Simple Menus of Cost-based Contracts with Monotone Optimal Effort*

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Abstract

This paper extends the Fixed-Price Cost-Reimbursement (FPCR) menu (Rogerson, 2003) by allowing the agent's cost-reducing optimal effort to be monotone in the agent's cost type. We show that the performance of the optimal FPCR menu relies crucially on the monotonicity of optimal effort. In particular, in an optimal FPCR menu if the optimal effort is increasing in type and only a portion of cost types are induced to exert effort, the performance of the optimal FPCR menu can be very poor relative to the fully optimal contract (Laffont and Tirole, 1986). Our results suggest that in designing an optimal FPCR menu it is important for the principal to take into account the cost structure or more exactly the monotonicity of optimal effort in type.

Keywords: Public Procurement, Fixed-Price Cost-Reimbursement Menu, Cost-Reducing Effort.
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1 Introduction

Because procurement by federal, state, and local government accounts for at least 10% of gross domestic product in the U.S. (Bajari and Tadelis, 2001), public procurement of goods and services has attracted much attention in the economics literature. The modern economic theories of procurement use mechanism design to model the procurement problem as one of ex ante asymmetric information coupled with moral hazard. Basically, the agent has private information about type of production cost that is unknown to the principal, and the principal screens the agent by offering a menu of contracts from which the agent selects a particular contract, thus (partially) revealing his private information. In practice, most contracts are variants of either fixed-price (FP) contracts for which the payment to the agent is a fixed price regardless of the agent's realized cost or cost-reimbursement (CR) contracts for which the agent is reimbursed exactly the realized cost. Prominent examples include the Indian customized software industry (Banerjee and Duflo, 2000), the U.S. Air Force engine procurement (Bajari and Tadelis, 2001), and the French transport industry (Gagnepain et al., 2013). One fundamental difference between FP contracts and CR contracts lies in the fact that FP contracts are high-incentive in the sense that under FP contracts the agent has incentive to make cost-reducing investment (effort) to reduce its production cost, while under CR contracts the agent has little incentive to exert cost-reducing effort.[1]

Motivated by the practical prevalence, the seminal paper Rogerson (2003) demonstrates that when the agent's cost type is uniformly distributed, a simple menu of contracts consisting of FP contracts and CR contracts captures at least three-fourths of the surplus that a fully optimal contract achieves, where the fully optimal contract is proposed by Laffont and Tirole (1986) to analyze the principal-agent problem of procurement and regulation, and can be implemented by the principal offering a menu consisting of a continuum of linear contracts.[2] Nevertheless, the powerful conclusions on the FPCR menu are associated with the key property that the agent's optimal effort is independent of cost type, i.e., the optimal effort is constant across cost type. This independence relies on the assumption that the cost function is additively separable in cost type and effort. However, the related empirical evidence suggests that the agent's optimal effort may increase in type, such as in the French transport industry by Gagnepain and Ivaldi (2002)

[1] The cost-reducing investment represented by the cost-reducting effort is typically used in procurement contracts (Ma, 2003).

[2] This fully optimal contract is too complex to be implemented in practice, whereas the aforementioned simple FPCR menu may secure a substantial share of the surplus that the fully optimal contract can secure.

and in the U.S. electricity industry by (Abito, 2017). Indeed, the monotone optimal effort is also used in the related theoretical literature (e.g., Laffont and Tirole (1993) and Ma (2003)).

In this paper, we analyze the performance of FPCR menus by allowing cost functions to be not additively separable in type and effort. We consider two specifications of cost in which the agent's optimal effort in an FPCR menu is weakly increasing in type. The first specification is a convex cost function and it has been widely used in the procurement contract theory such as Laffont and Tirole (1993) and Perrigne and Vuong (2012). The second specification is closer to the cost specification in Rogerson (2003) in the sense that for both specifications the production cost equals cost type when the agent exerts no effort. Under both cost specifications, we follow the literature to compute the reduction in the principal's expected cost under the optimal FPCR menu relative to the CR contract, which is used for the measure of contracts' performances. We also compute this relative reduction of the principal's expected cost for the fully optimal contract, and then analyze the performance of an optimal FPCR menu by calculating the ratio of relative reduction of the optimal FPCR menu to that of the fully optimal contract. The main finding of our paper is that the performance of an optimal FPCR menu can be very poor relative to the fully optimal contract. This is in contrast to the result in Rogerson (2003), where the ratio of relative reduction of the optimal FPCR menu to that of the fully optimal contract is at least 0.75.

Our analysis shows that the poor performance of an optimal FPCR menu is closely related to the increasing optimal effort in type. Specifically, we find that an optimal FPCR menu performs poorly when (1) the level of incentives for cost reduction is lower in an optimal FPCR menu in the sense that the optimal FPCR effort is induced only for a portion of types while in the fully optimal contract any type is induced to exert effort,[3] and the optimal effort is increasing in type for both contracts, or (2) both the fully optimal contract and the optimal FPCR menu induce only a portion of types to exert effort, and the optimal effort decreases with type for the former contract but increases with type for the latter contract, respectively. Intuitively, if the optimal FPCR effort increases with type, a less efficient type exerts more effort and incurs higher disutility, thus raising the fixed price and leaving more informational rent to the agent when the principal wants

[3]In the related literature, the FP contract is a higher-powered incentive scheme than the CR contract in terms of more incentives for cost-reducing effort in the FP contract than the CR contract (e.g., Rogerson (2003)). In the same spirit, we compare the level of incentives for cost reduction between the FPCR menu and the fully optimal contract in terms of the proportion of types associated with cost-reducing effort. Relatedly, in the literature on moral hazard a contract is high-powered if the compensation is very sensitive to the agent's performance (e.g., Roger (2016)).

to induce less efficient types to exert effort, and possibly making the FPCR menu worse when the optimal effort decreases with type in the fully optimal contract. Nevertheless, when both the optimal FPCR menu and the fully optimal contract induce all types to exert effort and the effort is increasing in type, then the performance of the optimal FPCR menu is close to that of a fully optimal contract.

We further explore the implication of shapes of cost functions to the relative performance of an optimal FPCR menu based on the relationship between monotonicity of optimal FPCR effort and shapes of cost functions. On the one hand, a submodular cost function tends to induce increasing optimal FPCR effort, and the relative performance of the optimal FPCR menu could be very poor based on the conjecture that the optimal FPCR menu might perform poorly with increasing effort. On the other hand, a supermodular cost function tends to induce decreasing optimal FPCR effort and the optimal FPCR menu could perform well based on the conjecture that the optimal FPCR menu might perform well with decreasing effort. Our results suggest that in designing an optimal FPCR menu it is important for the principal to take into account the cost structure or more exactly the monotonicity of optimal FPCR effort.

The remaining of this paper is organized as follows. Section 2 presents the basic model and analyzes the unique optimal FPCR menu. Section 3 compares the performance of the optimal FPCR menu with the fully optimal contract. In section 4, we calculate the relative performance of the optimal FPCR menu under a more comparable cost specification with Rogerson (2003) and extend our analysis to more general cost functions. Finally, Section 5 concludes and proofs are included in the Appendix.

2 The Model

A principal wishes to procure a project by offering a menu of two simple contracts to an agent. This menu consists of a fixed-price (FP) contract and a cost-reimbursement (CR) contract, and is therefore called fixed-price-cost-reimbursement (FPCR) menu. Formally, in an FPCR menu, a principal provides a fixed price p as the payment to the agent regardless of the agent's realized cost if the agent chooses an FP contract. If the agent chooses a CR contract, the agent is reimbursed exactly the realized cost. Let $c = c(\theta, e)$ be the agent's realized cost of production with θ and $e \geq 0$ being the agent's private cost type and unobservable cost-reducing effort, respectively[4]. Exerting effort e incurs disutility $\psi(e)$, and thus the agent's total cost for the project is $c(\theta, e) + \psi(e)$. An agent

[4]This general cost function specification is called a canonical model by Laffont (1994) with the unit production.

who chooses the CR contract only gets the realized cost of production c reimbursed, and hence makes zero profit under the assumption $\psi(0) = 0$, whereas an agent who chooses the FP contract makes profit $\pi(\theta, e) \equiv p - c(\theta, e) - \psi(e)$. For ease of providing our main findings under the monotone optimal effort, we first consider the following model specification.

Assumption 1 *(i) $c(\theta, e) = \beta(\theta - e)^2$ with $\beta > 0$, and $\psi(e) = e^2$. (ii) θ is uniformly distributed on its support $[\underline{\theta}, \bar{\theta}] \subset \mathbb{R}_+$.*

In part (i), the cost of production is a convex function of $(\theta - e)$, and the convexity of cost functions and disutility functions is widely assumed in related literature (e.g., Laffont and Tirole (1993) and Perrigne and Vuong (2011)). Under Assumption 1, the agent's profit is always zero due to $\psi(0) = 0$ in the CR contract, which implies that the optimal effort is always zero and the cost is reduced to $c = \beta\theta^2$. Note that the project is not shut down just because the effort is zero. The assumption of uniform distribution of the agent's type is imposed for the ease of the comparison with Rogerson (2003), where a uniform distribution is also imposed.

