Agenda Manipulation and Policymaking under Media Bias*

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Abstract

Policymakers often try to push certain agendas while being concerned about their careers and reputations. Even though the media provide policymakers an opportunity to influence public opinion, potential bias could distort these efforts. We investigate how media bias interacts with agenda manipulation and the implications of such interactions on policymaking and media reports. We show the importance of the public in shaping the policy outcomes through their desire of truthful information under different market structure. Consistent with Pigouvian theory, state-owned media could be less biased; however, the independent media’s slant toward the public opinion could distort the policymakers’ signal. We find state-owned media could actually lead to the optimal policy outcome, while independent media may induce worse policy outcome than the state-owned media, (ironically) when the public strongly care about the truth. Importantly, market competition can indeed generate more accurate information and efficient transmission of information, but not necessarily improves the policy outcome. Surprisingly, market competition may backfire and not always lead to a better policy outcome.

JEL classification: D72, L82

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*The usual disclaimer applies.
1 Introduction

Policymakers address public issues such as education, health care and foreign affairs by instituting regulations, executive orders or laws. These policies often benefit some groups of people at the expense of others. Consequently, policymakers might take certain stands depending on their political view to push an agenda. On the other hand, their concerns about career or reputation restrict their means of doing so, especially if the agenda turns out to be unpopular among their constituency. Therefore, policymakers often turn to the media as the intermediaries to convey certain information in order to test and gather public response and mitigate the possible aftermath of blame. However the media frequently act as gatekeepers determining the information the public see or hear, and then control or even slant the content the public finally access. As a result, the policy outcome could be affected or distorted from the optimal one.

Such practices of the policymakers and the media are common in regard to policy issues and in many countries with different democratic systems varying from state-owned media in China and Russia, monopoly media in Italy and Turkey,\(^1\) to competitive media market in India, UK, and the US.\(^2\) For example, Chinese policymakers in recent years frequently test public opinions through their mouthpieces by sending some signals about policy issues ranging from tightening regulations on ride sharing to hospital reforms to loosing one-child policy. In western democracies such as US and Italy, we often see policy makers release all kinds of signals on almost every policy issue. Digital media such as Twitter and Facebook further enable such ability. Media actively engage in such signals and publish their own interpretation and stories to help shape public opinion.

This paper provides a theoretical framework to investigate how the policymakers strategically use the media to push certain agendas, how the media could influence the policy through their reports to achieve their own goals, and finally, how policy outcomes and political accountability will be impacted accordingly. We build upon the information transmission theory (Sobel (1985) and Li (2010)) and media slant theory (Mullainathan and Shleifer 2005) and develop a model of agenda manipulation through the strategic intermediaries under different media market structures. We endogenize the impact of media reporting and public opinion, and explicitly formulate the policymakers’ decision making process regarding public issues.

In our model, the policymakers disclose certain information to the media in order to influence the public and push a certain agenda, while the public might hold specific opinions toward the

\(^1\)Companies controlled by the Berlusconi family have a 45% share of the Italian commercial TV audience, and 60% of advertising sales (www.ketupa.net/berlusconi.htm). Putting his own three stations together with the public RAI stations put the Berlusconi share at a approximately 90% of the television audience when Berlusconi was in power (Stille 2006; Anderson and McLaren 2012).

\(^2\)Six major companies dominate the U.S. media: Time Warner, Viacom, General Electric, Walt Disney, CBS and News Corp. Those six companies own 90% of all the media that Americans consumer every day.
policy issue. The information conveyed by the policymakers could truly reflect their own understandings of the issue or could be biased toward their own political stands. After receiving the signal from the policymakers, the media might self-censor to either cater to their core readers or avoid antagonizing advertisers. In the meanwhile, the media may spend considerable effort to figure out the truth partly to preserve their reputation, even though in the end they may still choose not to report that truth. Upon reading the news, the public could update their beliefs about the policy issue. After collecting public opinion, the policymakers decide their actions on the issue. The public will eventually observe the truth.

While influencing public opinion helps push a certain agenda, policymakers need to weigh this influence against consumers’ opinion toward the policy issue because of their career concerns such as reelection. They also have an incentive to do the “right” thing for their electorate, particularly when public opinion does not necessarily converge to the truth. Interestingly, even though the biased signal could help policymakers push a certain agenda, the media’s possible distortion could not only restrict their impact, but also hurt policymakers’ reputations.

Serving as the bridge between the policymakers and the public, the media try to build reputations as providers of accurate information while often conforming to readers’ beliefs (Gentzkow and Shapiro 2006). The so-called public interest (Pigouvian) theory claims government ownership of the media is desirable because as a public good, state media can expose the public to less biased information than privately-owned media. In contrast, the public choice theory argues that a competitive media market assures that consumers obtain, on average, less biased information (Djankov et al. 2003). This distinction suggests that the way the media industry is organized (e.g., state-owned in China, monopolistic in Italy or oligopolistic in the US), has profound impacts on how the public obtain information. Consequently, the media market structure could have significant influence on how information is transmitted from policymakers to the public.

The public forge their beliefs about and evaluations of policy issues mainly through the media in modern society. Ex ante, they may hold very different prior opinions. For example, for issues like health care, public opinion is divergent; on the other hand, the public might be relatively easy to converge on military or counter-terrorism issues. The issue-specific heterogeneity of prior beliefs toward policy issues among the public could trigger very different responses to media reports, and ultimately impact the final implemented policies and the future careers of these policymakers.

Our first set of results illustrates the importance of the public in shaping the policy outcomes through their desire of truthful information. Using the media as the messengers, the policymakers reduce the possible reputation cost of releasing inaccurate information while achieving their goal.

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3Gehlbach and Sonin (2014) show that media ownership influences media bias. Durante and Knight (2012) find that viewers respond to the change of ownership of media firms by adjusting their favorite programs. In a different context, Chan and Stone (2013) analyze the effects of media proliferation on electoral outcomes.
of pushing certain agendas. This shares a similarity with Li (2010)’s role of the intermediaries. However, since the policymakers act as the decision maker and the signal-sender here, our model captures the policy making process while hers focuses on whether the intermediary should be used. Since the decision maker does not participate in information transmission in Li (2010), the distortion of information in her paper is only from the senders. However, in our model the policymaker as a signal sender could distort the policy through mis-signaling or direct policy distortion. More importantly, prior beliefs of the public help shape the policymakers’ strategic signal and the final chosen policy in our model, but were left out in her setting.

Our second main result is that consistent with the public interest (Pigouvian) theory, state-owned media could be less biased. The rationale behind this is that without the biased intermediary, state-owned media can provide the public with more complete and less biased information without incurring additional cost. On the other hand, the independent media’s slant toward the public opinion could distort the policymaker’s strategic signal. Interestingly, we find state-owned media could actually lead to the optimal policy outcome. Ironically, when the public strongly care about the truth, the independent media may actually induce worse policy outcomes than the state-owned media. Importantly, competition in the media market does not always lead the public to the better policy outcomes in a democratic society. Our result resonates with the cautious suggestion of the expansion of Western news outlets by Mullainathan and Shleifer (2005).

The further analysis of information transmission foreshadows our third result: competition in the media market can indeed generate more accurate information and efficient transition of information. Unlike Gentzkow and Shapiro (2006) where lower bias in a competitive media market is conditional on the ex post verification of the true state, our result could survive the possible non-existence of the ex post verification. This could be very important for many policy issues where the ultimate truth may only be revealed over a long time period, but not in the near future. This finding complements the recent work (Gentzkow et al. 2011) that competition enhances ideological diversity by showing that diverse beliefs are necessary for competition to work in order to facilitate policy making and benefit the public. However, such media market competition does not always improve the policy outcome.

Our work is closely related to the literature on media’s impact on public policy and government accountability. Strömberg (2001) conjectures that the media could influence policy by creating more informed voters, who consequently vote in policies favorable to the media. In a series of empirical papers, Strömberg and his coauthors apply this argument and estimate the impacts of media coverage on public spending (Strömberg 2004), government reaction to natural disasters (Strömberg 2007), and politicians’ responsiveness to their constituencies (Snyder and Strömberg 2010). DellaVigna and Kaplan (2007) and Gentzkow and Shapiro (2006) investigate how the
political elections are influenced by the media. In contrast to those papers where the policymakers seem to be passive actors, we focus on the policymaker’s active role of utilizing media rather than only responding to it, especially when the policymakers are better informed than the public and, hence, act on the basis of privileged information. In the similar spirit of emphasizing policymakers’ active role, Besley and Prat (2006) study the endogenous media capture of democratic politics and focus on the interplay of government accountability and media bias induced by the political capture, but not the reputation concern and the demand from the public. Fu and Li (2010) is the only paper to our best knowledge which explicitly models the policy making process as ours does, but did not incorporate the role of the media on policy making processes and the media’s impact on political accountability.

The remainder of the paper is organized as follows. Section 2 presents the basic model and justifies the assumptions. Section 3 analyzes the case of state-owned media and discusses how the policy and signal are manipulated by the policymakers. Section 4 deals with the monopoly media market and studies the implication of introducing an independent media on equilibrium report and policy outcome. Section 5 considers the competitive media market and compares the policy outcomes in three different market structures of media ownership. Section 6 concludes. All the proofs are are relegated to the Appendix.

2 Model Setup

Consider a policymaking process described by a game with three types of players: the policymaker, the public, and the media firms. The nature chooses the state \( \theta \in \{-1, 0, 1\} \) with equal probability. We may interpret the states \(-1, 0\) and \(1\) as left, neutral and right, respectively. The policymaker observes the state of nature \( \theta \) and sends a signal \( s(\theta) \) to the media firms. The public have no access to the signal. Before receiving the signal, each media firm develops an investigation strategy \( e(\cdot) \), a reporting strategy \( r(\cdot) \) and a pricing strategy \( p(\cdot) \). Upon receiving the signal \( s \), the media firms simultaneously decide whether to investigate the state of nature, and then publish their reports on the issue based on the signal \( s \) and the investigation outcome \( z \). After learning the rivals’ reports, the media firms simultaneously choose their prices. The consumers who have opinions on the state of nature make their purchase decisions upon observing the report \( r \) and price \( p \), and they

\footnote{In a related setting, Chan and Suen (2008, 2009) study the role of the media in electoral competition in a cheap talk framework, while we focus on the policymaking process as the outcome of interplay among policymakers, the media and the public.}

\footnote{For the reasons of both analytical tractability and our focus of media bias on policy outcome (where left, neutral and right are often discussed), we follow Sobel (1985) and Li (2010)’s discrete setup of “the state of nature”, which is different from Zhu and Dukes (2015).
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\footnote{Throughout the remainder of the paper the terms public and consumers are used interchangeably.
}

\footnote{If the media firm chooses not to investigate, then \( z = \emptyset \).}
update their opinions after reading the report $r$. Consumers’ posterior beliefs help determine the perceived state of nature, therefore their desired optimal policy. Based on consumers’ such opinion, the policymaker chooses the implemented policy. Throughout the process, the policymaker and media firms could be strategic or truthful in sending their message to the receivers.

