \\ &= f_{X^*|X^*\in\omega(\alpha^*(\tilde{x})),X=\alpha^*(\tilde{x})}(t) \times \Pr(X^* \in \omega(\alpha^*(\tilde{x}))|X = \alpha^*(\tilde{x})) \\ &= f_{X^*|D=1,X=\alpha^*(\tilde{x})}(t) \times \Pr(D = 1|X = \alpha^*(\tilde{x})),\end{aligned}$$

where the first equality is due to monotonicity of α^* and the second and third equalities hold because "$D = 1$ and $X = x$" if and only if "$X^* \in \omega(x)$ and $X = x$" for all $x \in \mathcal{X}_e$. Second, note $\varphi'(\alpha^*(\tilde{x});\beta^*) \equiv \frac{\partial}{\partial x}\varphi(x;\beta^*)|_{x=\alpha^*(\tilde{x})}$ by definition and is identified for all $\tilde{x} \in (0,1)$. Third, $y_1(\alpha^*(\tilde{x}),t) \equiv \frac{\partial}{\partial x} y(x,t)|_{x=\alpha^*(\tilde{x})}$ is identified for all $\tilde{x} \in (0,1)$ and t such that $t \in \omega(\alpha^*(\tilde{x}))$. Finally,

$$p(\alpha^*(\tilde{x}),\tilde{x}) \equiv \Pr\{D = 1 \mid X = \alpha^*(\tilde{x}), \tilde{X} = \tilde{x}\} = \Pr\{D = 1 \mid X = \alpha^*(\tilde{x})\} \equiv p^*(\alpha^*(\tilde{x})),$$

where the equality is due to the monotonicity of α^*. This implies the right-hand is identifiable from the data, because $\alpha^*(\tilde{x})$ equals the $100 \times \tilde{x}$-th percentile of X, given our normalization of the marginal distribution of $\tilde{X}$. It then follows that $\pi'(\cdot)$ is identified over $\mathcal{X}_e$. Using a location normalization $\pi(\underline{x}) = \pi_0$ where $\underline{x}$ is the infimum of $\mathcal{X}_e$, we obtain a solution for π:

$$\pi(x) = \pi_0 + \int_{\underline{x}}^{x}\left\{[1 - p^*(z)]^{-1}\left(\varphi'(z;\beta^*) + \int_{\omega(z)} y_1(z,t) f_{X^*|X=z}(t)dt\right)\right\} dz,$$

for all $x \in \mathcal{X}_e$. □

Corollary A.1. (Theorem A.1) Suppose that Assumption A.5 in the Appendix, conditions A1 and A2 in the text hold and π is differentiable. Then, (a) $a(x,x^*)$ is identified for all (x,x^*) with $x \in \mathcal{X}_e$ and $s(x,x^*) > 0$, and $\pi(x)$ is identified for all $x \in \mathcal{X}_s$; and (b) the holdup $\delta(x;\lambda^*)$ and the cost distribution $F_{C|X=x}$ are identified for all $x \in \mathcal{X}_e$.

Proof of Corollary A.1. Part (a). That γ is identified under conditions A1 and A2 in the text is shown in the text of Section 3. The social surplus $\pi(x)$ is identified for all $x \in \mathcal{X}_e$, whereas $a(x,t)$ is identified for all $x \in \mathcal{X}_e$ and $t \in \omega(x) \cap \mathcal{X}_e$. Next, consider any $x \in \mathcal{X}_e$ and $t \in \omega(x)$ (but t may not belong to $\mathcal{X}_e$). Let y_j denote the partial derivative of $y(.,.)$ with respect to its j-th argument. It follows from (1) that

$$y_1(x,t) = -\gamma\pi'(x) + (1-\gamma)a_1(x,t);$$
$$y_2(x,t) = \gamma\pi'(t) + (1-\gamma)a_2(x,t).$$