Let e^* be the optimal cost-reducing effort exerted by an agent who accepts the FP contract. According to the first-order condition of optimal effort, i.e., the marginal cost reduction of effort equals the marginal disutility of effort, we obtain the optimal effort $e^*(\theta) = \beta\theta/(\beta + 1)$ with the increasing monotonicity $e^{*'}(\theta) = \beta/(1 + \beta) > 0$. Hence, the less efficient agent is induced to exert more effort in an FP contract. In most of the existing studies on cost-based contracts, however, the optimal effort is independent of type, i.e., e^* is a constant. This is because $c(\theta, e)$ is specified to be additively separable in type and effort (e.g., $c(\theta, e) = \theta - e$ in Laffont and Tirole (1988), Laffont and Tirole (1990), Rogerson (2003), and Chu and Sappington (2007)). That is, the marginal cost of effort is the same for any cost type. In other words, when $c(\theta, e)$ is not additively separable in θ and e, the optimal effort may depend on type.

We turn to the problem of the principal. Let $\pi^*(\theta) = \pi(\theta, e^*(\theta))$. Using the envelope theorem leads to

$$\partial\pi^*(\theta)/\partial\theta = \partial\pi(\theta, e^*(\theta))/\partial\theta = -2\beta\theta/(1 + \beta) < 0.$$

Suppose that the principal offers the fixed-price p such that the cut-off type is θ^*, i.e., an agent with $\theta^* \in [\underline{\theta}, \bar{\theta}]$ is indifferent between choosing the FP contract and the CR contract. Hence, $\pi^*(\theta^*) = 0$. Therefore, if the agent's type is $\theta \leq \theta^*$, he accepts the FP contract and makes positive profit; if the type is $\theta > \theta^*$, he accepts the CR contract and earns zero profit. Let $F(\cdot)$ be the distribution function of cost type θ. For an FPCR menu with the cut-off type $\theta^* \in [\underline{\theta}, \bar{\theta}]$, the principal's expected cost for the project is:

$$\begin{aligned} C(\theta^*) &= \int_{\underline{\theta}}^{\theta^*} [c(\theta^*, e^*(\theta^*)) + \psi(e^*(\theta^*))]\, dF(\theta) + \int_{\theta^*}^{\bar{\theta}} c(\theta, 0) dF(\theta) \\ &= \frac{\beta\theta^{*2}}{\beta+1} F(\theta^*) + \beta \int_{\theta^*}^{\bar{\theta}} \theta^2 dF(\theta)\ , \end{aligned} \tag{1}$$

where the two terms in the second equality are the payment to types associated with FP contracts and CR contracts, respectively. The following lemma provides the unique optimal FPCR menu (or equivalently the unique optimal cut-off type θ^*) in terms of minimizing the principal's expected cost of the project. Define $\gamma \equiv \bar{\theta}/\underline{\theta} \in (1, \infty)$. A larger γ implies a more dispersed distribution of cost type.

Lemma 1 *Suppose Assumption 1 holds. In the optimal FPCR menu, (i) when* $0 < \beta < 2(\gamma - 1)/\gamma$*, if the agent's cost type is* $\theta \leq \theta^* = 2\underline{\theta}/(2 - \beta)$*, he chooses the FP contract with fixed price* $p = 4\beta(\beta + 1)^{-1}(2 - \beta)^{-2}\underline{\theta}^2$*, otherwise he chooses the CR contract; (ii) when* $2(\gamma - 1)/\gamma \leq \beta$*, the cut-off type is* $\theta^* = \bar{\theta}$*, and the agent of any cost type chooses the FP contract with fixed price* $p = \beta(\beta + 1)^{-1}\bar{\theta}^2$*.*

Part (i) of Lemma 1 states that when β is relatively small, i.e., $\beta < 2(\gamma - 1)/\gamma$, the optimal FPCR menu induces a positive portion of types to exert effort; and part (ii) states that when β is relatively large, i.e., $\beta \geq 2(\gamma - 1)/\gamma$, the optimal FPCR menu induces an agent of any type to choose the FP contract and exert effort. Note that β can be interpreted as the ratio of cost parameter to the parameter of disutility since the coefficient of $\psi(e) = e^2$ is assumed to be one, which implies that the net benefit (cost saving minus incurred disutility) of a certain level of effort is larger when β is larger. Accordingly, it is optimal for the principal to induce larger types to exert effort when β is larger. In particular, when β is sufficiently large, the principal will induce any type to accept the FP contract and exert effort.

To further understand the result in Lemma 1, we suppose that the cut-off type is an interior θ such that the fixed price is $p = p(\theta) = c(\theta, e^*(\theta)) + \psi(e^*(\theta)) = \beta\theta^2/(1 + \beta)$. If the principal wants to increase the cut-off type from θ to $\theta + \delta\theta$ with $\delta > 0$, the principal's expected cost associated with the FP contract goes up by

$$\Delta p = F(\theta + \delta\theta)p(\theta + \delta\theta) - F(\theta)p(\theta) = \frac{\beta}{1+\beta}\frac{\delta\theta^2}{\bar{\theta} - \underline{\theta}}(\delta^2\theta + 3\delta\theta + 3\theta - \delta\underline{\theta} - 2\underline{\theta}).$$

Meanwhile, the types belonging to $[\theta, \theta + \delta\theta]$ switch from the CR contract to the FP contract, and the induced saving of the principal's expected cost is

$$\Delta s = \int_{\theta}^{\theta+\delta\theta} s(t)dF(t) = \frac{\beta^2}{3(1+\beta)}\frac{\delta\theta^2}{\bar{\theta} - \underline{\theta}}(\delta^2\theta + 3\delta\theta + 3\theta),$$

where $s(\theta) = c(\theta, 0) - c(\theta, e^*(\theta)) - \psi(e^*(\theta)) = \beta^2\theta^2/(1+\beta)$ is the saving of the principal's payment from such a switch for type θ. It is easy to check that $\Delta s > \Delta p$ when β is relatively large, e.g., $\beta > 3$. Therefore, it is optimal for the principal to set a fixed price such that an agent of any type exerts effort when β is sufficiently large. Similarly, when β is relatively small, we have $\Delta p > \Delta s$, and then only a portion of more efficient types are induced to exert effort in the optimal FPCR menu.

3 Comparison of Performance

In this section, we compare the performance of the optimal FPCR menu with the fully optimal complex contract in Laffont and Tirole (1986, 1993) by following the procedure in Rogerson (2003). Laffont and Tirole (1986, 1993) characterize the fully optimal mechanism that can be implemented by the principal offering a menu consisting of a continuum of linear contracts. First, we follow the standard practice to obtain the optimal effort $e^*(\theta)$ in their fully optimal contract.

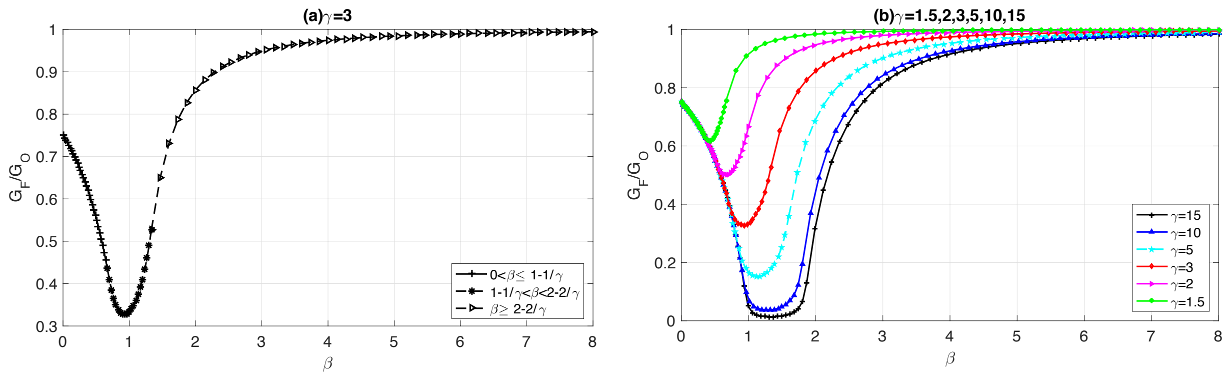
Lemma 2 *Under Assumption 1, the optimal cost-reducing effort* $e^*(\theta)$ *in the fully optimal contract is:*

$$e^*(\theta) = \begin{cases} 0, \textit{if } \theta \geq \underline{\theta}/(1-\beta) \textit{ and } \beta < 1. \\ \\ \frac{(\beta-1)\theta+\underline{\theta}}{\beta+1}, \textit{if } \theta \leq \underline{\theta}/(1-\beta) \textit{ and } \beta < 1, \textit{ or } \beta \geq 1. \end{cases} \tag{2}$$

In terms of the parameter γ, Lemma 2 can be restated as follows. When $0 < \beta < (\gamma-1)/\gamma$, $e^*(\theta) = 0$; when $\beta > (\gamma-1)/\gamma$, $e^*(\theta) = 0$ for $\theta \geq \underline{\theta}/(1-\beta)$ and $e^*(\theta) = [(\beta-1)\theta + \underline{\theta}]/(\beta+1)$ for $\theta \leq \underline{\theta}/(1-\beta)$. Lemma 2 states that for the fully optimal contract, as in the optimal FPCR menu, when β is relatively small, there exists a cut-off type such that an agent of more efficient types exerts effort; when β is relatively large, an agent of any type exerts effort. It is worth noting that under $\beta < 1$, the optimal effort in the fully optimal contract is weakly decreasing in type, i.e., more efficient agent exerts more effort. This is consistent with the conclusion in Laffont and Tirole (1986) but opposite to the optimal FPCR menu. Again, the project is not shut down just because the effort is zero.