The timing of the game is as follows: At time period 0, the policymaker observes media firms’ strategy $(e(\cdot), r(\cdot), p(\cdot))$ and the state of nature $\theta$ of the policy issue. At time period 1, the policymaker sends a message $s$ about the state of nature to the media firms, which is not directly observed by the public even though the consumers might have some prior opinion on the issue. At time period 2, the media outlets simultaneously decide whether to investigate the state of nature, and then publish reports $r$ on the policy issue. After learning the rivals’ reports, the media firms simultaneously choose their prices. At period 3, the public make their purchase decisions and read the report if choosing to buy. After reading the report, the public form their effective public opinion on the policy issue. At period 4, the policymaker announces the instituted policy. At the end, the state of nature $\theta$ and the signal $s$ are revealed to all players. Our solution concept is the Perfect Bayesian Equilibrium.

2.1 The policymaker

The policymaker is concerned about her career and reputation while motivated to make the right decision for her constituency. She chooses the signal $s(\cdot)$ revealed to the media, and following that she decides the implemented policy $y \in \{-1, 0, 1\}$ based on the gathered information from the public after the media reports. Her preference is represented by the utility function

$$U^p(y, s; \theta, (\phi^*, \alpha)) = -(y - \hat{y}(s; \phi^*, \alpha))^2 - \lambda_1 (y - \theta)^2 - \lambda_2(s - \theta)^2,$$  \hfill (1)

where the first term on the R.H.S. captures the policymaker’s career concern, defined as the difference between the implemented policy $y$, and the effective public opinion, $\hat{y}(\cdot, \cdot)$. The effective public opinion $\hat{y}(\cdot, \cdot)$ summarizes how the policymaker aggregates individual opinions about the policy issue in society. The effective public opinion depends on the distribution of the public’s innate preference over the focal policy issue. As we will discuss later, we assume that this distribution is summarized by a pair $(\phi^*, \alpha)$, where $\phi^* \in \{-1, 0, 1\}$ is the mainstream policy preference and $\alpha$ is the fraction of consumers who have this preference on the focal policy issue (to be described later). The second term, $\lambda_1 (y - \theta)^2$, reflects the policymaker’s incentive to serve her constituency by setting up the policy close to the state of nature. The third term highlights the policymaker’s

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8Alberto and Tabellini (2007), Dewatripont et al. (1999a) and Dewatripont et al. (1999b) have extensive discussions about the policymaker’s objectives and concerns.
reputation concern while she tries to manipulate the policy agenda. Essentially the signal $s$ sent by the policymaker cannot be too far away from the state of nature.

The degrees of how the policymaker cares about her constituency and her own reputation relative to her career concern are measured by the positive coefficients, $\lambda_1$ and $\lambda_2$. With these two parameters, we explicitly take into account the political accountability of the policymaker. The larger $\lambda_1$ and $\lambda_2$, the more the policymaker cares about her constituency and the reputation.

When the policymaker strongly cares about her reputation, she always tells the truth. To make our analysis interesting, we assume that the policymaker weakly cares about her constituency

$$0 < \lambda_1 < 1,$$

and her reputation

$$0 < \lambda_2 < (1 - \rho)\lambda_1.$$  

where $0 < \rho < 1$ is the probability that a media firm learns the state of nature after investigation (to be discussed later). In this case, reputation concern cannot deter the policymaker from strategic mis-signaling and agenda manipulation.

### 2.2 The Media Firms

In order to gather or even manipulate public opinion regarding certain policy issues, the policymaker discloses the signal $s$ to the media firms. These media firms desire to maximize the viewership and the corresponding profits through conforming to the public’s policy preference in their reports. However, complete conformity to the public’s policy preference might hurt media’s reputation if the state of nature turns out to be different at the end. This difference could drive away the readers in the future events, and consequently reduce the future profit. Therefore, depending on the informativeness of the signal $s$, the media firms may invest a costly effort $e_j \in \{0, 1\}$ to investigate the issue in order to minimize the difference between their report and the “truth” they find out. We incorporate this “value of reputation” in the media’s payoff function. We

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9While $\lambda_1$ and $\lambda_2$ are exogenously given, we illustrate their roles in the analysis, i.e., the higher $\lambda_1$, the policy maker is more likely to directly intervene the policy making process. The higher $\lambda_2$, the policymaker is less likely to cheat. We leave the fully endogenous effect of media on political accountability in the future research. Conditions C1-C4 shown below restrict our attention to the more interesting cases here.

10The first two terms of equation (1) are bounded, so the policymaker will always choose $s(\theta) = \theta$ if $\lambda_2$ is sufficiently large.

11Gentzkow and Shapiro (2006) also consider the case that the media are motivated by firms’ desire to build up a reputation of accuracy.

12Even though consumers prefer to read the report close to their prior beliefs, they also demand the “truth” as discussed in Mullainathan and Shleifer (2005). The “value of reputation” term characterizes the media firms’
assume that media’s investigation is imperfect such that the investigation leads to the outcome, $z_j$, where the outcome is the true state $z_j = \theta$ with probability $0 < \rho < 1$, and nothing $z_j = \emptyset$ otherwise.

The media firms, either independent or owned by the government, could have their own economic or political motivation (Anderson and McLaren 2012). They usually have their own stances on certain issues, which are characterized by their specific reporting strategies, $r_j(\cdot)$ for firm $j$, $j = 1, 2, ..., J$. Once receiving the signal and the investigation outcome\(^\text{13}\), based on their own interpretation and the consumers’ prior opinions on the issue, media firms choose their optimal report $r_j$ to maximize their profit while minimizing the possible reputation contamination if too far away from the state of nature.

The media firms may charge different consumers different prices. This practice is common in the digital media market, and is facilitated by the wide adoption of big data analytic. In the big data era, the media firms may utilize the rich information of consumer behavior to infer their reading habits and policy preference. The knowledge of these consumer traits helps the media firms price discriminate among consumers. In our analysis, we assume that the media firms charge different prices to each consumer. We denote $p_{ij}$ as the price that the media $j$ charges consumer $i$ for its news report.\(^\text{14}\)

In summary, the expected payoff of a media firm $j$ consists of the revenue from its report $r_j(\cdot)$, the value of reputation, and the cost of investigation, specified as the following,

$$
\Pi_j(r_j(\cdot), e_j, \{p_{ij}\}) = E_i[p_{ij} \cdot 1_{\text{consumer } i \text{ buys}}] - \gamma E_\theta[r_j(s, z) - \theta|s, z|^2 - ce_j^2],
$$

where

$$
z_j = \begin{cases} 
\theta, & \text{with probability } \rho, \text{ if } e_j = 1; \\
\emptyset, & \text{with probability } 1 - \rho, \text{ if } e_j = 1; \\
\emptyset, & \text{if } e_j = 0.
\end{cases}
$$

In the above payoff function, the first term summarizes the media firm’s revenue. The revenue depends on the market size and the average revenue. The former is normalized to one (to be discussed later), and the latter depends on the media $j$’s pricing scheme, $p_{ij}$, and the proportion of consumers who purchase and read the report, based on their policy preference about the issue. The second term captures the value of reputation for the media. Even though the state of nature, $\theta$, is unknown, a media firm could improve the accuracy of its report by investing efforts in incentives to find out and consequently report the truth.

\(^\text{13}\)If the media does not investigate, then the outcome is $\emptyset$.

\(^\text{14}\)This term loosely captures both the subscription fee and advertising fee received by the media firm proportional to its overall media viewership.
the investigation of the issue.\textsuperscript{15} The third term is the cost of a media outlet with effort level \( e_j \in \{0, 1\} \).\textsuperscript{16} To make our analysis interesting, we assume that the media investigation is efficient.

\[ 0 < c < \frac{\rho \gamma}{3}. \]  

(C3)

In summary, the media firm \( j \)'s problem is

\[
\max_{r_j(\cdot), e_j, \{p_{ij}\}} \Pi_j(r_j(\cdot), e_j, \{p_{ij}\}) = \max_{r_j(\cdot), e_j, \{p_{ij}\}} E_i[p_{ij} \cdot \mathbb{1}_{\text{consumer buys}}] - \gamma E_{\theta}[r_j(s, z) - \theta|s, z]^2 - ce_j^2. 
\]

(3)

### 2.3 The Public

There is a unit mass of consumers who have heterogeneous preferences over the policy issue in question. We denote consumer \( i \)'s policy preference as \( \phi_i \in \{-1, 0, 1\} \). For simplicity, we assume that the distribution of consumers’ policy preferences is summarized by a pair \((\phi^*, \alpha)\), where \( \phi^* \in \{-1, 0, 1\} \) is a mainstream preference and \( \frac{1}{3} < \alpha < 1 \) is the fraction of consumers who have this policy preference. The remaining \( 1 - \alpha \) fraction of consumers are evenly divided into two groups, each of which shares one of the other two policy preferences \( \phi \in \{-1, 0, 1\} \setminus \{\phi^*\} \).\textsuperscript{17} The fraction \( \alpha \) measures how strong the public policy preference is clustered at the mainstream policy preference \( \phi^* \). We say that the mainstream policy preference is left, neutral, or right if \( \phi^* \) is -1, 0, or 1, respectively.

Once the media publish the reports on the issue, consumers decide whether and which report to buy. Consumers could observe the general stance of a media firm say through the title of a report, but they have to pay for reading the content.\textsuperscript{18} Consumers derive utility only from reading the content of a report. Specifically, consumer \( i \)'s utility of reading media firm \( j \)'s report is

\[
U_{ij} = R_i + v_i \cdot \mathbb{1}_{r_j = \phi_i} - p_{ij} 
\]

In the utility function above, the first term \( R_i \) is consumer \( i \)'s willingness to pay for reading a genuine report. We assume that \( R_i \) is i.i.d., and denote \( \bar{R} = E[R_i] \) as the expectation of \( R_i \). The second term describes the additional surplus that yields to consumer \( i \) when media firm \( j \)'s report

\textsuperscript{15}Comparing with no investigation, investigation allows the media firm to learn the true state \( \theta \) with probability \( \rho \).

\textsuperscript{16}The cost of the media firms could include things like supporting materials and reporters’ compensation.

\textsuperscript{17}For example, if \( \phi^* = 0 \), then \( \alpha \) fraction of consumers prefer policy 0, \( \frac{1-\alpha}{2} \) fraction of consumers prefer policy -1, and \( \frac{1+\alpha}{2} \) fraction of consumers prefer policy 1. Our results still hold even when consumers with alternative preferences are not evenly divided but the difference between the two fractions is small.

\textsuperscript{18}For example, consumers are usually free to read the title and the first few lines of a news article, but they have to pay for reading the whole article.
r_j conforms to her policy preference \( \phi_i \in \{-1, 0, 1\} \). Our specification is in the spirit of Calvert (1985) and Suen (2004) in which they show rational consumers optimally demand information that is biased towards their prior opinions.\(^{19}\) The parameter \( v_i > 0 \) captures how much consumer \( i \) values the conformity of the media report to her policy preferences \( \phi_i \). We assume that \( v_i \) is i.i.d., and denote \( \bar{v} = E[v_i] \) as the expectation of \( v_i \). In our analysis, we assume that on average, consumers have high willingness to pay for the media report that is conformed to their policy preferences.

\[
\bar{v} > \max\{\frac{2\gamma}{1-\alpha}, \frac{8\gamma}{3\alpha - 1}\}. \tag{C4}
\]

The third term is the media firm \( j \)'s price changed to consumer \( i \).