Adding the two equations and using the fact that $a_1(x,t) + a_2(x,t) = 0$ under (A2), we get

$$\pi'(t) = \pi'(x) + [y_1(x,t) + y_2(x,t)]/\gamma \text{ for all } t \in \omega(x),$$

where the right-hand side is identified. By the smoothness of s in x^* given any x, the set $\mathcal{X}_e \cup \omega(x)$ is convex for all $x \in \mathcal{X}_e$. Therefore, under the location normalization $\pi(\underline{x}) = \pi_0$, $\pi(t)$, is identified for all $t \in \mathcal{X}_e \cup \omega(x)$. Thus, $a(x,t)$ is identified as

$$a(x,t) = \frac{y(x,t) - \gamma\pi(t) + \gamma\pi(x)}{1-\gamma}$$

for such a pair of (x,t) with $x \in \mathcal{X}_e$ and $t \in \omega(x)$.

Part (b). By definition, the holdup on the buyer conditional on any $x \in \mathcal{X}_e$ is

$$\delta(x;\lambda^*) = \gamma \int_{\omega(x)} [\pi(t) - \pi(x) - a(x,t)]\, dF_{X^*|X=x}(t), \tag{A14}$$

where we have used the fact that $F_{X^*|\tilde{X}=\alpha^{*-1}(x)}(.) = F_{X^*|X=x}(.)$ for all $x \in \mathcal{X}_e$. Part (a) showed that the integrand on the right-hand side of (A14) is identified for all $x \in \mathcal{X}_e$ and $t \in \omega(x)$. Besides, for any $x \in \mathcal{X}_e$, the density $f_{X^*|X=x}(t)$ is identified at any $t \in \omega(x)$ as

$$f_{X^*|X=x}(t) = f_{X^*|D=1,X=x}(t) \times \Pr\{D=1|X=x\}.$$

Thus, $\delta(x;\lambda^*)$ is identified for all $x \in \mathcal{X}_e$. It then follows from Lemma A.2 that the inverse of bidding strategy $\beta^*(.,x)$, and therefore the cost distribution $F_{C|X=x}$, is identified for all $x \in \mathcal{X}_e$. □

□ **Buyer surplus in cost-plus and fixed-price contracts.** We now explain how to use estimates from Section 6 to calculate the buyer surplus under cost-plus and fixed-price contracts. Recall the expected surplus for buyers under a cost-plus contract is

$$E[\pi(X_j^*, job_j)|w_j] - E(C_{j,i}^* + \zeta_{j,i}|w_j), \tag{A15}$$

where i denotes the winner, and $w_j \equiv (auti_j, afri_j, adis_j, nbid_j, job_j)$.

By construction, *ex ante* costs from contractor i is

$$E(C_{j,i}^*|\tilde{z}_{j,i}) = E(C_{j,i}|\tilde{z}_{j,i}) + E[a(X_j, X_j^*, \tilde{z}_{j,i})|\tilde{z}_{j,i}]. \tag{A16}$$

That is, the actual costs for implementing the new design X^* is decomposed into the sum of costs based on the initial design $C_{j,i}$ and the incremental costs $a(X_j, X_j^*, \tilde{z}_{j,i})$. We assume X_j^* and the initial design in equilibrium $X_j = \alpha^*(\tilde{X}_j)$ are independent from the contractor-specific information once conditional on the project-level characteristic job_j in $\tilde{z}_{j,i}$.

Our first step is to estimate the expected profit margin quoted by the winner in equilibrium. We begin by estimating the *ex ante* actual costs by the mean of $\hat{c}_{j,i} + \hat{a}_{j,i}$ conditional on $\tilde{z}_{j,i}$, where $\hat{c}_{j,i}$ are estimated costs under the initial design in Section 4 and $\hat{a}_{j,i} \equiv a(x_j, x_j^*, \tilde{z}_{j,i}; \hat{\theta})$ are estimates for contractor-specific incremental costs in Section 4. For contracts with no transfers in the data, $\hat{a}_{j,i} = 0$ by construction. In what follows, we discretize the support of $\tilde{z}_{j,i}$ into disjoint bins, and estimate *ex ante* actual costs in (A16) by the sample average of $\hat{c}_{j,i} + \hat{a}_{j,i}$ conditional on discretized values of $\tilde{z}_{j,i}$. Denote the estimates by $\hat{\upsilon}_{j,i}$. In each auction j, we find contractors i and i' who have the two lowest estimates for *ex ante* actual costs, and calculate the winner's quote for profit margin in equilibrium as $\hat{\upsilon}_{j,i'} - \hat{\upsilon}_{j,i}$.