Now we provide our main findings on the comparison of performances between the optimal FPCR menu and the fully optimal contract. Let G_S denote the reduction in the principal's expected cost under the optimal FPCR menu relative to the cost reimbursement contract, and G_O denote the reduction in the principal's expected cost under the

Figure 1: The relationship between G_S/G_O and β



fully optimal contract relative to the CR contract. We focus on the analysis of the ratio G_S/G_O, which describes the performance of the FPCR menu relative to the fully optimal contract.

Theorem 1 *Suppose Assumption 1 holds. For any given* γ*, (i) if* $0 < \beta \leq (\gamma - 1)/\gamma$*,* G_S/G_O *is strictly decreasing in* β *and* $G_S/G_O < 3/4$*; (ii) if* $(\gamma - 1)/\gamma < \beta < 2(\gamma - 1)/\gamma$*,* G_S/G_O *decreases to its minimum then increase in* β*; (iii) if* $\beta \geq 2(\gamma - 1)/\gamma$*,* G_S/G_O *is strictly increasing in* β *and* $\lim_{\beta \to \infty} G_S/G_O = 1$*.*

As expected, the performance of the optimal FPCR menu depends on distribution parameter γ and cost parameter β. To visualize our primary findings in Theorem 1, in Figure 1 we illustrate the relationship between G_S/G_O and β for a given γ ($\gamma = 3$) in panel (a) and for different values of γ in panel (b). As panel (a) shows, G_S/G_O strictly decreases in β when β is relatively small, and strictly increases to one in β when β is relatively large. As illustrated in panel (b), the minimum of G_S/G_O decreases as γ increases, which is similar to Rogerson (2003). The main difference from Rogerson (2003), however, lies in that the minimum of G_S/G_O maybe arbitrarily close to zero for a sufficiently large γ, which implies that the performance of the optimal FPCR menu maybe very poor relative to the fully optimal contract; while G_S/G_O is at least 0.75 in Rogerson (2003), which implies that the relative performance of the optimal FPCR menu is always desirable.

The dependence of G_S/G_O on the magnitude of β can be interpreted as follows. When β is relatively large, as the intuition for Lemma 1 states, the saving from cost-reducing effort dominates the incremental informational rent if an agent of any type switches from a CR contract to a FP contract. Hence, the optimal FPCR menu induces all types to exert effort (so does a fully optimal contract). Moreover, the net saving increases in β, thus leaving less information rent to the agent. Accordingly, the performance of the optimal FPCR menu is closer to the fully optimal contract as β is sufficiently large.

When β is small, as panel (a) shows, G_S/G_O decreases for $\beta < 1 - 1/\gamma$. In this case, both the optimal FPCR menu and the fully optimal contract induce a portion of types to exert effort. For those types of exerting effort, the optimal effort decreases in type for the fully optimal contract, while the optimal FPCR effort increases in type. The decreasing monotonicity of optimal effort leads to that the disutility of effort deceases in type for the fully optimal contract, thus suggesting smaller informational rent to an agent with a larger type. However, the increasing monotonicity of optimal FPCR effort implies that the disutility of effort increases in type. Therefore, if the principal wants to induce more types to exert optimal FPCR effort, the fixed price must increase by a larger amount to cover the increasing disutility of effort, thus leaving more information rent to all types already accepting the FP contract. In addition, for any type θ with FP contract the disutility of effort $\psi(e^*(\theta)) = (\beta\theta)^2/(1+\beta)^2$ is larger when β is larger. Therefore, the fixed price must also increase by a larger amount to cover the larger disutility of effort when β is larger, thus leaving more information rent to all types already accepting the FP contract. The above arguments suggests that G_S/G_O decreases in β.

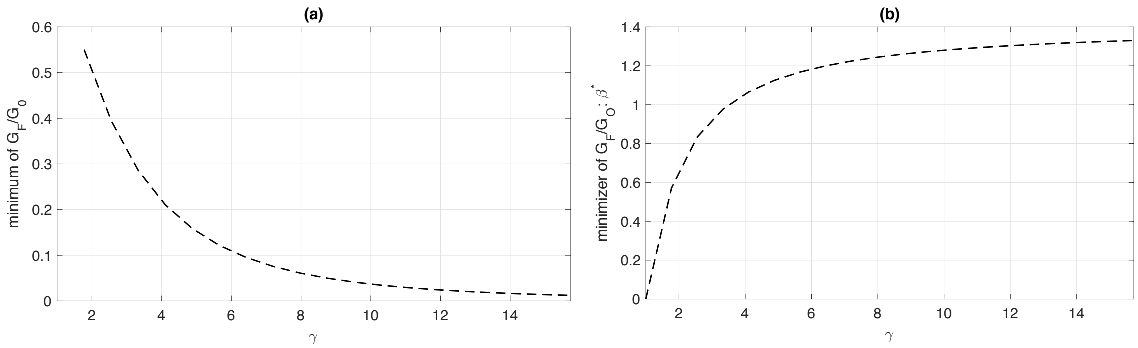
When β is in the interval $(1-1/\gamma, 2-2/\gamma)$, G_S/G_O reaches its minimum. Especially, this ratio can be arbitrarily close to zero for a large γ as illustrated in panel (b) of Figure 1. For instance, for $\gamma = 1.5$ the optimal FPCR menu at least secures 63% of the gain secured by the fully optimal contract, whereas the number is only 1.4% for $\gamma = 15$. Since tractably analytical expressions of the minimum of G_S/G_O and its minimizer are not available, we illustrate them in Figure 2. Panel (a) shows that the minimum of G_S/G_O converges to zero as γ goes to infinity, and panel (b) indicates that the minimizer of G_S/G_O, denoted by β^*, strictly increases in γ. In the following, we summarize that the performance of the optimal FPCR menu can be extremely poor relative to the fully optimal contract as long as $\beta \in (1-1/\gamma, 2-2/\gamma)$ for a given sufficiently large γ.

Theorem 2 *Suppose Assumption 1 holds. For any given $\gamma > 1$, if $\beta \in (1-1/\gamma, 2-2/\gamma)$, then G_S/G_O is strictly decreasing in γ and $\lim_{\gamma\to\infty} G_S/G_O = 0$.*

It is worth noting that the theorem above allows the interval $(1-1/\gamma, 2-2/\gamma)$ to vary with $\gamma > 1$, i.e., β belongs to a different interval for different $\gamma > 1$. Our result above does not require the existence of some fixed value of β such that $\beta \in \bigcap_{\gamma>1}(1-1/\gamma, 2-2/\gamma)$.

The extremely poor performance of the optimal FPCR menu for a large γ can be understood in terms of incentive. Note that the interval $(1-1/\gamma, 2-2/\gamma)$ converges to $(1, 2)$ for a sufficiently large γ, and we provide intuition for $\beta \in (1, 2)$. In this case, the optimal effort, if exerted, increases in type for both contracts. However, any type in the fully optimal contract is induced to exert effort while only a portion of more efficient types are induced to exert optimal FPCR effort. As a result, the optimal FPCR menu suffers

Figure 2: The relationship between minimum (and minimizer) of G_S/G_O and γ



loss of gain due to lower incentive compared with the fully optimal contract. Therefore, as γ goes up, although the gains of both contracts decrease, G_S decreases faster than G_O, which suggests that G_S/G_O goes to zero.

The results in Theorems 1 and 2 suggest that the relative performance of an optimal FPCR menu depends crucially on the increasing monotonicity of optimal effort. To summarize, our findings are that the optimal FPCR menu can perform poorly relative to the fully optimal contract if (1) optimal FPCR effort is induced only for a portion of types while all types are induced to exert effort in the fully optimal contract, or (2) both the fully optimal contract and the FPCR menu induce only a portion of types to exert effort while the optimal effort is decreasing and increasing in type for the optimal FPCR menu and the fully optimal contract, respectively. Nevertheless, when both the optimal FPCR menu and the fully optimal contract induce all types to exert effort, and the effort is increasing in type, their performances can be close to each other.