Consumer \( i \) chooses the report that yields her the highest surplus, so she makes her purchase decision based on the following utility maximization problem,

\[
\max_{j \in \{1,2,...,J\}} U_{ij} = \max_{j \in \{1,2,...,J\}} \{ R_i + v_i \cdot 1_{r_j = \phi_i} - p_{ij} \}. \tag{5}
\]

We maintain the following tie-breaking rules: Among the reports that yield her the highest surplus, i) consumer \( i \) chooses one with equal probability from the set of reports that are conformed to her policy preference (e.g., \( r_j = \phi_i \)), and ii) if i) is infeasible, then consumer \( i \) chooses one with equal probability from the set of reports that are not conformed to her policy preference (e.g., \( r_j \neq \phi_i \)). In other words, consumer \( i \) prefers a news report that is conformed to her policy preference to an otherwise news report that yields her the same surplus.

After reading the media report, consumers update their beliefs about the state of nature using Bayes Rule. Then consumers generate their effective public opinion based on their posterior beliefs about the state of nature and their policy preferences. Consumer \( i \)'s utility of the policy \( y \) is

\[
-\eta E_{\theta}[(y - \theta)^2|r_1, r_2, ..., r_J] - (1 - \eta)(y - \phi_i)^2
\]

where \( 0 < \eta < 1 \) \((0 < 1 - \eta < 1)\) measures the extent that consumer \( i \) cares about the truth (her own policy preference \( \phi^* \)). The larger (smaller) is \( \eta \), the stronger consumer \( i \) cares about the truth (her own policy preference \( \phi^* \)). The effective public opinion \( \hat{y} \) maximizes the total consumer surplus of the policy

\[
\hat{y} \in \arg \max_{y \in \{-1,0,1\}} E_i[-\eta E_{\theta}[(y - \theta)^2|r_1, r_2, ..., r_J] - (1 - \eta)(y - \phi_i)^2] \tag{6}
\]

\(^{19}\)This conformity toward consumers’ beliefs is also studied in Gentzkow and Shapiro (2006) and Gentzkow and Shapiro (2010) where the former assumes full coverage and the latter finds this type of readers’ preference is economically significant.
2.4 Policy Outcome

The policy outcome is the instituted policy \( y^* \) that maximizes (1), and is evaluated based on the ex-post consumer policy surplus. The ex-post consumer policy surplus is the overall consumer policy surplus under the instituted policy \( y^* \) after the state of nature \( \theta \) is revealed to everyone

\[
PS(y^*, \theta) = E_i[-\eta(y^* - \theta)^2 - (1 - \eta)(y^* - \phi_i)^2].
\]  

The implemented policy \( y^*_1 \) is better than \( y^*_2 \) in the state \( \theta \) if \( PS(y^*_1, \theta) > PS(y^*_2, \theta) \). Moreover, the instituted policy \( y^*_1 \) is overall better than \( y^*_2 \) if \( PS(y^*_1, \theta) \geq PS(y^*_2, \theta) \), \( \forall \theta \) and \( PS(y^*_1, \theta_0) > PS(y^*_2, \theta_0) \), for some \( \theta_0 \in \{-1, 0, 1\} \).

The policy surplus \( PS(y^*, \theta) \) is the same as the consumer policy surplus under the instituted policy \( y^* \) when there is perfect information. We compare \( PS(y^*, \theta) \) to the highest consumer policy surplus under perfect information when the state of nature is \( \theta \)

\[
PS^*(\theta) = \max_{y \in \{-1, 0, 1\}} E_i[-\eta(y - \theta)^2 - (1 - \eta)(y - \phi_i)^2].
\]  

If \( PS(y^*, \theta) = PS^*(\theta) \), then the instituted policy \( y^* \) is the optimal policy under the state of nature \( \theta \). In this case, the ex-post consumer policy surplus reaches its first best.

3 The State-owned Media

We start our analysis of the policymaking process in the case of state-owned media to shed some lights on the effects of government ownership. Public Interest (Pigouvian) theory argues that government ownership is beneficial, whereas Public Choice theory suggests that government ownership undermines economic and political freedoms Djankov et al. 2003). Often the state-owned media act as the mouthpiece for the government or the ruling party. Consequently, the media will not devote any effort to investigating the issue (i.e., \( e_j = 0 \)) but just convey the signal sent from the policymaker to the public. Using the media as the perfect agent, the policymaker tries to manipulate the policy agenda in certain direction.

The public often have different preferences on certain policy issues. If the policymaker takes into account these heterogeneous opinions, she has to aggregate them into “effective public opinion” (Kennamer 1992) in certain ways. The effective public opinion summarizes the public stance of the issue, and is given by the effective public opinion \( \hat{y} \) in (6). Our specification of the aggregated consumers’ desired optimal policy allows us to analyze how consumers with different policy preferences are differently influenced by the policymaker’s behavior of aggregating public opinion. The policymaker often pays more attention to the majority of the public because of their career
or reputation concerns. Therefore, when the policymaker aggregates public opinion, she focuses on the mainstream preference $\phi^*$ and the extent $\alpha$ that the public lean toward the mainstream preference.

Consequently, the policymaker chooses the signal and the instituted policy by taking into account the effective public opinion. We first look at the separating equilibrium in the state-owned media market, and then discuss the non-separating equilibrium.\textsuperscript{20}

\textbf{Lemma 1.} Under (C1) - (C4), there exists a fully separating equilibrium in the state-owned media market if any of the following holds

i) the mainstream policy preference is either left or right (e.g., $\phi^* = -1, 1$ and $\frac{1}{3} < \alpha < 1$), and the public strongly cares about the truth (e.g., $\max\{0, 1 - \frac{1}{3\alpha - 1}\} < \eta < 1$);

ii) the mainstream policy preference is either left or right (e.g., $\phi^* = -1, 1$ and $\frac{1}{3} < \alpha < 1$), and the public strongly cares about their own policy preferences (e.g., $0 < \eta < \max\{0, 1 - \frac{3}{3\alpha + 1}\}$);

iii) the mainstream policy preference is neutral (e.g., $\phi^* = 0$ and $\frac{1}{3} < \alpha < 1$).

In equilibrium, the policymaker always signals the state of nature $s^* = \theta$, and the state-owned media never investigates $e^* = 0$ and always reports the signal $r^* = s$.

Lemma 1 shows that there exists a fully separating equilibrium if the mainstream preference $\phi^*$ is sidewise and the public strongly cares about the truth or their own policy preferences, or the mainstream preference $\phi^*$ is neutral. In equilibrium, the state-owned media chooses not to investigate (e.g., $e^* = 0$) and always tells the truth (e.g., $r^* = s^* = \theta$). The state-owned media usually acts as the mouthpiece for the government or the ruling party, so they have no incentive to devote any effort to the investigation. The policymaker uses it as a perfect agent to convey the signal to the public.

The policymaker also have incentives to tell the truth if the three conditions in lemma 1 is true. First, when the mainstream policy preference is either left or right (e.g., $\phi^* = -1, 1$ and $\frac{1}{3} < \alpha < 1$), and the public strongly cares about the truth (e.g., $\max\{0, 1 - \frac{1}{3\alpha - 1}\} < \eta < 1$), the public’s posterior belief about the state of nature determines the effective public opinion $\hat{y}$. The effective public opinion $\hat{y}$ will be close to or even coincide with the state of nature $\theta$ if the public learns the truth from the media report. In other words, the policymaker’s concerns for career, constituency and reputation are almost or even already perfectly aligned, so strategic mis-signaling cannot further improve the implemented policy $y^*$ for the policymaker. Therefore, in equilibrium, the policymaker tells the truth.

Second, when the mainstream policy preference is either left or right (e.g., $\phi^* = -1, 1$ and $\frac{1}{3} < \alpha < 1$), and the public strongly cares about their own policy preferences (e.g., $0 < \eta < \frac{1}{3}$, $\alpha < 1$).

\textsuperscript{20}The state-owned media tells the truth if and only if the policymaker tells the truth. In this section, we use the state-owned media and the policymaker interchangeably when describing the truth-telling behavior.
max\(\{0, 1 - \frac{3}{3\alpha + 1}\}\), the mainstream preference \(\phi^*\), determines the effective public opinion \(\hat{y}\). In this case, the public simply ignore their posterior belief about the state of nature in forming the effective public opinion \(\hat{y}\). The policymaker is unable to influence the public opinion by strategic mis-signaling as they are too fanatical about their own policy preference. Since sending a dishonest signal is costly to the policymaker, she tells the truth in the equilibrium.

Third, when the public policy preference is toward central (e.g., \(\phi^* = 0\)), then there is an equilibrium in which the state-owned media chooses not to investigate (e.g., \(e^* = 0\)) and always tells the truth (e.g., \(s^* = \theta\)). The state-owned media usually acts as the mouthpiece for the government or the ruling party, so it has no incentive to devote any effort to investigating the issue. The policymaker uses it as a perfect agent to convey the signal to the public. If the state of nature is also neutral (e.g., \(\theta = 0 = \phi^*\)), then the policymaker has no incentive to send a misleading signal to the media, as doing so incurs a reputation cost \(\lambda_2 > 0\) and may direct the effective public opinion away from the state of nature. Both consequences decrease the policymaker’s utility in (1). In fact, in this case, the policymaker receives her maximal utility zero when she tells the truth. If the policymaker sends a misleading signal to the public, then it not only incurs a reputation cost to the policymaker, but also may push the effective public opinion further away from the state of nature. Both of them make the policymaker worse-off, so the policymaker will report the state of nature even if it is not consistent with the mainstream preference.

**Corollary 1.** Under \((C1) - (C4)\), in equilibrium, the state-owned media leads to the optimal policy outcome if any of the following holds

i) the mainstream policy preference is either left or right (e.g., \(\phi^* = -1, 1\) and \(\frac{1}{3} < \alpha < 1\), and the public strongly cares about the truth (e.g., \(\max\{0, 1 - \frac{1}{\alpha + 1}\} < \eta < 1\));

ii) the mainstream policy preference is either left or right (e.g., \(\phi^* = -1, 1\) and \(\frac{1}{3} < \alpha < 1\), the public strongly cares about their own policy preferences (e.g., \(0 < \eta < \max\{0, 1 - \frac{3}{3\alpha + 1}\}\)), and the policymaker’s constituency concern is trivial (e.g., \(0 < \lambda_1 < \frac{1}{3}\));

iii) the mainstream policy preference is neutral (e.g., \(\phi^* = 0\) \(\frac{1}{3} < \alpha < 1\)).

Corollary 1 shows that in the state-owned media market, the public could obtain the optimal policy outcome if they learn the state of nature from the news report and the policymaker highly respects the public opinion. Comparing (6) with (8), the effective public opinion \(\hat{y}\) is the same as the optimal policy if the public learn the state of nature \(\theta\) from the news report. Then the public will enjoy the optimal policy outcome if the implemented policy \(y^*\) is consistent with the effective public opinion \(\hat{y}\). There are two cases in which the policymaker follows the effective public opinion. One is that policy distortion is unnecessary as the effective public opinion \(\hat{y}\) is close to or even coincide with the state of nature \(\theta\) (e.g., condition i) and iii)), and the other one is that policy distortion is unpopular as the policymaker highly respects the effective public opinion (e.g., case
Both cases highlight the importance of efficient information transmission and transparency in achieving the optimal policy outcome.

Lemma 1 and Corollary 1 show that the state-owned media will lead to the optimal policy outcome if the public has an extreme preference for the truth or their own opinions, or the mainstream policy preference is central. In practice, however, the mainstream policy preference is often sidewise. Moreover, majority of consumers might be sophisticated and try to balance between the truth and their own opinion. Instead, they moderately care about the truth, and generate the effective public opinion after full considerations of their posterior belief about the state of nature and their innate policy preference. The following lemma characterizes the equilibrium in this case.