We then estimate the second term in (A15) by the average of $\hat{\upsilon}_{j,i'}$ across contracts conditional on discretized values of w_j (because $C_{j,i}^* + \zeta_{j,i} = C_{j,i'}^*$ in equilibrium). To estimate the first term in (A15), we calculate $\hat{\pi}_j^* \equiv \pi(x_j^*, job_j; \hat{\theta})$ for auctions with transfers; and $\hat{\pi}_j^* \equiv \pi(x_j, job_j; \hat{\theta})$ otherwise. We then calculate the average of $\hat{\pi}_j^*$ conditional on (discretized) values of w_j. The sum of these two conditional averages are our estimates for the buyer surplus in (A15).

Next, we turn to the estimation of buyer surplus in equilibrium under fixed-price contracts, which is equal to

$$E[\pi(\alpha^*(\tilde{X}_j), job_j) + \mu(\alpha^*(\tilde{X}_j), \tilde{X}_j, W_j) - \varphi(\alpha^*(\tilde{X}_j), W_j; \beta^*)|W_j = w_j],$$

where μ and φ are defined in Section 2. The first term $E[\pi(\alpha^*(\tilde{X}_j), job_j)|w_j]$ is estimated by the average of $\hat{\pi}_j \equiv \pi(x_j, job_j; \hat{\theta})$ conditional on (discretized) values of w_j. To estimate $E[\mu(\alpha^*(\tilde{X}_j), \tilde{X}_j, W_j)|w_j]$, we first calculate $\hat{m}_j \equiv \widehat{\Delta}_{j,i}\hat{q}_j[1-\gamma(w_j;\hat{\varrho})]/\gamma(w_j;\hat{\varrho})$ for each contract j, with i being the winning contractor, and then take the average of $\hat{m}_{j,i}$ across contracts with w_j.[14] The last term $E[\varphi(\alpha^*(\tilde{X}_j), W_j; \beta^*)|w_j]$ is estimated by a kernel regression of auction prices given $W_j = w_j$. Alternatively, if components in w_j are discrete, we can estimate it by a simple sample average.

[14] We use $\widehat{\Delta}_{j,i} \times \hat{q}_j$ as an estimator for $\delta(.;\lambda^*)$, and use the fact that $y - a = \gamma s$ in the Nash Bargaining solution.

References

ARYAL, G., PERRIGNE, I. AND QUANG, V. "Identification of Insurance Models with Multidimensional Screening." *SSRN 2094995*, 2012.

ATHEY, S. AND HAILE, P.A. "Nonparametric Approaches to Auctions." *Handbook of Econometrics*, Vol. 6 (2007), pp. 3847–3965.

BAJARI, P., HOUGHTON, S., AND TADELIS, S. "Bidding for Incomplete Contracts: An Empirical Analysis of Adaptation Costs." *American Economic Review*, Vol. 104 (2014), pp. 1288–1319.

BAJARI, P., MCMILLAN, R., AND TADELIS, S. "Auctions versus Negotiations in Procurement: An Empirical Analysis." *Journal of Law, Economics, and Organization*, Vol. 25 (2009), pp. 372–399.

CALIFORNIA DEPARTMENT OF TRANSPORTATION. *Project Risk Management Handbook: A Scalable Approach*. Sacramento, California: State of California, 2012.

CALIFORNIA DEPARTMENT OF TRANSPORTATION. *Highway Design Manual*. Sacramento, California: State of California, 2015.

CROCKER, K.J. AND REYNOLDS, K.J. "The Efficiency of Incomplete Contracts: An Empirical Analysis of Air Force Engine Procurement." *RAND Journal of Economics*, Vol. 24 (1993), pp. 126–146.