4 Alternative Cost Functions

We show that the optimal FPCR menu can perform very poorly in Section 3. A natural question is whether the poor performance can be inscribed to different cost specifications. In this section, we investigate how alternative cost specifications affect the performance of an optimal FPCR menu. First, we consider $c(\theta, e) = \theta(1 - e)$. It is comparable to $c(\theta, e) = \theta - e$ in Rogerson (2003) in the sense that $c(\theta, 0) = \theta$, i.e., the cost of production with no effort is the same for both cost specifications.[5] We find that the optimal FPCR menu can still perform very poorly. Second, we extend our analysis to the

[5]More generally, we can consider $c(\theta, e) = \beta\theta(1 - e)$. We normalize the coefficient β to be one for the purpose of comparison with Rogerson (2003).

general cost specification $c(\theta, e)$. We discuss that a submodular cost function tends to induce increasing optimal FPCR effort, thus suggesting a poor performance of the optimal FPCR menu; while a supermodular cost function may induce decreasing optimal FPCR effort, thus suggesting a desirable performance of the optimal FPCR menu. The following assumption is similar to Assumption 1.

Assumption 2 *(i)* $c(\theta, e) = \theta(1-e)$ *and* $\psi(e) = e^2$. *(ii)* θ *is uniformly distributed on its support* $[\underline{\theta}, \bar{\theta}] \subset [0, 2]$.

Under part (i) of Assumption 2, the optimal effort $e^*(\theta) = \theta/2$ increases with type in the FP contract. The support $[\underline{\theta}, \bar{\theta}] \subset [0, 2]$ in Part (ii) is implied by the requirement that $e^*(\theta) \geq 0$ and $c(\theta, e^*(\theta)) \geq 0$. we analyze the optimal FPCR menu in Lemma 3 and the optimal effort in the fully optimal contract in Lemma 4, respectively.

Lemma 3 *Suppose Assumption 2 holds. In an optimal FPCR menu, (i) when* $\underline{\theta} < 4 - 2\sqrt{3}$ *and* $\bar{\theta} > [\underline{\theta} + 2 - (4 + \underline{\theta}^2 - 8\underline{\theta})^{1/2}]/3$, *if the agent's cost type is* $\theta \leq \theta^* = [\underline{\theta} + 2 - (4 + \underline{\theta}^2 - 8\underline{\theta})^{1/2}]/3$, *he chooses the FP contract with fixed price* $p = \theta^* - \theta^{*2}/4$, *otherwise he chooses the CR contract; (ii) for all other* $\underline{\theta}$ *and* $\bar{\theta}$, *the optimal cut-off type is* $\theta^* = \bar{\theta}$, *and the agent of any type chooses the FP contract with fixed price* $p = \bar{\theta} - \bar{\theta}^2/4$.

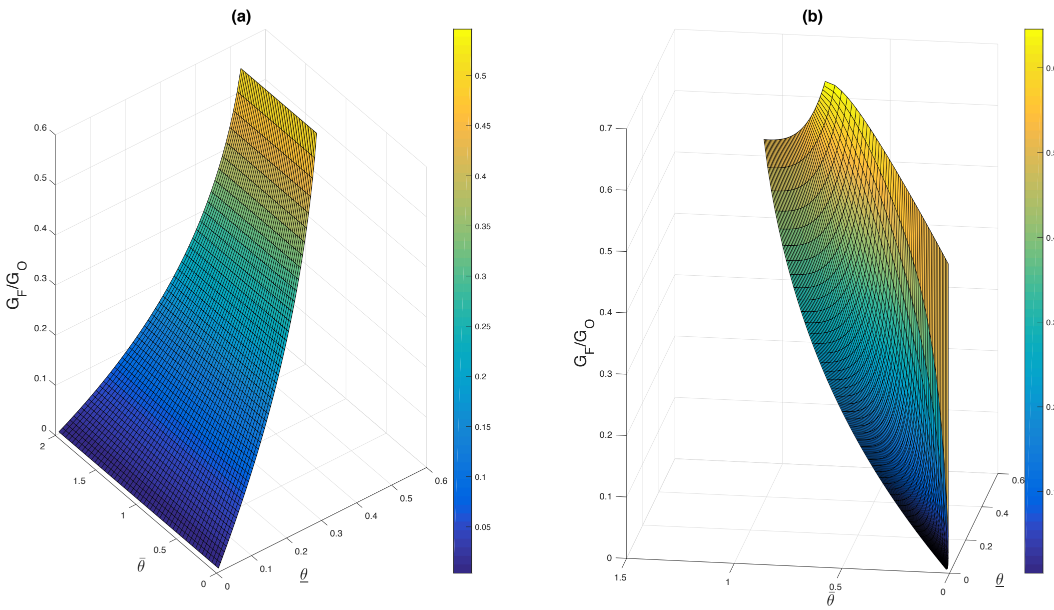
As in Lemma 1, Lemma 3 states that when $c(\theta, e) = \theta(1 - e)$, either all types or a portion of types are induced to exert optimal FPCR effort, depending on the values of $\underline{\theta}$ and $\bar{\theta}$. Note that in contrast to $c(\theta, e) = \beta(\theta - e)^2$ in Lemma 1, the coefficient of $c(\theta, e) = \theta(1 - e)$ is assumed to be one.

Lemma 4 *Under Assumption 2, the optimal cost-reducing effort* $e^*(\theta)$ *in the fully optimal contract is:*

$$e^*(\theta) = \begin{cases} 0, & \text{if } \bar{\theta} > \theta \geq 2\underline{\theta}, \\ \\ \underline{\theta} - \theta/2, & \text{if either } \bar{\theta} \leq 2\underline{\theta} \text{ or } \theta \leq 2\underline{\theta} < \bar{\theta}. \end{cases} \tag{3}$$

Lemma 4 is similar to Lemma 2. It is worth noting that the optimal effort in the fully optimal contract is weakly decreasing in type, which is the same as the case of Lemma 2 for $\beta < 1$. Based on the two lemmas above, we discuss the performance of the optimal FPCR menu by analyzing G_S/G_O. Since the goal of this section is to show that an optimal FPCR menu may also perform poorly under $c(\theta, e) = \theta(1 - e)$, we will focus on the cases in which G_S/G_O can be arbitrarily close to zero, which are summarized in the theorem below.

Figure 3: The impact of $\underline{\theta}$ and $\bar{\theta}$ on G_S/G_O



Theorem 3 *Suppose Assumption 2 holds. Then* $\lim_{\underline{\theta}\to 0} G_S/G_O = 0$ *if* $\underline{\theta} < 4 - 2\sqrt{3}$ *and* $\bar{\theta} > [\underline{\theta} + 2 - (4 + \underline{\theta}^2 - 8\underline{\theta})^{1/2}]/3$.

As we show in the proof of Theorem 3, there are two cases in which $\lim_{\underline{\theta}\to 0} G_S/G_O = 0$: (1) $\underline{\theta} < 4-2\sqrt{3}$ and $\bar{\theta} > 2\underline{\theta}$, and (2) $\underline{\theta} < 4-2\sqrt{3}$ and $[\underline{\theta}+2-(4+\underline{\theta}^2-8\underline{\theta})^{1/2}]/3 < \bar{\theta} < 2\underline{\theta}$. The expression of G_S/G_O differs in these two cases. Figure 3 illustrates how the ratio G_S/G_O depends on $\underline{\theta}$ and $\bar{\theta}$, where panels (a) and (b) are for the first and second case, respectively. In both panels, G_S/G_O is close to zero when $\underline{\theta}$ is sufficiently close to zero. Specifically, given that $\bar{\theta} > 2\underline{\theta}$, G_S/G_O in panel (a) does not depend on $\bar{\theta}$. It is worth noting that in both cases, the maximum of G_S/G_O is less than 0.75, which is the minimum of G_S/G_O in Rogerson (2003).

The relative poor performance of the optimal FPCR menu Theorem 3 is consistent with that in Theorems 1 and 2, and it can also explained in terms of the monotonicity of optimal effort and the level of incentives for cost reduction. Specifically, when $\underline{\theta} < 4-2\sqrt{3}$ and $\bar{\theta} > 2\underline{\theta}$, both of the optimal FPCR menu and the fully optimal contract induce only a portion of the types to exert effort. However, the optimal effort is increasing and decreasing in type for the optimal FPCR menu and a fully optimal contract, respectively. This is similar to the case $\beta < 1-1/\gamma$ in Section 3. Following our prior intuitive explanations, the increasing monotonicity of optimal effort can lead to poor performances of an FPCR menu relative to the fully optimal contract. When $\underline{\theta} < 4-2\sqrt{3}$ and $[\underline{\theta}+2-(4+\underline{\theta}^2-8\underline{\theta})^{1/2}]/3 < \bar{\theta} < 2\underline{\theta}$, any type in the fully optimal contract is induced to exert effort while only a portion of types are induced to exert optimal FPCR effort. This is similar to the case $\beta \in (1-1/\gamma, 2-2/\gamma)$ in Section 3. By combining the lower level of incentives for cost reduction, therefore, the optimal FPCR menu is more likely to perform poorly.