**Lemma 2.** Under \((C1) - (C4)\), there exists a semi-separating equilibrium in the state-owned media market if the mainstream policy preference is either left or right (e.g., \(\phi^* = -1, 1\) and \(\frac{1}{3} < \alpha < 1\)), and the public moderately cares about the truth (e.g., \(\max\{0, \frac{3\alpha - 2}{3\alpha}\} < \eta < \max\{0, \frac{3\alpha - 2}{3\alpha - 1}\}\)).

In equilibrium, the policymaker signals \(s^*\), where

\[
  s^*(\theta) = \begin{cases} 
  \phi^*, & \text{if } \theta = \phi^*; \\
  -\phi^*, & \text{otherwise.} 
  \end{cases}
\]

and the state-owned media never investigates \(e^* = 0\) and always reports the signal \(r^* = s\).

Clearly, the fully separating equilibrium no longer exists in the state-owned media market if the public is sophisticated and the mainstream policy preference is sidewise. When the public moderately cares about the truth, the discrepancy between the public policy preference and their posterior belief about the state of nature determines the effective public opinion. The effective public opinion \(\hat{y}\) will be the mainstream preference \(\phi^*\) (the neutral 0) if the public has a strong preference towards the side and they believe that the discrepancy is small (large). This effective public opinion generating process provides the policymaker an opportunity to manipulate the agenda via strategic mis-signaling. For example, when the state of nature is neutral 0, the policymaker can overstate the discrepancy by signaling \(-\phi^*\). Then the effective public opinion \(\hat{y}\) moves from \(\phi^*\) to 0, and the policymaker is better-off.

Although the fully separating equilibrium does not exist under this scenario, lemma 2 shows that there exists a semi-separating equilibrium in the state-owned media market if the public is sophisticated and the mainstream policy preference is sidewise. In equilibrium, the policymaker sends a pooling signal if the state of nature \(\theta\) is 0 or \(-\phi^*\). Interestingly, the policymaker never wants the public to learn the truth if the state of nature is neutral 0. Instead, the policymaker misleads the public into overstating the discrepancy, and manipulates the effective public opinion into moving towards the neutral 0. Such strategic mis-signaling resolves the disparity between
the policymaker’s career concern and her constituency, and improves the policymaker’s surplus. Thus, the policymaker has incentives to manipulate the signal.

Upon reading the pooling news report, the public is not informed about the state of nature \( \theta \), so the effective public opinion \( \hat{y} \) may not maximize the ex-post consumer policy surplus. Therefore, the instituted policy \( y^* \) is not optimal. For example, when the state of nature is neutral 0, the equilibrium implemented policy \( y^* \) is 0 whereas the optimal policy is \(-1\). The following corollary summarizes this result.

**Corollary 2.** Under (C1) - (C4), in equilibrium, the state-owned media leads to a suboptimal policy outcome if the mainstream policy preference is either left or right (e.g., \( \phi^* = -1, 1 \) and \( \frac{1}{3} < \alpha < 1 \)), and the public moderately cares about the truth (e.g., \( \max\{0, \frac{3\alpha - 2}{3\alpha}\} < \eta < \max\{0, \frac{3\alpha - 2}{3\alpha - 1}\} \)).

Comparing corollary 2 to corollary 1, the state-owned media may lead to a worse policy outcome if consumers become sophisticated. In other words, the efficiency of information transmission in the state-owned media market depends on the market preference for the truth \( \eta \) (and individual opinions \( 1 - \eta \)). The public learns the state of nature from the equilibrium news report if they strongly care about the truth or their own opinions. The public’s extreme preference for the truth or their own opinions eliminates the policymaker’s incentive to strategically mis-signal as doing so is neither necessary nor effective. Therefore, in equilibrium, the public obtains the optimal policy outcome. On the other hand, if the public moderately cares about the truth, then the policymaker will exploit the moderateness of the preference and overstate the discrepancy between the state of nature \( \theta \) and the mainstream preference \( \phi^* \). As a result, the public is misled by the pooling news report, and the effective public opinion \( \hat{y} \) moves to the actual state of nature \( \theta \). Such strategic mis-signaling benefits the policymaker, but lead to a suboptimal policy outcome. The following proposition summarizes these results.

**Proposition 1.** Under (C1) - (C4), in equilibrium,

i) the state-owned media may lead to the optimal policy outcome if the public strongly cares about the truth or their own policy preferences;

ii) the state-owned media may lead to a suboptimal policy outcome if the public moderately cares about the truth.

Proposition 1 suggests why the state-owned media may not be popular in general. For example, in most developed and democratic economies, independent media rather than state-owned media dominate the market. In fact, if the public is mature and sophisticated that they have a balanced preference for the truth and individual opinions, then the policymaker can exploit this complexity of the public philosophy and manipulate the effective public opinion by strategic mis-signaling. The pooling news report (offered by the state-owned media) makes the public uninformed about
the state of nature, and therefore, results in a suboptimal policy outcome. On the other hand, however, in the extreme situations, such as the focal policy issue being extremely important or completely irrelevant to the public, the public will have a simple extreme preference for the truth or strong preference about their own opinion. With this simple public philosophy, the policymaker finds it either unnecessary or ineffective to manipulate the agenda, so she tells the truth and this leads to the optimal policy outcome. For example, in 2003, the China state-owned media’s honest reports of SARS helped the public quickly learn the truth and minimized the public panic.

Clearly, the state-owned media may be inefficient in reporting general policy issues in mature markets. Then a natural question is that whether independent media can lead to a better policy outcome under the same market conditions. We explore this question in our remaining analysis and start with the independent media monopoly.

4 The Media Monopoly

The media are often independently-owned rather than government-owned in democratic societies to ensure press freedom; however, media consolidation in recent years has been dramatic. In the United States, 25% of metropolitan standard areas are served by only one newspaper (George and Waldfogel 2003). Countries like Italy and Turkey are also known for having a highly concentrated media market. We study in this section how a media monopoly could distort the policymaker’ agenda manipulation and consequently influence the policy outcome. In particular, we want to understand whether the media monopoly can lead to a better policy outcome than the state-owned media does.

The media monopoly first chooses its reporting strategy and investigation effort level based on the signal received from the policymaker, then decides the price for the report. For simplicity, we assume that the market is fully covered in equilibrium. The following lemma describes the equilibrium.

**Lemma 3.** Under (C1) - (C4), in equilibrium, the policymaker always signals the state of nature \( s^* = \theta \), and the media monopoly never investigates (e.g., \( e^* = 0 \)), always reports \( r^* = \phi^* \) and charges \( p^*_i = R_i + v_i \cdot 1_{\phi_i = r^*} \).

Lemma 3 shows that in equilibrium, the media monopoly never investigates and simply aligns its report with the mainstream preference \( \phi^* \) if consumers have high willingness to pay for the conformity of the media report to their policy preference. Consumers’ demand for conformity to their policy preferences divides the market into three consumer segments, based on the public’s preferences on the focal policy issue. When the media monopoly chooses its stance, it faces the
tradeoff between the profit from the consumers’ demand for conformity and the possible reputation cost. If consumers have high willingness to pay for the conformity, then the media monopoly will find it profitable to target the consumer segment that consumers have the mainstream preference $\phi^*$. In this case, the media monopoly always fits its report into the mainstream preference $\phi^*$ even when it knows that doing so would incur a reputation cost up to $4\lambda_2$. Moreover, media investigation is unnecessary as the media monopoly always reports $\phi^*$, so in equilibrium, the media monopoly never investigates.

The policymaker always tells the truth as strategic mis-signaling no longer helps shape the effective public opinion. Due to the media monopoly, all signals from the policymaker are translated into reports that fit the mainstream preference $\phi^*$. Thus, the policymaker tells the truth to avoid the reputation cost. In contrast to the second case of lemma 1, lemma 3 provides an alternative mechanism that explains why the policymaker always tells the truth if strategic mis-signaling does not help manipulate the effective public opinion. This new mechanism comes from the failure of information transmission from the policymaker to the public when the media monopoly translates the signals into reports that are conformed to the mainstream preference $\phi^*$, whereas the former one relies on the receiver’s (public’s) no response to the signal as the public strongly cares about their own opinion, so they do not take the policymaker’s signal seriously in generating the effective public opinion.

In equilibrium, due to the media monopoly’s pooling news report, the public remains uninformed about the state of nature even when the policymaker always tells the truth. When the effective public opinion $\hat{y}$ critically depends on the state of nature $\theta$, the public’s ignorance of the true state may lead to a suboptimal policy outcome. The following corollary summarizes the claim.

**Corollary 3.** Under $(C1) - (C4)$, in equilibrium, the media monopoly leads to a suboptimal policy outcome if any of the following holds

i) the mainstream policy preference is either left or right (e.g., $\phi^* = -1, 1$ and $\frac{1}{3} < \alpha < 1$), and the public strongly cares about the truth (e.g., $\max\{\frac{3\alpha-2}{3\alpha-1}, \frac{3\alpha-2}{3\alpha-3}\} < \eta < 1$);

ii) the mainstream policy preference is neutral (e.g., $\phi^* = 0$ and $\frac{1}{3} < \alpha < 1$), and the public strongly cares about the truth (e.g., $\frac{1}{2} < \eta < 1$).

Corollary 3 shows that the media monopoly leads to a suboptimal policy outcome if the public strongly cares about the truth. When the public strongly cares about the truth, the public’s posterior belief about the state of nature $E[\theta|r^*]$ determines the effective public opinion $\hat{y}$. However, due to the media monopoly’s pooling news report, in equilibrium, the public remains uninformed about the state of nature $\theta$, so they form the effective public opinion based on their prior beliefs.
about the state of nature $E[\theta]$. As long as there is a discrepancy between the actual state of nature $\theta$ and the public’s prior belief $E[\theta]$, the implemented policy $y^*$ is never optimal.

Conversely, if the public strongly cares about their own policy preferences and the policymaker’s constituency concern is trivial, then the media monopoly may lead to the optimal policy outcome. When the public strongly cares about their own policy preferences, the mainstream preference $\phi^*$ could determine the effective public opinion $\hat{y}$. The effective public opinion $\hat{y}$ coincides with the the optimal policy, so the public is indifferent between a pooling report and an informative one as learning the actual state of nature $\theta$ no longer helps the public obtain a better policy outcome.

With a weak concern for constituency, the policymaker follows the effective public opinion $\hat{y}$ and implements the optimal policy. The follow corollary summarizes this result.

**Corollary 4.** Under (C1) - (C4), in equilibrium, the media monopoly leads to the optimal policy outcome if any of the following holds

i) the mainstream policy preference is either left or right (e.g., $\phi^* = -1, 1$ and $\frac{1}{3} < \alpha < 1$), the public strongly cares about their own policy preferences (e.g., $0 < \eta < \max\{\frac{3\alpha-2}{3\alpha+1}, \frac{3\alpha-2}{3\alpha-3}\}$), and the policymaker’s constituency concern is trivial (e.g., $0 < \lambda_1 < \frac{1}{3}$);

ii) the mainstream policy preference is neutral (e.g., $\phi^* = 0$ and $\frac{1}{3} < \alpha < 1$), and the public strongly cares about their own policy preferences (e.g., $0 < \eta < \frac{1}{2}$).