DE SILVA, D.G., DUNNE, T., KOSMOPOULOU, G., AND LAMARCHE, C. "Project Modifications and Bidding in Highway Procurement Auctions." Federal Reserve Bank of Atlanta Working Paper Series, no. 2015-14, 2016.

D'HAULTFOEUILLE, X. AND FÉVRIER, P. "Identification and Estimation of Incentive Problems: Adverse Selection." Unpublished Manuscript, CREST-INSEE. no. 1216, 2007.

ELYAKIME, B., LAFFONT, J.J., LOISEL, P., AND VUONG, Q. "Auctioning and Bargaining: An Econometric Study of Timber Auctions with Secret Reservation Prices." *Journal of Business & Economic Statistics*, Vol. 15 (1997), pp. 209–220.

GOWRISANKARAN, G., NEVO, A., AND TOWN, R. "Mergers When Prices Are Negotiated: Evidence from the Hospital Industry." *American Economic Review*, Vol. 105 (2014), pp. 172–203.

GRENNAN, M. "Price Discrimination and Bargaining: Empirical evidence from medical devices." *American Economic Review*, Vol. 103 (2013), pp. 145–177.

GROSSMAN, S.J. AND HART, O.D. "The Costs and Benefits of Ownership: A Theory of Vertical and Lateral Integration." *Journal of Political Economy*, Vol. 94 (1986), pp. 691–719.

GUERRE, E., PERRIGNE, I., AND VUONG, Q. "Optimal Nonparametric Estimation of First-Price Auctions." *Econometrica*, Vol. 68 (2000), pp. 525–574.

HART, O. AND MOORE, J. "Property Rights and the Nature of the Firm." *Journal of Political Economy*, Vol. 98 (1990), pp. 1119–1158.

HO, K. AND LEE, R.S. "Insurer Competition in Health Care Markets." *Econometrica*, Vol. 85 (2017), pp. 379–417.

ICHIMURA, H. "Semiparametric Least Squares (SLS) and Weighted SLS Estimation of Single-Index Models." *Journal of Econometrics*, Vol. 58 (1993), pp. 71–120.

JUNG, H., KOSMOPOULOU, G., LAMARCHE, C., AND SICOTTE, R. "Strategic Bidding and Contract Renegotiation." International Economic Review, forthcoming, 2018.

KLEIN, B., CRAWFORD, R.G., AND ALCHIAN, A.A. "Vertical Integration, Appropriable Rents, and the Competitive Contracting Process." *Journal of Law and Economics*, Vol. 21 (1978), pp. 297–326.

LARSEN, B. "The Efficiency of Dynamic, Post-Auction Bargaining: Evidence from Wholesale Used-Auto Auctions." NBER Working Paper, w20431, 2014.

LEWIS, G. AND BAJARI, P. "Moral Hazard, Incentive Contracts, and Risk: Evidence from Procurement." *Review of Economic Studies*, Vol. 81 (2014), pp. 1201–1228.

MYERSON, R.B. "Two-Person Bargaining Problems with Incomplete Information." *Econometrica*, Vol. 52 (1984), pp. 461–487.

PERRIGNE, I. AND VUONG, Q. "Nonparametric Identification of a Contract Model with Adverse Selection and Moral Hazard." *Econometrica*, Vol. 79 (2011), pp. 1499–1539.

———. "On the Identification of the Procurement Model." *Economics Letters*, Vol. 114 (2012), pp. 9–11.

SIMON, H.A. "A Formal Theory of the Employment Relationship." *Econometrica*, Vol. 19 (1951), pp. 293–305.

SONG, U. "Nonparametric Identification and Estimation of a First-Price Auction Model with an Uncertain Number of Bidders." Working Paper, University of British Columbia, 2006.

SPENCER, B.J. "International Outsourcing and Incomplete Contracts." *Canadian Journal of Economics/Revue canadienne d'èconomique*, Vol. 38 (2005), pp. 1107–1135.

TIROLE, J. "Cognition and Incomplete Contracts." *American Economic Review*, Vol. 99 (2009), pp. 265–294.

WILLIAMSON, O. *The Economic Institutions of Capitalism: Firms, Markets, and Relational Contracting*. New York: Free Press, 1985).