Discussions on general cost functions. Now we discuss how the monotonicity of optimal FPCR effort varies with the property of the cost function $c(\theta, e)$, i.e., whether the cost function is submodular or supermodular. The submodularity or supermodularity of cost functions depends on the way in which the effort interacts with cost type to affect the cost. Specifically, suppose that the cost function is strictly submodular in θ and e, i.e., $\partial^2 c(\theta, e)/\partial e \partial \theta < 0$, implying that the magnitude of marginal cost-reduction of effort is larger for an agent with a higher (less efficient) type. Intuitively this implies that the optimal effort increases in type. The submodularity of cost functions is supported by some empirical analyses, e.g., the cost function is estimated to be proportional to exponential of $(\theta - e)$, which is strictly submodular in θ and e, for the transport industry in Gagnepain and Ivaldi (2002) and the electric utilities industry in Abito (2017). Formally, for the type associated with an FP contract, the agent's objective function $\pi(\theta, e) = p - c(\theta, e) - \psi(e)$ is strictly supermodular in θ and e, and hence the optimal effort e^* is (weakly) increasing in type θ by Topkis's theorem.[6] Moreover, under the additional assumption $\partial^2 c(\theta, e)/\partial e^2 > 0$ and $\psi''(e) > 0$, the optimal effort is single-valued due to the fact that the agent's objective function $\pi(\theta, e)$ is strictly concave in e. In addition, $\pi(\theta, e)$ is continuously differentiable in e, then the optimal FPCR effort is strictly increasing in type for which the effort is positive.

If the cost function is strictly supermodular in θ and e, i.e., $\partial^2 c(\theta, e)/\partial e \partial \theta > 0$, which implies that the magnitude of marginal cost-reduction of effort is smaller for an agent with a higher type, then intuitively the optimal effort decreases in type. Such cost functions may occur in the biotechnology industry, where more efficient manufacturers in terms of higher plant inspection intensity are induced to exert more effort to raise the product quality (Mayer et al., 2004). Similar to the analysis of submodular cost functions above, if $\psi''(e) > -\partial^2 c(\theta, e)/\partial e^2$, then the optimal effort is single-valued and (weakly) decreasing in type θ. The continuous differentiability of $\pi(\theta, e)$ further implies that the optimal FPCR effort is strictly decreasing in type whenever effort is positive.[7]

The intuition behind the poor performance of FPCR contracts in our examples with increasing effort leads to a conjecture that the FPCR contracts might perform well and poorly with decreasing and increasing efforts, respectively. This conjecture is worth pur-

[6] See Theorem 2.3 in Vives (2001).

[7] For example, $c(\theta, e) = \beta_1(\theta - e) - \beta_2(\theta - e)^2$ with $\beta_1 > 0$ and $0 < \beta_2 < 1$, $\psi(e) = e^2$. Then in the FP contract $e^*(\theta) = (\beta_1 - 2\beta_2\theta)/2$ with $e^{*\prime}(\theta) < 0$.

suing in future work. If the conjecture is true, one may assess the performance of an optimal FPCR menu according to cost properties: a submodular cost function may result in the poor relative performance of an optimal FPCR menu, while an optimal FPCR menu may perform relatively well when the cost function is supermodular. An intermediate case is that the optimal FPCR effort is independent of type, i.e., $e^{*\prime}(\theta) = 0$, and this happens when type θ and effort e are additively separable in the cost function $c(\theta, e)$. As discussed in Rogerson (2003), the optimal FPCR menu performs very well in the sense that G_S/G_O is no less than 0.75.

5 Conclusions

We extended the FPCR menu to allow for the optimal cost-reducing effort to be monotone in the agent's cost type, and found that the performance of an optimal FPCR menu relies crucially on the monotonicity of optimal FPCR effort. In contrast to Rogerson (2003) where the optimal FPCR effort is independent of type, an optimal FPCR menu can perform very poorly relative to the fully optimal contract proposed by Laffont and Tirole (1986) when (1) the level of incentives for cost reduction is lower in an optimal FPCR menu in the sense that the optimal FPCR effort is induced only for a portion of types while in the fully optimal contract any type is induced to exert effort, and the optimal effort is increasing in type for both contracts, or (2) both the fully optimal contract and the optimal FPCR menu induce only a portion of types to exert effort, and the optimal effort decreases with type for the former contract but increases with type for the latter contract, respectively. We also found that when both the optimal FPCR menu and the fully optimal contract induce all types to exert effort for both contracts, and their optimal effort increases with type, then their performances can be close to each other. Our findings suggest that the information on the agent's cost structure is essential to the principal when an FPCR menu is implemented, and it will be interesting to explore under the monotone optimal effort whether another simple menu exists such that it can secure a substantial portion of expected surplus secured by the fully optimal contract.

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Appendix

Proof of Lemma 1. Let $\Delta \equiv \bar{\theta} - \underline{\theta}$. Suppose that $\hat{\theta} \in [\underline{\theta}, \bar{\theta}]$ is the cut-off type. By plugging the optimal effort level $e^*(\hat{\theta}) = \beta\hat{\theta}/(\beta+1)$, the principal's expected cost for the project is calculated as

$$\begin{aligned} C(\hat{\theta}) &= \Big[\beta(\hat{\theta} - e^*(\hat{\theta}))^2 + e^{*2}(\hat{\theta})\Big]F(\hat{\theta}) + \int_{\hat{\theta}}^{\bar{\theta}} \beta\theta^2 dF(\theta) \\ &= \frac{\beta}{\Delta(\beta+1)}\hat{\theta}^2(\hat{\theta} - \underline{\theta}) + \frac{\beta}{3\Delta}(\bar{\theta}^3 - \hat{\theta}^3), \end{aligned}$$

with the first-order condition of an interior solution being:

$$C'(\hat{\theta}) = \frac{\beta\hat{\theta}}{\Delta(\beta+1)}\big[(2-\beta)\hat{\theta} - 2\underline{\theta}\big].$$

The interior solution for the optimal cut-off type θ^*, if exists, is given by $C'(\theta^*) = 0$,

$$\theta^* = \frac{2\underline{\theta}}{2-\beta}, \text{ if } \beta \neq 2.$$

When $0 < \beta < 2(\gamma-1)/\gamma$, $\theta^* = 2\underline{\theta}/(2-\beta) \in (\underline{\theta}, \bar{\theta})$, and the corresponding second-order condition $C''(\theta^*) = 2\beta\underline{\theta}/[(\beta+1)\Delta] > 0$ satisfies. Hence, $\theta^* = 2\underline{\theta}/(2-\beta)$ is an interior solution of the principal's cost minimization problem when $0 < \beta < 2(\gamma-1)/\gamma$. The fixed price is $p = \beta(\theta^* - e^*(\theta^*))^2 + e^{*2}(\theta^*) = \frac{\beta}{1+\beta}(\frac{2}{2-\beta})^2\underline{\theta}^2$.

When $\beta \geq 2$ or $\gamma \leq 2/(2-\beta)$, i.e., $2(\gamma-1)/\gamma \leq \beta$ due to $\gamma \geq 1$, $\theta^* = 2\underline{\theta}/(2-\beta) \notin (\underline{\theta}, \bar{\theta})$. Thus the cut-off type is not an interior point of the support $[\underline{\theta}, \bar{\theta}]$ and it is necessary to compare $C(\underline{\theta})$ with $C(\bar{\theta})$ to determine the FPCR contract. Considering that

$$C(\underline{\theta}) = \frac{\beta}{3}(\bar{\theta}^2 + \bar{\theta}\underline{\theta} + \underline{\theta}^2);\ C(\bar{\theta}) = \frac{\beta}{\beta+1}\bar{\theta}^2.$$

It is straightforward to verify that the ratio $C(\underline{\theta})/C(\bar{\theta})$ is greater than one whenever $2(\gamma-1)/\gamma \leq \beta$. Thus, the optimal cut-off type is $\theta^* = \bar{\theta}$ and the principal sets the fixed-price to be $p = \beta(\bar{\theta} - e^*(\bar{\theta}))^2 + e^{*2}(\bar{\theta}) = \beta\bar{\theta}^2/(\beta+1)$, and an agent with any cost type chooses the FP contract. The proof is complete.

Proof of Lemma 2. For the optimal contract in Laffont and Tirole (1986), the principal solves the following optimization problem:

$$\begin{aligned} &\min_{t(\theta),e(\theta)} \int_{\underline{\theta}}^{\bar{\theta}} t(\theta)dF(\theta) \\ &\text{s.t. } U(\theta|\theta) \geq 0 \\ &\qquad U(\theta|\theta) \geq U(\hat{\theta}|\theta) \\ &\qquad c(\theta, e(\tilde{\theta}|\theta)) = c(\tilde{\theta}, e(\tilde{\theta})), \end{aligned}$$

where $U(\tilde{\theta}|\theta) \equiv t(\tilde{\theta}) - c(\theta, e(\tilde{\theta}|\theta)) - \psi(e(\tilde{\theta}|\theta))$, $e(\tilde{\theta}|\theta)$ and $t(\tilde{\theta})$ are the agent's effort and the principal's payment, respectively, when the agent's true type is θ but he announces $\tilde{\theta}$.