Corollary 3 and 4 suggest that the public could obtain the optimal policy outcome in the monopoly media market if they do not care about the informativeness of a media report. Conversely, if learning the state of nature helps the public form a better opinion on the focal policy issue, then the media monopoly will lead to a suboptimal policy outcome. However, this is just one side of the story. The following proposition shows that the public could enjoy the optimal policy outcome even when they care the informativeness of a media report.

**Proposition 2.** Under (C1) - (C4), in equilibrium, the media monopoly may lead to the optimal policy outcome if the public cares about the informativeness of a media report.

In contrast to corollary 3, proposition 2 shows that the public may achieve the optimal policy outcome in the monopoly media market even if they care the informativeness of a media report. To see why, consider the case in which the mainstream policy preference is either left or right (e.g., $\phi^* = -1, 1$ and $\frac{1}{3} < \alpha < 1$), the public moderately cares about the truth (e.g., $\max\{0, \frac{3\alpha-2}{3\alpha+1}\} < \eta < \max\{0, \frac{3\alpha-2}{3\alpha-3}\}$), and the policymaker’s constituency concern is not trivial (e.g., $\frac{1}{3} < \lambda_1 < 1$). By lemma 2, with a moderate market preference for the truth, the discrepancy between the state of nature $\theta$ and the mainstream preference $\phi^*$ influences the optimal policy. The optimal policy moves from the mainstream preference $\phi^*$ towards the state of nature $\theta$ as the discrepancy between them increases. For example, the optimal policy will be central 0 if the state of nature $\theta = -\phi^*$ is the
opposite to the mainstream preference $\phi^*$. On the other hand, the same discrepancy also influences the instituted policy $y^*$. With a non-trivial constituency concern, the policymaker implements the policy $y^*$ that balances the her career concern and constituency concern. Surprisingly, the implemented policy $y^*$ coincides with the optimal policy. The policymaker corrects the mistake made by the uninformed public, and secures the optimal policy outcome. Therefore, the public may still obtain the optimal policy outcome in the monopoly media market even if they care the informativeness of a media report.

Proposition 2 unveils the possibility that the media monopoly may lead to a better policy outcome than the state-owned media does. However, the comparison between the media monopoly and the state-owned media is more complicated. The following proposition provides the comparison and offers an answer to our beginning question that whether the media monopoly can lead to a better policy outcome than the state-owned media does.

**Proposition 3.** Under $(C1) - (C4)$, in equilibrium,

i) the media monopoly may lead to a better policy outcome than the state-owned media does if the public moderately cares about the truth;

ii) the media monopoly may lead to a worse policy outcome than the state-owned media does if the public strongly cares about the truth;

iii) the media monopoly may lead to the same optimal policy outcome as the state-owned media does if the public strongly cares about their own policy preferences.

Proposition 3 compares the policy outcome in the monopoly media market to that in the state-owned media market. First, the media monopoly may lead to a better policy outcome if the public moderately cares about the truth. This follows from the comparison between proposition 2 and corollary 2. When the public moderately cares about the truth, the state-owned media strategically mis-signals and manipulates the effective public opinion. The public prefers no information (e.g., an uninformative report) to false information (e.g., a misleading report), so they obtain a better policy outcome in the monopoly media market. Second, the media monopoly may lead to a worse policy outcome if the public strongly cares about the truth. This follows from the comparison between corollary 3 and corollary 1. When the public strongly cares about the truth, the state-owned media always tells the truth. The public prefers the correct information (e.g., an informative report) to no information (e.g., an uninformative report), so they achieve a better policy outcome in the state-owned media market. Third, both the media monopoly and the state-owned media may lead to the (same) optimal policy outcome if the public strongly cares about their own opinions. In contrast to the second case, now the public is indifferent between the correct information (e.g., an informative report) and no information (e.g., an uninformative report). Therefore, the public
is indifferent between the media monopoly and the state-owned media as they obtain the optimal policy outcome in both media markets.

Interestingly, with the introduction of the independent media monopoly, consumers are not necessarily better off with a better policy outcome. This indicates that independent media brings new incentives to the market that may further distort the information transmission between the policymaker and the public. The public, in the end, may be worse-off as compared to what they might have been in the state-owned media market. To further understand the impact of the independent media, we study whether market competition could mitigate the potential negative effect from these new distortions and help the public achieve a better policy outcome in the independent media market. We address this question in the next section.

5 The Media Duopoly

Most democratic countries are characterized by multiple media outlets which often have clear but different stands on certain policy issues. Mergers among media firms often ignite fears of diminished diversity of political opinions and weakened public discourse, besides mere profit motive (Anderson and McLaren 2012). We explore in this section whether and how market competition influences the incentives in the media market and in the policy making process, when competitive media firms exert divergent influence on consumers through the news reports and consequently the policy outcome.

Similar to the media monopoly, there are two forces that influence the media duopoly’s stances on the focal policy issue. One is the media firm’s reputation concern, and the other one is the profit motive. The former encourages the media duopoly to conduct necessary investigations and tell the truth, whereas the latter tempts the media duopoly to align their reports with the mainstream preference $\phi^*$. In contrast to the media monopoly case, the profit motive brings a new incentive to the duopoly media market that the media duopoly choose different stances if consumers have high willingness to pay for the conformity of the media report to their policy preference. In fact, when both media firms choose the same stance on the focal policy issue, their reports become homogenous, and price competition drives their profits to zero. Therefore, in duopoly media market, the profit objective motivates product differentiation, and induces some media firms to fit their report into the mainstream preference $\phi^*$. As long as consumers have high willingness to pay for the conformity to their preference, the media firm that chooses different stance can still be profitable by serving certain groups of consumers even when the stance is neither the state of nature $\theta$ nor the mainstream preference $\phi^*$.

The media firm’s profit motive and reputation concern unveil a dimension called the media’s
stance, along which competitive media firms can differentiate themselves. In duopoly media market, one media firm feeds consumers with what they want to hear, and the other one conduct necessary investigations and tell the truth. In this case, both media firms make positive profits if consumer have high willingness to pay for the conformity to their preference. In equilibrium, the public may learn the state of nature from the news report offered by one media firm. Consequently, the information transmission in the duopoly media market may be as efficient as that in the state-owned media market if the policymaker stays honest. The following lemma summarizes these conjectures.

**Lemma 4.** Under (C1) - (C4), there exists a fully separating equilibrium in the duopoly media market if any of the following holds

i) the mainstream policy preference is either left or right (e.g., $\phi^* = -1$, 1 and $\frac{1}{3} < \alpha < 1$), and the public strongly cares about the truth (e.g., $\max\{0, 1 - \frac{1}{3\alpha - 1}\} < \eta < 1$);

ii) the mainstream policy preference is either left or right (e.g., $\phi^* = -1$, 1 and $\frac{1}{3} < \alpha < 1$), and the public strongly cares about their own policy preferences (e.g., $0 < \eta < \max\{0, 1 - \frac{1}{3\alpha - 1}\}$);

iii) the mainstream policy preference is neutral (e.g., $\phi^* = 0$ and $\frac{1}{3} < \alpha < 1$).

In equilibrium, the policymaker always signals the state of nature $s^* = \theta$, the media firm 1 never investigates (e.g., $e_1^* = 0$), reports $r_1^*$ and charges $p_{11}^* = v_i \cdot 1_{\phi_i=r_1^*}$, and the media firm 2 never investigates (e.g., $e_2^* = 0$), reports $r_2^*$ and charges $p_{12}^* = v_i \cdot 1_{\phi_i=r_2^*}$ where

$$r_1^*(s, \phi^*) = s; \quad r_2^*(s, \phi^*) = \begin{cases} \phi^*, & \text{if } s \neq \phi^*; \\ -1, & \text{if } s = \phi^* = 0; \\ 0, & \text{if } s = \phi^* \text{ and } \phi^* \neq 0. \end{cases}$$

Lemma 4 shows that there exists a fully separating equilibrium if the mainstream preference $\phi^*$ is sidewise and the public strongly cares about the truth or their own opinions, or the mainstream preference $\phi^*$ is neutral. Compared to the state-owned media case, in equilibrium, the public learns the state of nature $\theta$ in the duopoly media market if they do so in the state-owned media market. The introduction of competitive independent media does not distort the information transmission in these scenarios. This result highlights the importance of market competition in mitigating distortions to the information transmission in the media market. Market competition leads to product differentiation, which prevents media firms from clustering and feeding consumers with what they want to hear. As long as a media firm chooses to deliver honest reports, the public may learn the state of nature from the news report.

From the policymaker’s perspective, the media duopoly delivers the same amount of information to the public as the state-owned media does if she is honest. In equilibrium, the policymaker
is indifferent between the media duopoly and the state-owned media, so she stays honest as she
does in the state-owned media market under the same extreme conditions. Upon receiving the
signal $s^*$ from the honest policymaker, both media firms learn the state of nature $\theta$, so they have
no incentive to conduct any costly and unnecessary investigation (e.g., $e_1^* = e_2^* = 0$).

In equilibrium, the public learns the state of nature $\theta$ from the (honest) media firm 1’s news
report $r_1^*$, so they obtain the same optimal policy outcome as they achieve in the state-owned
media market.\textsuperscript{21} We characterize this result in the following corollary.

\textbf{Corollary 5.} Under (C1) - (C4), in equilibrium, the media duopoly leads to the optimal policy
outcome if any of the following four conditions holds,

i) the mainstream policy preference is either left or right (e.g., $\phi^* = -1, 1$ and $\frac{1}{3} < \alpha < 1$), and
the public strongly cares about the truth (e.g., $\max\{0, 1 - \frac{1}{3\alpha - 1}\} < \eta < 1$);

ii) the mainstream policy preference is either left or right (e.g., $\phi^* = -1, 1$ and $\frac{1}{3} < \alpha < 1$), the
public strongly cares about their own policy preferences (e.g., $0 < \eta < \max\{0, 1 - \frac{3}{3\alpha + 1}\}$), and the
policymaker’s constituency concern is trivial (e.g., $0 < \lambda_1 < \frac{1}{3}$);

iii) the mainmainstream policy preference is neutral (e.g., $\phi^* = 0$ and $\frac{1}{3} < \alpha < 1$).

Corollary 5 shows that the public achieves the optimal policy outcome in the duopoly media
market if they learn the state of nature from the news report and the policymaker highly respects
the public opinion. The resemblance of the fully separating equilibrium and the policy outcome in
the competitive market to those in the state-owned media market illustrates that the efficiency of
information transmission in extreme situations could be unified across free and regulated societies
as long as market competition enhances the independent media firm’s incentive to be honest.
With a strong incentive to be honest, some competitive media firms report the truth and become
trustworthy. As shown in the next lemma, these (honest) firms are the key to keep the public
staying informed about the state of nature when market becomes mature and sophisticated.

\textbf{Lemma 5.} Under (C1) - (C4), there exists a semi-separating equilibrium in the duopoly media
market if the mainstream policy preference is either left or right (e.g., $\phi^* = -1, 1$ and $\frac{1}{3} < \alpha < 1$),
the public moderately cares about the truth (e.g., $\max\{0, \frac{3\alpha - 2}{3\alpha + 1}\} < \eta < \max\{0, \frac{3\alpha - 2}{3\alpha - 1}\}$).