We first consider the agent's optimization problem, by using the same arguments in Laffont and Tirole (1986), we have the first order condition

$$U'(\theta) = -\psi'(e(\theta)), \tag{A.1}$$

where $U(\theta) = U(\theta|\theta)$ is the equilibrium utility for an agent with type θ, $e(\theta)$ is the optimal effort for type θ, and the second-order incentive constraint

$$e'(\theta) < 1. \tag{A.2}$$

Next, we derive the optimal effort by analyzing the principal's problem. Using (A.1) and $U(\bar{\theta}) = 0$ leads to

$$U(\theta) = \int_{\theta}^{\bar{\theta}} -\psi'(e(\tau))d\tau.$$

As a result,

$$\int_{\underline{\theta}}^{\bar{\theta}} U(\theta)dF(\theta) = U(\theta)F(\theta)\Big|_{\underline{\theta}}^{\bar{\theta}} - \int_{\underline{\theta}}^{\bar{\theta}} F(\theta)dU(\theta) = \int_{\underline{\theta}}^{\bar{\theta}} \psi'(e(\theta))\frac{F(\theta)}{f(\theta)}dF(\theta).$$

Since $t(\theta) = c(\theta, e(\theta)) + \psi(e(\theta)) + U(\theta)$, the principal's expected cost is

$$\begin{aligned}\int_{\underline{\theta}}^{\bar{\theta}} t(\theta)dF(\theta) &= \int_{\underline{\theta}}^{\bar{\theta}} \left[c(\theta, e(\theta)) + \psi(e(\theta)) + U(\theta)\right] dF(\theta) \\ &= \int_{\underline{\theta}}^{\bar{\theta}} \left[c(\theta, e(\theta)) + \psi(e(\theta)) + \psi'(e(\theta))\frac{F(\theta)}{f(\theta)}\right] dF(\theta).\end{aligned}$$

Hence, the level of cost-reducing effort is given by:

$$e(\theta) = \arg\min_{e \geq 0} \left\{c(\theta, e) + \psi(e) + \psi'(e)\frac{F(\theta)}{f(\theta)}\right\}, \tag{A.3}$$

where $c(\theta, e) = \beta(\theta - e)^2$, $\psi(e) = e^2$, and $F(\theta)$ is the uniform distribution function of type on the support $[\underline{\theta}, \bar{\theta}]$. Then the first-order condition implies

$$\beta(\theta - e) - e - (\theta - \underline{\theta}) = 0 \Rightarrow (1 + \beta)e = (\beta - 1)\theta + \underline{\theta} \text{ and } e \geq 0.$$

Note that $e^{*\prime}(\theta) = (\beta - 1)/(\beta + 1) < 1$, which implies that in the fully optimal contract the optimal effort can be decreasing in type. Specifically, due to $e \geq 0$, when $\beta \geq 1$, the optimal effort is $e^*(\theta) = \frac{(\beta-1)\theta+\underline{\theta}}{\beta+1}$ for any type; when $0 < \beta < 1$, the agent exerts effort $e^*(\theta) = \frac{(\beta-1)\theta+\underline{\theta}}{\beta+1}$ if $(1 - \beta)\theta \leq \underline{\theta}$, and exerts zero effort if $(1 - \beta)\theta \geq \underline{\theta}$. It is easy to verify that e^* is the minimum because the second order condition is $2(\beta + 1) > 0$ for $\beta > 0$. Clearly, the above optimal effort satisfies the second-order incentive constraint (A.2). The proof is complete.

Proof of Theorem 1. We first write down the surplus under the FPCR menu G_S,

$$\begin{aligned}
G_S &\equiv C(\underline{\theta}) - C(\theta^*) \\
&= \int_{\underline{\theta}}^{\bar{\theta}} c(\theta,0)dF(\theta) - \Big[c(\theta^*, e^*(\theta^*)) + \psi(e^*(\theta^*))\Big]F(\theta^*) - \int_{\theta^*}^{\bar{\theta}} c(\theta,0)dF(\theta) \\
&= \begin{cases} \frac{\beta}{3}\left(\frac{\beta-2}{\beta+1}\gamma^2+\gamma+1\right)\underline{\theta}^2, & 2(\gamma-1)/\gamma \le \beta, \\ -\frac{\beta^3(\beta-3)}{3(\gamma-1)(\beta-2)^2(\beta+1)}\underline{\theta}^2, & 0<\beta<2(1-\gamma)/\gamma, \end{cases} \qquad \text{(A.4)}
\end{aligned}$$

Next, G_O can be obtained according to its definition:

$$G_O = \begin{cases} \int_{\underline{\theta}}^{\bar{\theta}} c(\theta,0)dF(\theta) - \int_{\underline{\theta}}^{\bar{\theta}} \left\{c(\theta,e^*(\theta)) + \psi(e^*(\theta)) + \psi'(e^*(\theta))\frac{F(\theta)}{f(\theta)}\right\} dF(\theta), \ \beta > (\gamma-1)/\gamma, \\ \int_{\underline{\theta}}^{\bar{\theta}} c(\theta,0)dF(\theta) - \int_{\underline{\theta}}^{\underline{\theta}/(1-\beta)} \left\{c(\theta,e^*(\theta)) + \psi(e^*(\theta)) + \psi'(e^*(\theta))\frac{F(\theta)}{f(\theta)}\right\} dF(\theta) \\ \qquad - \int_{\underline{\theta}/(1-\beta)}^{\bar{\theta}} c(\theta,0)dF(\theta), \ 0<\beta\le(\gamma-1)/\gamma. \end{cases}$$

Based on the optimal effort level in Lemma 2, we discuss G_O in the following two cases.

Case 1: $0<\beta<1$ and $(1-\beta)\bar{\theta} \ge \underline{\theta}$, i.e., $0<\beta\le(\gamma-1)/\gamma$.

$$\begin{aligned}
G_O &= \int_{\underline{\theta}}^{\bar{\theta}} c(\theta,0)dF(\theta) - \int_{\underline{\theta}}^{\underline{\theta}/(1-\beta)} \left\{c(\theta,e^*(\theta)) + \psi(e^*(\theta)) + \psi'(e^*(\theta))\frac{F(\theta)}{f(\theta)}\right\} dF(\theta) \\
&\qquad - \int_{\underline{\theta}/(1-\beta)}^{\bar{\theta}} c(\theta,0)dF(\theta) \\
&= \int_{\underline{\theta}}^{\underline{\theta}/(1-\beta)} \left\{c(\theta,0) - c(\theta,e^*(\theta)) - \psi(e^*(\theta)) - \psi'(e^*(\theta))\frac{F(\theta)}{f(\theta)}\right\} dF(\theta) \\
&= \frac{\beta^3\underline{\theta}^3}{3(1-\beta^2)}\frac{1}{\bar{\theta}-\underline{\theta}} = \frac{\beta^3\underline{\theta}^2}{3(1-\beta^2)}\frac{1}{\gamma-1}. \qquad \text{(A.5)}
\end{aligned}$$

Case 2: $0<\beta<1$ and $(1-\beta)\bar{\theta} \le \underline{\theta}$, or $\beta>1$; i.e., $\beta>(\gamma-1)/\gamma$.

$$\begin{aligned}
G_O &= \int_{\underline{\theta}}^{\bar{\theta}} c(\theta,0)dF(\theta) - \int_{\underline{\theta}}^{\bar{\theta}} \left\{c(\theta,e^*(\theta)) + \psi(e^*(\theta)) + \psi'(e^*(\theta))\frac{F(\theta)}{f(\theta)}\right\} dF(\theta) \\
&= \int_{\underline{\theta}}^{\bar{\theta}} \left\{\beta\theta^2 - \beta\left(\frac{2\theta-\underline{\theta}}{1+\beta}\right)^2 - \left(\frac{(\beta-1)\theta+\underline{\theta}}{1+\beta}\right)^2 - 2\frac{(\beta-1)\theta+\underline{\theta}}{1+\beta}(\theta-\underline{\theta})\right\} dF(\theta) \\
&= \frac{[(\beta-1)\bar{\theta}+\underline{\theta}]^3 - (\beta\underline{\theta})^3}{3(\beta^2-1)}\frac{1}{\bar{\theta}-\underline{\theta}} \\
&= \frac{1}{3(1+\beta)}\left\{(\beta-1)^2\bar{\theta}^2 + (\beta+2)(\beta-1)\bar{\theta}\underline{\theta} + (\beta^2+\beta+1)\underline{\theta}^2\right\} \\
&= \frac{(\beta-1)^2\gamma^2 + (\beta^2+\beta-2)\gamma + (\beta^2+\beta+1)}{3(1+\beta)}\underline{\theta}^2 \qquad \text{(A.6)}
\end{aligned}$$

The results in (A.5) and (A.6) allow us to obtain G_S/G_O in both cases discussed above.

Case 1: $0 < \beta \leq 1 - 1/\gamma$:

$$\frac{G_S}{G_O} = \frac{(\beta-1)(\beta-3)}{(\beta-2)^2}. \tag{A.7}$$

Case 2: $1 - 1/\gamma < \beta < 2(1 - 1/\gamma)$:

$$\frac{G_S}{G_O} = -\frac{\beta^3(\beta-3)}{(\gamma-1)(\beta-2)^2[(\beta-1)^2\gamma^2 + (\beta^2+\beta-2)\gamma + (\beta^2+\beta+1)]}. \tag{A.8}$$

Case 3: $2(1 - 1/\gamma) \leq \beta$:

$$\frac{G_S}{G_O} = 1 - \frac{\gamma^2 + 1 - 2\gamma}{(\beta-1)^2\gamma^2 + (\beta^2+\beta-2)\gamma + (\beta^2+\beta+1)}. \tag{A.9}$$

It is easy to show that for case 1 the ratio G_S/G_O is strictly decreasing in β and $G_S/G_O < 3/4$ for any β. For case 3, the denominator of the second term on the right-hand-side of (A.9) is increasing in β. As a result,

$$\frac{d}{d\beta}\frac{G_S}{G_O} > 0, \text{ if } 2(1-1/\gamma) \leq \beta.$$

In addition, it is straightforward to verify that the limit of G_S/G_O is 1 as β approaches infinity, i.e, $\lim_{\beta\to\infty} \frac{G_S}{G_O} = 1$. Hence, for any given $\gamma > 1$ G_S/G_O is decreasing in $\beta \in (0, 1-1/\gamma]$ and increasing in $\beta \in [2-2/\gamma, \infty)$, and the continuity of G_S/G_O in γ implies there must be a minimum in the interval $\beta \in (1-1/\gamma, 2-2/\gamma)$. The proof is complete.

Proof of Theorem 2. For a given $\gamma > 1$, let $\beta \in (1-1/\gamma, 2-2/\gamma)$. Then according to (A.8), we have

$$\begin{aligned}
\frac{G_S}{G_O} &= \frac{\beta^3(3-\beta)}{(\gamma-1)(\beta-2)^2[(\beta-1)^2\gamma^2 + (\beta^2+\beta-2)\gamma + (\beta^2+\beta+1)]} \\
&= \frac{\beta^3(3-\beta)}{(\beta-2)^2}\frac{1}{(\gamma-1)[(\beta-1)^2\gamma^2 + (\beta^2+\beta-2)\gamma + (\beta^2+\beta+1)]},
\end{aligned}$$

where the first term $\beta^3(3-\beta)/(\beta-2)^2$ is a positive constant. Thus we focus on the denominator of the second term, denoted by $\xi(\gamma,\beta)$. For a given $\beta \in (1-1/\gamma, 2-2/\gamma)$,

$$\begin{aligned}
\frac{d\xi(\gamma,\beta)}{d\gamma} &= 3\gamma^2(\beta-1)^2 - 2\gamma(\beta-1)^2 + 2\gamma(\beta^2+\beta-2) + (\beta^2+\beta-2) + (\beta^2+\beta+1) \\
&= 3\gamma^2\beta^2 + (6\gamma - 6\gamma^2)\beta + 3\gamma^2 - 6\gamma + 3.
\end{aligned}$$

It is easy to show that for any $\beta > 1 - 1/\gamma$, the quadratic form of β is positive, i.e., $d\xi(\gamma,\beta)/d\gamma > 0$ for any $\beta \in (1-1/\gamma, 2-2/\gamma)$. This completes the proof that G_S/G_O is strictly decreasing in γ for any $\beta \in (1-1/\gamma, 2-2/\gamma)$. Since $\xi(\gamma,\beta) \to \infty$ as $\gamma \to \infty$ for any given $\beta \in (1-1/\gamma, 2-2/\gamma)$, it follows that $\lim_{\gamma\to\infty} G_S/G_O = 0$. The proof is complete.

Proof of Lemma 3. Similar to the proof of Lemma 1, suppose that $\hat{\theta} \in [\underline{\theta}, \bar{\theta}]$ is the cut-off type. By plugging the optimal effort level $e^*(\hat{\theta}) = \hat{\theta}/2$, the principal's expected cost for the project is calculated as

$$C(\hat{\theta}) = [c(\hat{\theta}, e^*(\hat{\theta})) + \psi(e^*(\hat{\theta}))]F(\hat{\theta}) + \int_{\hat{\theta}}^{\bar{\theta}} c(\theta,0)dF(\theta) = \frac{4\hat{\theta} - \hat{\theta}^2}{4}\frac{\hat{\theta} - \underline{\theta}}{\Delta} + \frac{(\bar{\theta}^2 - \hat{\theta}^2)}{2\Delta},$$

with the first-order condition of an interior solution being:

$$C'(\hat{\theta}) \;=\; \frac{-3\hat{\theta}^2 + (2\underline{\theta} + 4)\hat{\theta} - 4\underline{\theta}}{4\Delta}.$$

The second-order condition for minimization is

$$C''(\hat{\theta}) \;=\; \frac{-6\hat{\theta} + (2\underline{\theta} + 4)}{4\Delta} \geq 0,$$

which implies that $\hat{\theta} \leq (\underline{\theta} + 2)/3$. Hence, the interior solution for the optimal cut-off type may be $\theta^* = [\underline{\theta} + 2 - (4 + \underline{\theta}^2 - 8\underline{\theta})^{1/2}]/3$ given by $C'(\theta^*) = 0$. In addition, $2 > \bar{\theta} > [\underline{\theta} + 2 - (4 + \underline{\theta}^2 - 8\underline{\theta})^{1/2}]/3 > \underline{\theta}$ implies $\underline{\theta} < 4 - 2\sqrt{3}$. Hence, when $\underline{\theta} < 4 - 2\sqrt{3}$ and $\bar{\theta} > [\underline{\theta} + 2 - (4 + \underline{\theta}^2 - 8\underline{\theta})^{1/2}]/3$, $\theta^* = [\underline{\theta} + 2 - (4 + \underline{\theta}^2 - 8\underline{\theta})^{1/2}]/3$ is the interior optimal cut-off type.

When $\underline{\theta} < 4 - 2\sqrt{3}$ and $\bar{\theta} < [\underline{\theta} + 2 - (4 + \underline{\theta}^2 - 8\underline{\theta})^{1/2}]/3$ or $\underline{\theta} \geq 4 - 2\sqrt{3}$, the optimal cut-off type is not an interior point of the support $[\underline{\theta}, \bar{\theta}]$ and it is necessary to compare $C(\underline{\theta})$ with $C(\bar{\theta})$ to determine the FPCR contract. Considering that

$$C(\underline{\theta}) \;=\; \frac{(\bar{\theta}^2 - \underline{\theta}^2)}{2\Delta};\;\; C(\bar{\theta}) = \frac{(4\bar{\theta} - \bar{\theta}^2)(\bar{\theta} - \underline{\theta})}{4\Delta};\;\; \frac{C(\underline{\theta})}{C(\bar{\theta})} = \frac{2(\bar{\theta} + \underline{\theta})}{\bar{\theta}(4 - \bar{\theta})}.$$

It can be shown that $C(\underline{\theta})/C(\bar{\theta}) \geq 1$. Specifically, when $\underline{\theta} < 4 - 2\sqrt{3}$ and $\bar{\theta} < [\underline{\theta} + 2 - (4 + \underline{\theta}^2 - 8\underline{\theta})^{1/2}]/3$, $C(\underline{\theta})/C(\bar{\theta}) \geq 1$ is equivalent to $-\bar{\theta}^2 + 2\bar{\theta} \leq 2\underline{\theta}$. Due to $\underline{\theta} - (4 + \underline{\theta}^2 - 8\underline{\theta})^{1/2}$ is strictly increasing for $0 < \underline{\theta} < 4 - 2\sqrt{3}$, the maximum of $\underline{\theta} - (4 + \underline{\theta}^2 - 8\underline{\theta})^{1/2}$ is $4 - 2\sqrt{3} < 1$, i.e., $[\underline{\theta} + 2 - (4 + \underline{\theta}^2 - 8\underline{\theta})^{1/2}]/3 \leq 1$. Then the maximizer of $-\bar{\theta}^2 + 2\bar{\theta}$ is $\bar{\theta} = [\underline{\theta} + 2 - (4 + \underline{\theta}^2 - 8\underline{\theta})^{1/2}]/3$, and thus $C(\underline{\theta})/C(\bar{\theta}) \geq 1$ is equivalent to $-([\underline{\theta} + 2 - (4 + \underline{\theta}^2 - 8\underline{\theta})^{1/2}]/3)^2 + 2([\underline{\theta} + 2 - (4 + \underline{\theta}^2 - 8\underline{\theta})^{1/2}]/3) \leq 2\underline{\theta}$ for $0 < \underline{\theta} < 4 - 2\sqrt{3}$, which is true.