In equilibrium, the policymaker signals $s^*$, the media 1 exerts investigation effort $e_1^*$, reports $r_1^*$
and charges $p_{11}^* = v_i \cdot 1_{\phi_i=r_1^*}$, and the media 2 never investigates (e.g., $e_2^* = 0$), reports $r_2^*$ and

\textsuperscript{21}By (6), the effective public opinion $\hat{y}$ is the same in both markets. Then the same policy outcome immediately
follows from the policy making process (6).
chages $p_{i2}^* = v_i \cdot 1_{\phi_i = r_2^*}$, where

$$s^*(\theta) = \begin{cases} 
\phi^*, & \text{if } \theta = \phi^*; \\
-\phi^*, & \text{otherwise.}
\end{cases}$$

$$e_1^*(s, \phi^*) = \begin{cases} 
0, & \text{if } s = \phi^*; \\
1, & \text{otherwise.}
\end{cases}$$

$$r_1^*(s, \phi^*) = \begin{cases} 
\phi^*, & \text{if } s = \phi^*; \\
-\phi^*, & \text{if } s = 0 \text{ and } z = -\phi^*; \\
-\phi^*, & \text{if } s = -\phi^* \text{ and } z = -\phi^* \text{ or } \emptyset; \\
0, & \text{otherwise.}
\end{cases}$$

Lemma 5 shows that there exists a semi-separating equilibrium in the duopoly media market if the public balance between their desire of the truth and their own opinion, and their policy preference is strongly biased towards the side. This semi-separating equilibrium is more efficient than the one in lemma 2 in information transmission as the public now can learn the state of nature neutral 0 with a positive probability $0 < \rho < 1$ from the (honest) media 1’s equilibrium report. This advantage of efficiency helps discipline the policymaker, and makes the media duopoly more popular in sophisticated markets than the state-owned media.

In contrast to lemma 4, the (honest) media firm 1 exerts efforts in investigating the state of nature $\theta$ upon receiving the pooling signal $-\phi^*$. The investigation helps the media firm 1 detect the state of nature $\theta$ and maintain its trustworthiness in the market. On the other hand, the possible imperfect investigation also leaves an opportunity to the policymaker to manipulate the agenda as strategic mis-signaling still works, though less effectively. In comparison to the state-owned media market, although less likely, the public may still obtain a suboptimal policy outcome in the duopoly media market if the state of nature is neutral $\theta$. The following corollary shows this result.

Corollary 6. Under (C1) - (C4), in equilibrium, the media duopoly leads to a suboptimal policy outcome if the mainstream policy preference is either left or right (e.g., $\phi^* = -1, 1$ and $\frac{1}{3} < \alpha < 1$), the public moderately cares about the truth (e.g., $\max\{0, \frac{3\alpha - 2}{3\alpha}\} < \eta < \max\{0, \frac{3\alpha - 2}{3\alpha - 1}\}$).

Corollary 6 shows that the public could obtain a suboptimal policy outcome in the duopoly media market if they are sophisticated and have a strong preference towards certain side. Although the suboptimal instituted policy $y^*$ in the duopoly media market is the same as that in the state-
owned media market, the policymaker is less likely to implement it in the duopoly media market. Market competition generates more accurate information for the consumers, and benefits them as indicated by the more truthful implemented policy.

The interplay between media competition and political institutions leads to government accountability that the policymaker becomes more responsible and provides a better policy outcome. The wishful-thinking type of incentives of the policymakers, such as constituency and reputation concern, are less important to hold them accountable in competitive media markets. Instead, the market competition and the effective media investigation are the very forces that discipline the policymakers and guarantee a better policy outcome. Therefore, the introduction of competitive independent media mitigates the policymaker’s distortion to the implemented policy, and benefits consumers overall.

We present this result in the following proposition.

**Proposition 4.** Under (C1) - (C4), in equilibrium, the media duopoly never leads to a worse policy outcome than the state-owned media does.

Proposition 3 and 4 illustrate the impact of independent media on policy outcomes, and highlight the importance of market heterogeneity in choosing the optimal market structure in media industry. In fact, both the independent media monopoly and the independent media duopoly may lead to a better policy outcome than the state-owned media does, so which one is more efficient in information transmission is subject to further study. In the following subsection, we perform a careful comparison between the three market structures of media ownership to address this question.

### 5.1 The Impact of Different Democratic Political Systems

In many democratic countries such as the United States, Australia and New Zealand, the public opinion on the focal policy issue more or less influences the implemented policy. The extent of public influence on the implemented policy varies across different democratic political systems.

How efficiently the information transmits in these democratic societies is important to shape public’s opinion toward the issue (e.g. \( E[\theta|\pi^*] \)), therefore help achieve a better policy outcome. Our analysis shows that with high willingness to pay for the conformity to their preference, the public may obtain a more informative equilibrium in the duopoly media market than in the state-owned media market, and have an uninformative equilibrium in the monopoly media market. Therefore, the public may prefer the duopoly media market to the state-owned media market, and consider the monopoly media market as the worst market structure of media ownership. We present this result in the following proposition.
Proposition 5. Under (C1) - (C4), there exist the equilibrium outcomes across different markets which rank as below,

\[ \text{Duopoly Media Ownership} \succeq \text{State-Owned Media Ownership} \succeq \text{Monopoly Media Ownership}. \]

Proposition 5 unveils a ranking of the market structures of media ownership that prevails in many democratic political systems. Such ranking suggests that the monopoly media ownership may be unpopular in democratic societies if consumers have high willingness to pay for the conformity to their preference.

However, this is only one side of the story. In the so-called populist democracy, the policymaker’s constituency concern is trivial (e.g., \(0 < \lambda_1 < \frac{1}{3}\)), while the implemented policy often follows the public opinion. On the other hand, in so-called conscientious democracy, the policymaker’s constituency concern is not trivial (e.g., \(\frac{1}{3} < \lambda_1 < 1\)) as she not only pay attention to the public opinion but also tries to maintain her conscience in serving public truthfully. In this case, the implemented policy follows the public opinion only if the public opinion is not a huge mistake. For example, consider the case in which the effective public opinion \(\hat{y}\) is the sidewise mainstream preference \(\phi^* \in \{-1, 1\}\), and the state of nature \(\theta\) is \(-\phi^*,\) the opposite of it. In populist democracy, the implemented policy \(y^*\) follows the effective public opinion \(\hat{y}\) and is \(\phi^*,\) whereas in the conscientious democracy, the implemented policy \(y^*\) balances the public opinion and the policymaker’s constituency concern and is neutral 0. Surprisingly, the monopoly media ownership may be the best in generating policy outcomes if the political system is conscientious democracy (e.g., \(\frac{1}{3} < \lambda_1 < 1\)). The following corollary shows this result.

Corollary 7. Under (C1) - (C4), under conscientious democracy (e.g., \(\frac{1}{3} < \lambda_1 < 1\)), in equilibrium, the public may rank the policy outcomes across markets as follows

\[ \text{Monopoly Media Ownership} \succeq \text{Duopoly Media Ownership} \succeq \text{State-Owned Media Ownership}. \]

Corollary 7 reveals another ranking of the market structures of media ownership in the elite democratic political system (e.g., \(\frac{1}{3} < \lambda_1 < 1\)). At first glance, this ranking seems counterintuitive as it suggests that the public may prefer the uninformative news report from the media monopoly. In fact, the public will find it optimal to stay ignorant by reading the media monopoly’s pooling news report if the alternative is being fooled by the policymaker through a mix of honest and misleading reports made by the media duopoly and the state-owned media.\(^{22}\) The public knows that the elite policymaker will correct their mistake if the mistake is significant, so they have

\(^{22}\)By lemma 2 and 5, both the state-owned media and the media duopoly may deliver a semi-separating equilibrium report to the public.
no incentive to play the agenda manipulation game with the policymaker and infer the state of
nature from the semi-separating equilibrium report. As a result, the public leaves the policymaker
no opportunity to manipulate the agenda, and achieves the optimal policy outcome through the
policymaker’s beneficial policy distortion.

In addition to information transmission efficiency, corollary 7 suggests that the political system
is also an important factor that determines the ranking of the market structures of media own-
ership. Indeed, the optimal market structures of media ownership varies across markets. With
market heterogeneity and the rich variation of political systems, it is possible to observe the
co-existence of state-owned media and independent media in a market.

6 Conclusions

We analyze the policymaker’s decision making process with the intervention of the media. We
show that a policymaker’s effort to push a certain agenda could be distorted by media bias and
the public’s prior opinions on the policy issue. Our results suggest that state-owned media could
be less biased and actually generate optimal policy outcome. The introduction of the independent
media could distort the policymaker’s signal when the media slant toward the public opinion.
This is in part due to the fact that the reputation concern of the media is not aligned with
the policymaker’s reputation and career concern. This creates friction between the media and
the policymaker, which could lead worse policy outcome than the state-owned media when the
public strongly care about the truth. Competition can indeed induce more accurate information
and efficient information transmission. However, the information accuracy and efficiency do not
always translate to better policy outcome.

Our theoretical analysis generates some interesting hypotheses which could be tested empiri-
cally if certain data are available. For example, the effects of media on policymaking could differ
across countries with different market structures. Furthermore, the consolidation of media markets
in recent years across the world presents us the opportunity of utilizing these natural events to
study not only the economic impact of the media mergers but also the political/social implications
of these mergers.

Even though our study is among the first to formally study the policymaker’s agenda ma-
ipulation when facing media bias, several modeling simplifications offer some avenues for future
research on the policy making process and political accountability. Consistent with the equal
voting rules, we investigate the aggregate effect of consumer prior beliefs on policy making, while
leaving out the possible heterogenous impact on different interest groups discussed in Strömberg
(2001). We incorporate the policymaker’s career and reputation concerns; while endogenizing
these effects across different market structure and the consequent policymaking process could enable us to understand the interplay between the policymaker and the public, and allow researchers to further pin down the underlying mechanisms.

References


Durante, Ruben and Brian Knight, “Partisan control, media bias, and viewer responses: Evidence from Berlusconi’s Italy,” Journal of the European Economic Association, 2012, 10 (3), 451–481.


Appendix

A Proofs of Lemmas

Lemma 1: By symmetry, we only show the result for $\phi^* = -1, 0$. Consider the strategy profile in which the policymaker always signals the state of nature $s = \theta$ and chooses the instituted policy $y^*$ that maximizes (1) given $\hat{y}$ and $\theta$ (to be specified in each case below), and the state-owned media never investigates $e = 0$ and always reports the signal $r = s$. The public’s posterior belief about the state of nature is

| Report $r$ | Public Posterior Belief $E[\theta|r]$ |
|------------|--------------------------------------|
| $-1$       | $-1$                                 |
| $0$        | $0$                                  |
| $1$        | $1$                                  |

By the definition of the state-owned media, it has no incentive to deviate as it is forced to report the policymaker’s signal and investigation is costly. Moreover, by the definition of $y^*$, the policymaker does not want to implement alternative policy. Thus, it suffices to derive the instituted policy $y^*$ for every combination of $\hat{y}$ and $\theta$, which is displayed in the following table.

<table>
<thead>
<tr>
<th>$\theta = -1$</th>
<th>$\hat{y} = -1$</th>
<th>$\hat{y} = 0$</th>
<th>$\hat{y} = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y^* = -1$</td>
<td>$y^* = 0$</td>
<td>$y^* = 1$</td>
<td></td>
</tr>
<tr>
<td>$\theta = 0$</td>
<td></td>
<td>$y^* = 0$</td>
<td>$y^* = 1$</td>
</tr>
<tr>
<td>$\theta = 1$</td>
<td>$y^* = {\begin{cases} -1, &amp; \text{if } 0 &lt; \lambda_1 &lt; \frac{1}{3}; \ 0, &amp; \text{otherwise.} \end{cases}$</td>
<td>$y^* = 0$</td>
<td>$y^* = 1$</td>
</tr>
</tbody>
</table>

Then we need to derive the instituted policy $y^*$ for every state of nature $\theta$ under the prescribed strategy, and show that the policymaker does not want to deviate at the signaling stage. We denote $\hat{s} \in \{-1, 0, 1\} \setminus \{s\}$ as the policymaker’s deviation at the signaling stage.