When $\underline{\theta} \geq 4 - 2\sqrt{3}$, $C(\underline{\theta})/C(\bar{\theta}) \geq 1$ is equivalent to $-\bar{\theta}^2 + 2\bar{\theta} \leq 2(4 - 2\sqrt{3}) > 1$, which is true due to the fact that the maximum of $-\bar{\theta}^2 + 2\bar{\theta}$ is one at $\bar{\theta} = 1$. Hence, the optimal cut-off type is $\theta^* = \bar{\theta}$, and an agent with any type chooses the FP contract with fixed-price $p = \bar{\theta} - \bar{\theta}^2/4$. Therefore, the optimal cut-off type is $\theta^* = \bar{\theta}$, and an agent with any type chooses the FP contract with fixed-price $p = \bar{\theta} - \bar{\theta}^2/4$. The proof is complete.

Proof of Lemma 4. Similar to the proof of Lemma 2, by noting that here $c(\theta, e) = \theta(1 - e)$, the level of cost reduction chosen by the agent $e(\theta)$ is given by (A.3). Then the first-order condition implies $e = \underline{\theta} - \theta/2$ and $e \geq 0$. Since $e \geq 0$, the optimal effort is $e^*(\theta) = \underline{\theta} - \theta/2$ when $\underline{\theta} - \theta/2 \geq 0$, i.e., $\theta \leq \min\{2\underline{\theta}, \bar{\theta}\}$. More specifically, when $2\underline{\theta} \geq \bar{\theta}$, the agent with any type exerts optimal effort $e^*(\theta) = \underline{\theta} - \theta/2$; when $\bar{\theta} > 2\underline{\theta}$, the agent exerts $e^*(\theta) = \underline{\theta} - \theta/2$ if $\theta \leq 2\underline{\theta}$, and exerts no effort if $\theta > 2\underline{\theta}$. As in Lemma 2, the optimal effort also satisfies the second-order incentive constraint (A.2). The proof is complete.

Proof of Theorem 3. Now we use Lemma 3 and Lemma 4 to show the cases in which G_S/G_O is arbitrarily close to zero. To this end, we calculate G_S in an optimal FPCR menu according to Lemma 3. When $\underline{\theta} < 4 - 2\sqrt{3}$ and $\bar{\theta} > [\underline{\theta} + 2 - (4 + \underline{\theta}^2 - 8\underline{\theta})^{1/2}]/3$, we have

$$\begin{aligned} G_S &= \int_{\underline{\theta}}^{\bar{\theta}} c(\theta, 0) dF(\theta) - \left[(c(\theta^*, e^*(\theta^*)) + \psi(e^*(\theta^*))) F(\theta^*) + \int_{\theta^*}^{\bar{\theta}} c(\theta, 0) dF(\theta) \right] \\ &= \frac{(\theta^* - \underline{\theta})(\theta^{*2} - 2\theta^* + 2\underline{\theta})}{4(\bar{\theta} - \underline{\theta})}. \end{aligned}$$

When $\underline{\theta} < 4 - 2\sqrt{3}$ and $\bar{\theta} < [\underline{\theta} + 2 - (4 + \underline{\theta}^2 - 8\underline{\theta})^{1/2}]/3$ or $\underline{\theta} \geq 4 - 2\sqrt{3}$, we obtain

$$G_S = \int_{\underline{\theta}}^{\bar{\theta}} c(\theta, 0) dF(\theta) - [c(\bar{\theta}, e^*(\bar{\theta})) + \psi(e^*(\bar{\theta}))] = (\underline{\theta} + \bar{\theta})/2 - (\bar{\theta} - \bar{\theta}^2/4) = \frac{\bar{\theta}^2 - 2\bar{\theta} + 2\underline{\theta}}{4}.$$

Next, we calculate G_O in the fully optimal contract according to Lemma 4. When $\bar{\theta} > 2\underline{\theta}$, we have

$$G_O = \int_{\underline{\theta}}^{2\underline{\theta}} c(\theta, 0) dF(\theta) - \int_{\underline{\theta}}^{2\underline{\theta}} \left\{ c(\theta, e^*(\theta)) + \psi(e^*(\theta)) + \psi'(e^*(\theta)) \frac{F(\theta)}{f(\theta)} \right\} dF(\theta) = \frac{\underline{\theta}^3}{12(\bar{\theta} - \underline{\theta})}.$$

When $\bar{\theta} \leq 2\underline{\theta}$, we have

$$G_O = \int_{\underline{\theta}}^{\bar{\theta}} c(\theta, 0) dF(\theta) - \int_{\underline{\theta}}^{\bar{\theta}} \left\{ c(\theta, e^*(\theta)) + \psi(e^*(\theta)) + \psi'(e^*(\theta)) \frac{F(\theta)}{f(\theta)} \right\} dF(\theta) = \frac{7\underline{\theta}^2 + \bar{\theta}^2 - 5\underline{\theta}\bar{\theta}}{12}.$$

We only consider the case where $\bar{\theta}/\underline{\theta}$ can possibly converge to infinity, i.e., $\underline{\theta} < 4 - 2\sqrt{3}$, because if $\underline{\theta} \geq 4 - 2\sqrt{3}$, then $\bar{\theta}/\underline{\theta} \leq 2/(4 - 2\sqrt{3})$ by noting the condition $\bar{\theta} \leq 2$ imposed in Assumption 2. Based on the above results, we have the following result on the ratio G_S/G_O,

$$G_S/G_O = \begin{cases} 3\underline{\theta}^{-3}(\theta^* - \underline{\theta})(\theta^{*2} - 2\theta^* + 2\underline{\theta}), \text{if } \bar{\theta} > 2\underline{\theta}; \\ 3\frac{(\theta^* - \underline{\theta})(\theta^{*2} - 2\theta^* + 2\underline{\theta})}{(\bar{\theta} - \underline{\theta})(7\underline{\theta}^2 + \bar{\theta}^2 - 5\underline{\theta}\bar{\theta})}, \text{if } [\underline{\theta} + 2 - (4 + \underline{\theta}^2 - 8\underline{\theta})^{1/2}]/3 < \bar{\theta} < 2\underline{\theta}. \end{cases}$$

We first consider the limit of G_S/G_O when $0 < \underline{\theta} < 4 - 2\sqrt{3}$ and $\bar{\theta} > 2\underline{\theta}$. It is easy to check that when $\underline{\theta} \to 0$, both the numerator and the denominator go to zero. By using L'Hospital's rule, we have

$$\begin{aligned} \lim_{\underline{\theta} \to 0} \frac{G_S}{G_O} &= \lim_{\underline{\theta} \to 0} \frac{3\left(\theta^* - \underline{\theta}\right)\left(\theta^{*2} - 2\theta^* + 2\underline{\theta}\right)}{\underline{\theta}^3} \\ &= \lim_{\underline{\theta} \to 0} \frac{\left(\underline{\theta}^3 - \left(\sqrt{\underline{\theta}^2 - 8\underline{\theta} + 4} + 12\right)\underline{\theta}^2 + \left(8\sqrt{\underline{\theta}^2 - 8\underline{\theta} + 4} + 30\right)\underline{\theta} - 4\sqrt{\underline{\theta}^2 - 8\underline{\theta} + 4} + 8\right)}{9\left(\sqrt{\underline{\theta}^2 - 8\underline{\theta} + 4}\right)^{3/2}} \\ &= 0, \end{aligned} \tag{A.10}$$

where the third equality holds because the limits of the numerator and denominator are zero and 72, respectively.

Let $\bar{\theta}/\underline{\theta} \equiv \rho(\bar{\theta}, \underline{\theta})$, and we drop the arguments of ρ for ease of exposition. It is easy to show that $\theta^* = [\underline{\theta} + 2 - (4 + \underline{\theta}^2 - 8\underline{\theta})^{1/2}]/3 \geq \underline{\theta}$ for $0 < \underline{\theta} < 4 - 2\sqrt{3}$, and hence $1 < \rho < 2$ for $\theta^* < \bar{\theta} < 2\underline{\theta}$. Then

$$\lim_{\underline{\theta} \to 0} \frac{G_S}{G_O} = \lim_{\underline{\theta} \to 0} \frac{3(\theta^* - \underline{\theta})(\theta^{*2} - 2\theta^* + 2\underline{\theta})}{(\bar{\theta} - \underline{\theta})(7\underline{\theta}^2 + \bar{\theta}^2 - 5\underline{\theta}\bar{\theta})} = \lim_{\underline{\theta} \to 0} \frac{3\underline{\theta}^{-3}(\theta^* - \underline{\theta})(\theta^{*2} - 2\theta^* + 2\underline{\theta})}{(\rho - 1)(7 + \rho^2 - 5\rho)}. \tag{A.11}$$

Since we require that $\bar{\theta} > \underline{\theta}$ holds for any $\underline{\theta}$, $\rho > 1$ even when $\underline{\theta} \to 0$. Thus the denominator is positive. According to (A.10), the numerator in the last expression of (A.11) has limit zero as $\underline{\theta} \to 0$. This completes the proof.