Condition i): When $\phi^* = -1$ and $\max\{0, \frac{3\alpha - 2}{3\alpha + 1}\} < \eta < 1$, we have

i-a) $\phi^* = -1$ and $\left\{ \begin{array}{l} \frac{1}{3} < \alpha \leq \frac{2}{3} \\ \frac{2}{3} - \frac{3\alpha}{3\alpha + 1} < \eta < \frac{3\alpha}{3\alpha + 1} \end{array} \right.$

i-b) $\phi^* = -1$ and $\frac{1}{3} < \alpha < \frac{2}{3}$ and $0 < \eta < \frac{2 - 3\alpha}{3 - 3\alpha}$

i-c) $\phi^* = -1$ and $\frac{1}{3} < \alpha < 1$ and $\frac{3\alpha}{3\alpha + 1} < \eta < 1.$
Suppose condition i-a) holds, consider the stage at which the policymaker chooses the instituted policy. The policymaker’s instituted policy \( y^* \) under the prescribed strategy is

\[
\begin{array}{cccc}
\theta & \text{report } r & \text{effective public opinion } \hat{y} & \text{instituted policy } y^* \\
-1 & -1 & -1 & -1 \\
0 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 \\
\end{array}
\]

It remains to show that the policymaker does not want to deviate at the signaling stage. The policymaker’s payoffs of the underlying reporting strategy \( s \) and the deviating strategy \( \hat{s} \) are

\[
\begin{array}{cccc}
\theta & \text{public posterior belief } E[\theta|r] & \text{payoff of } s & \text{payoff of } \hat{s} = -1 & \text{payoff of } \hat{s} = 0 & \text{payoff of } \hat{s} = 1 \\
-1 & -1 & 0 & n/a & -\lambda_1 - \lambda_2 & -\lambda_1 - 4\lambda_2 \\
0 & 0 & 0 & -\lambda_1 - \lambda_2 & n/a & -\lambda_1 \\
1 & 1 & -\lambda_1 & -\lambda_1 - 4\lambda_2 & -\lambda_1 - \lambda_2 & n/a \\
\end{array}
\]

Clearly, the policymaker has no incentive to deviate if condition i-a) holds.

Suppose condition i-b) holds, consider the stage at which the policymaker chooses the instituted policy. The policymaker’s instituted policy \( y^* \) under the prescribed strategy is

\[
\begin{array}{cccc}
\theta & \text{report } r & \text{effective public opinion } \hat{y} & \text{instituted policy } y^* \\
-1 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 \\
\end{array}
\]

It remains to show that the policymaker does not want to deviate at the signaling stage. The policymaker’s payoffs of the underlying reporting strategy \( s \) and the deviating strategy \( \hat{s} \) are

\[
\begin{array}{cccc}
\theta & \text{public posterior belief } E[\theta|r] & \text{payoff of } s & \text{payoff of } \hat{s} = -1 & \text{payoff of } \hat{s} = 0 & \text{payoff of } \hat{s} = 1 \\
-1 & -1 & -\lambda_1 & n/a & -\lambda_1 - \lambda_2 & -\lambda_1 - 4\lambda_2 \\
0 & 0 & 0 & -\lambda_2 & n/a & -\lambda_2 \\
1 & 1 & -\lambda_1 & -\lambda_1 - 4\lambda_2 & -\lambda_1 - \lambda_2 & n/a \\
\end{array}
\]
Clearly, the policymaker has no incentive to deviate if condition i-b) holds.

Suppose condition i-c) holds, consider the stage at which the policymaker chooses the instituted policy. The policymaker’s instituted policy $y^*$ under the prescribed strategy is

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>report $r$</th>
<th>effective public opinion $\hat{y}$</th>
<th>instituted policy $y^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-1$</td>
<td>$-1$</td>
<td>$-1$</td>
<td>$-1$</td>
</tr>
<tr>
<td>$0$</td>
<td>$0$</td>
<td>$0$</td>
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<tr>
<td>$1$</td>
<td>$1$</td>
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<td>$1$</td>
</tr>
</tbody>
</table>

It remains to show that the policymaker does not want to deviate at the signaling stage. The policymaker’s payoffs of the underlying reporting strategy $s$ and the deviating strategy $\hat{s}$ are

| $\theta$ | public posterior belief $E[\theta|r]$ | payoff of $s$ | payoff of $\hat{s} = -1$ | payoff of $\hat{s} = 0$ | payoff of $\hat{s} = 1$ |
|----------|--------------------------------------|---------------|--------------------------|--------------------------|--------------------------|
| $-1$     | $-1$                                | $0$           | $-\lambda_1 - \lambda_2$ | $-\min\{1 + \lambda_1, 4\lambda_1\} - 4\lambda_2$ |
| $0$      | $0$                                 | $0$           | $-\lambda_1 - \lambda_2$ | $n/a$                      | $-\lambda_1 - \lambda_2$ |
| $1$      | $1$                                 | $0$           | $-\min\{1 + \lambda_1, 4\lambda_1\} - 4\lambda_2$ | $-\lambda_1 - \lambda_2$ | $n/a$                      |

Clearly, the policymaker has no incentive to deviate if condition i-c) holds.

Condition ii): When $\phi^* = -1$ and $0 < \eta < \max\{0, \frac{3\alpha - 2}{3\alpha + 1}\}$, we have

$$\phi^* = -1 \text{ and } \frac{2}{3} < \alpha < 1 \text{ and } 0 < \eta < \frac{3\alpha - 2}{3\alpha + 1}$$

Suppose condition ii) holds, consider the stage at which the policymaker chooses the instituted policy. The policymaker’s instituted policy $y^*$ under the prescribed strategy is

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>report $r$</th>
<th>effective public opinion $\hat{y}$</th>
<th>instituted policy $y^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-1$</td>
<td>$-1$</td>
<td>$-1$</td>
<td>$-1$</td>
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<tr>
<td>$0$</td>
<td>$0$</td>
<td>$-1$</td>
<td>$-1$</td>
</tr>
<tr>
<td>$1$</td>
<td>$1$</td>
<td>$-1$</td>
<td>$\begin{cases} -1, \text{ if } 0 &lt; \lambda_1 &lt; \frac{1}{3} \ 0, \text{ otherwise.} \end{cases}$</td>
</tr>
</tbody>
</table>

It remains to show that the policymaker does not want to deviate at the signaling stage. The policymaker’s payoffs of the underlying reporting strategy $s$ and the deviating strategy $\hat{s}$ are
Clearly, the policymaker has no incentive to deviate if condition ii) holds.

**Condition iii):** When $\phi^* = 0$, we have

iii-a) $\phi^* = 0$ and $\frac{1}{3} < \alpha < 1$ and $\frac{1}{2} < \eta < 1$;

iii-b) $\phi^* = 0$ and $\frac{1}{3} < \alpha < 1$ and $0 < \eta < \frac{1}{2}$.

Suppose condition iii-a) holds, consider the stage at which the policymaker chooses the instituted policy. The policymaker’s instituted policy $y^*$ under the prescribed strategy is

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>payoff of $s$</th>
<th>payoff of $\hat{s} = -1$</th>
<th>payoff of $\hat{s} = 0$</th>
<th>payoff of $\hat{s} = 1$</th>
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<tr>
<td>$-1$</td>
<td>$-1$</td>
<td>$-\lambda_2$</td>
<td>$-\lambda_2$</td>
<td>$-4\lambda_2$</td>
</tr>
<tr>
<td>$0$</td>
<td>$0$</td>
<td>$-\lambda_1 - \lambda_2$</td>
<td>$n/a$</td>
<td>$-\lambda_1 - \lambda_2$</td>
</tr>
<tr>
<td>$1$</td>
<td>$1 - \min{1 + \lambda_1, 4\lambda_1}$</td>
<td>$-\min{1 + \lambda_1, 4\lambda_1} - 4\lambda_2$</td>
<td>$-\min{1 + \lambda_1, 4\lambda_1} - \lambda_2$</td>
<td>$n/a$</td>
</tr>
</tbody>
</table>

It remains to show that the policymaker does not want to deviate at the signaling stage. The policymaker’s payoffs of the underlying reporting strategy $s$ and the deviating strategy $\hat{s}$ are

<table>
<thead>
<tr>
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<th>payoff of $s$</th>
<th>payoff of $\hat{s} = -1$</th>
<th>payoff of $\hat{s} = 0$</th>
<th>payoff of $\hat{s} = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-1$</td>
<td>$-1$</td>
<td>$n/a$</td>
<td>$-\lambda_1 - \lambda_2$</td>
<td>$-\min{1 + \lambda_1, 4\lambda_1} - 4\lambda_2$</td>
</tr>
<tr>
<td>$0$</td>
<td>$0$</td>
<td>$-\lambda_1 - \lambda_2$</td>
<td>$n/a$</td>
<td>$-\lambda_1 - \lambda_2$</td>
</tr>
<tr>
<td>$1$</td>
<td>$0 - \min{1 + \lambda_1, 4\lambda_1}$</td>
<td>$-\min{1 + \lambda_1, 4\lambda_1} - 4\lambda_2$</td>
<td>$-\lambda_1 - \lambda_2$</td>
<td>$n/a$</td>
</tr>
</tbody>
</table>

Clearly, the policymaker has no incentive to deviate if condition iii-a) holds.

Suppose condition iii-b) holds, consider the stage at which the policymaker chooses the instituted policy. The policymaker’s instituted policy $y^*$ under the prescribed strategy is
It remains to show that the policymaker does not want to deviate at the signaling stage. The policymaker’s payoffs of the underlying reporting strategy $s$ and the deviating strategy $\hat{s}$ are

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>report $r$</th>
<th>effective public opinion $\hat{y}$</th>
<th>instituted policy $y^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>-1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
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<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
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</tbody>
</table>

Clearly, the policymaker has no incentive to deviate if condition iii-b) holds.

In summary, the policymaker has no incentive to deviate if any of the three conditions of the lemma holds. The fully separating equilibrium exists.

**Lemma 2:** By symmetry, we only show the result for $\phi^* = -1$. Consider the strategy profile in which the policymaker signals $s^*$, where

$$s^*(\theta) = \begin{cases} 
\phi^*, & \text{if } \theta = \phi^*; \\
-\phi^*, & \text{otherwise}.
\end{cases}$$

and chooses the instituted policy $y^*$ that maximizes (1) given $\hat{y}$ and $\theta$ (to be specified in each case below), and the state-owned media never investigates $e^* = 0$ and always reports the signal $r^* = s$.

The public’s posterior belief about the state of nature is

| report $r$ | public posterior belief $E[\theta|r]$ |
|------------|-------------------------------------|
| -1         | -1                                  |
| 0          | $\frac{1}{3} \times (-1) + \frac{1}{3} \times 0 + \frac{1}{3} \times 1 = 0$ |
| 1          | $\frac{1}{2} \times 0 + \frac{1}{2} \times 1 = \frac{1}{2}$ |

By the definition of the state-owned media, it has no incentive to deviate as it is forced to
report the policymaker’s signal and investigation is costly. Moreover, by the definition of $y^*$, the policymaker does not want to implement alternative policy. Thus, it suffices to derive the instituted policy $y^*$ for every combination of $\hat{y}$ and $\theta$, which is displayed in the following table.

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>$\hat{y} = -1$</th>
<th>$\hat{y} = 0$</th>
<th>$\hat{y} = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta = -1$</td>
<td>$y^* = -1$</td>
<td>$y^* = 0$</td>
<td>( y^* = \begin{cases} 1, &amp; \text{if } 0 &lt; \lambda_1 &lt; \frac{1}{3}; \ 0, &amp; \text{otherwise.} \end{cases} )</td>
</tr>
<tr>
<td>$\theta = 0$</td>
<td>$y^* = -1$</td>
<td>$y^* = 0$</td>
<td>$y^* = 1$</td>
</tr>
<tr>
<td>$\theta = 1$</td>
<td>$y^* = \begin{cases} -1, &amp; \text{if } 0 &lt; \lambda_1 &lt; \frac{1}{3}; \ 0, &amp; \text{otherwise.} \end{cases}$</td>
<td>$y^* = 0$</td>
<td>$y^* = 1$</td>
</tr>
</tbody>
</table>

Then we need to derive the instituted policy $y^*$ for every state of nature $\theta$ under the prescribed strategy, and show that the policymaker does not want to deviate at the signaling stage. We denote $\hat{s} \in \{-1, 0, 1\} \setminus \{s\}$ as the policymaker’s deviation at the signaling stage.

When $\phi^* = -1$ and $\max\{0, \frac{3\alpha - 2}{3\alpha} \} < \eta < \max\{0, \frac{3\alpha - 2}{3\alpha - 1}\}$, we have

$$\phi^* = -1 \text{ and } \frac{2}{3} < \alpha < 1 \text{ and } \frac{3\alpha - 2}{3\alpha} < \eta < \frac{3\alpha - 2}{3\alpha - 1}$$

Suppose this condition holds, consider the stage at which the policymaker chooses the instituted policy. The policymaker’s instituted policy $y^*$ under the prescribed strategy is

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>report $r$</th>
<th>effective public opinion $\hat{y}$</th>
<th>instituted policy $y^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-1$</td>
<td>$-1$</td>
<td>$-1$</td>
<td>$-1$</td>
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<tr>
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<td>$1$</td>
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<tr>
<td>$1$</td>
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<td>$0$</td>
</tr>
</tbody>
</table>

It remains to show that the policymaker does not want to deviate at the signaling stage. The policymaker’s payoffs of the underlying reporting strategy $s$ and the deviating strategy $\hat{s}$ are

| $\theta$ | public posterior belief $E[\theta|r]$ | payoff of $s$ | payoff of $\hat{s} = -1$ | payoff of $\hat{s} = 0$ | payoff of $\hat{s} = 1$ |
|----------|--------------------------------------|---------------|--------------------------|-------------------------|--------------------------|
| $-1$     | $-1$                                 | $0$           | n/a                      | $-\lambda_2$            | $-\lambda_1 - 4\lambda_2$ |
| $0$      | $\frac{1}{2}$                        | $-\lambda_2$  | $-\lambda_1 - \lambda_2$| $-\lambda_1$           | n/a                      |
| $1$      | $\frac{1}{2}$                        | $-\lambda_1$  | $-\min\{1 + \lambda_1, 4\lambda_1\} - 4\lambda_2$ | $-\min\{1 + \lambda_1, 4\lambda_1\} - \lambda_2$ | n/a                      |
Clearly, the policymaker has no incentive to deviate if the condition holds.

In summary, the policymaker has no incentive to deviate if the condition of the lemma holds. The semi-separating equilibrium exists.

**Lemma 3:** By symmetry, we only show the result for $\phi^* = -1, 0$. Consider the strategy profile in which the policymaker always signals the state of nature $s = \theta$ and chooses the instituted policy $y^*$ that maximizes (1) given $\hat{y}$ and $\theta$ (to be specified in each case below), and the media monopoly never investigates $e = 0$, always reports the mainstream preference $r = \phi^*$ and charges consumer $i$ the monopolistic price $p_i = R_i + v_i \cdot 1_{\phi_i = r}$. The media and the public’s posterior beliefs about the state of nature are

| signal $s$ | investigation outcome $z$ | media monopoly belief $E[\theta|s, z]$ | report $r$ | public posterior belief $E[\theta|r]$ |
|------------|---------------------------|-------------------------------------|----------|----------------------------------|
| -1         | 0                         | -1                                  | -1       | $\frac{1}{3} \times (-1) + \frac{1}{3} \times 0 + \frac{1}{3} \times 1 = 0$ |
| 0          | 0                         | 0                                   | 0        | $\frac{1}{3} \times (-1) + \frac{1}{3} \times 0 + \frac{1}{3} \times 1 = 0$ |
| 1          | 0                         | 1                                   | 1        | $\frac{1}{3} \times (-1) + \frac{1}{3} \times 0 + \frac{1}{3} \times 1 = 0$ |
| all reports| -1                       | -1                                  |          |                                  |
| all reports| 0                        | 0                                   |          |                                  |
| all reports| 1                        | 1                                   |          |                                  |

We first show that the media monopoly has no incentive to deviate, and then show that the policymaker does not want to deviate either.

**Media monopoly:** Clearly, the media monopoly’s pricing strategy is optimal. The lowest profit that the media monopoly receives from reporting $\phi^*$ is $\bar{R} + \alpha \bar{v} - 4\gamma$, where $\bar{R}$ is the profit from genuine readership, $\alpha \bar{v}$ is the profit from conformity of the report to the consumers’ the policy preference $\phi^*$, and $4\gamma$ is the maximal reputation cost. The highest profit that the media monopoly receives from reporting alternative stance $\hat{r} \in \{-1, 0, 1\} \setminus \{\phi^*\}$ is $\bar{R} + \frac{1 - \alpha}{2} \bar{v}$, where $\bar{R}$ is the profit from genuine readership, and $\frac{1 - \alpha}{2} \bar{v}$ is the profit from conformity of the report to the consumers’ the policy preference $\phi_i = \hat{r}$. By (C4), we have

$$\bar{R} + \alpha \bar{v} - 4\gamma > \bar{R} + \frac{1 - \alpha}{2} \bar{v}.$$  \[(A.1)\]

Therefore, the media monopoly always reports $\phi^*$ even it knows the state of nature $\theta$. Given the media monopoly always reports $\phi^*$, investigation is unnecessary, so it never investigates.

**Policymaker:** By the definition of $y^*$, the policymaker does not want to implement alternative
policy. Thus, it suffices to derive the instituted policy $y^*$ for every combination of $\hat{y}$ and $\theta$, which is displayed in the following table.

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>$\hat{y} = -1$</th>
<th>$\hat{y} = 0$</th>
<th>$\hat{y} = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta = -1$</td>
<td>$y^* = -1$</td>
<td>$y^* = 0$</td>
<td>$\begin{cases} 1, &amp; \text{if } 0 &lt; \lambda_1 &lt; \frac{1}{3}; \ 0, &amp; \text{otherwise.} \end{cases}$</td>
</tr>
<tr>
<td>$\theta = 0$</td>
<td>$y^* = -1$</td>
<td>$y^* = 0$</td>
<td>$y^* = 1$</td>
</tr>
<tr>
<td>$\theta = 1$</td>
<td>$y^* = \begin{cases} -1, &amp; \text{if } 0 &lt; \lambda_1 &lt; \frac{1}{3}; \ 0, &amp; \text{otherwise.} \end{cases}$</td>
<td>$y^* = 0$</td>
<td>$y^* = 1$</td>
</tr>
</tbody>
</table>

Then we need to derive the instituted policy $y^*$ for every state of nature $\theta$ under the prescribed strategy, and show that the policymaker does not want to deviate at the signaling stage. We denote $\hat{s} \in \{-1, 0, 1\} \setminus \{s\}$ as the policymaker’s deviation at the signaling stage. There are three cases listed as follows.

Case i) $\phi^* = -1$ and $\begin{cases} \frac{1}{3} < \alpha \leq \frac{2}{3} \text{ and } 0 < \eta < 1; \\ \frac{2}{3} < \alpha < 1 \text{ and } \frac{3\alpha - 2}{3\alpha - 1} < \eta < 1 \end{cases}$;
Case ii) $\phi^* = -1$ and $\frac{2}{3} < \alpha < 1$ and $0 < \eta < \frac{3\alpha - 2}{3\alpha - 1}$;
Case iii) $\phi^* = 0$.

In case i), consider the stage at which the policymaker chooses the instituted policy. The policymaker’s instituted policy $y^*$ under the prescribed strategy is

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>report $r$</th>
<th>effective public opinion $\hat{y}$</th>
<th>instituted policy $y^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-1$</td>
<td>$-1$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$0$</td>
<td>$-1$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$1$</td>
<td>$-1$</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

It remains to show that the policymaker does not want to deviate at the signaling stage. The policymaker’s payoffs of the underlying reporting strategy $s$ and the deviating strategy $\hat{s}$ are
Clearly, the policymaker has no incentive to deviate in case i).

In case ii), consider the stage at which the policymaker chooses the instituted policy. The policymaker’s instituted policy $y^*$ under the prescribed strategy is

\[
\begin{array}{ccccc}
\theta & \text{report } r & \text{effective public opinion } \hat{y} & \text{instituted policy } y^* \\
\hline
-1 & -1 & -1 & -1 \\
0 & -1 & -1 & -1 \\
1 & -1 & -1 & \begin{cases} -1, & \text{if } 0 < \lambda_1 < \frac{1}{3}; \\ 0, & \text{otherwise.} \end{cases} \\
\end{array}
\]

It remains to show that the policymaker does not want to deviate at the signaling stage. The policymaker’s payoffs of the underlying reporting strategy $s$ and the deviating strategy $\hat{s}$ are

\[
\begin{array}{cccc}
\theta & \text{payoff of } s & \text{payoff of } \hat{s} = -1 & \text{payoff of } \hat{s} = 0 \\
\hline
-1 & 0 & -\lambda_1 & n/a \\
0 & 0 & 0 & -\lambda_2 \\
1 & 0 & -\lambda_1 & -\lambda_1 - 4\lambda_2 \\
\end{array}
\]

Clearly, the policymaker has no incentive to deviate in case ii).

In case iii), consider the stage at which the policymaker chooses the instituted policy. The policymaker’s instituted policy $y^*$ under the prescribed strategy is

\[
\begin{array}{cccc}
\theta & \text{report } r & \text{effective public opinion } \hat{y} & \text{instituted policy } y^* \\
\hline
-1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 \\
\end{array}
\]
It remains to show that the policymaker does not want to deviate at the signaling stage. The policymaker’s payoffs of the underlying reporting strategy \(s\) and the deviating strategy \(\hat{s}\) are

| \(\theta\) | public posterior belief \(E[\theta|r]\) | payoff of \(s\) | payoff of \(\hat{s} = -1\) | payoff of \(\hat{s} = 0\) | payoff of \(\hat{s} = 1\) |
|---------|----------------------------------|---------------|-----------------|-----------------|-----------------|
| -1      | 0                                | \(-\lambda_1\) | \(n/a\)        | \(-\lambda_1 - \lambda_2\) | \(-\lambda_1 - 4\lambda_2\) |
| 0       | 0                                | 0             | \(-\lambda_2\) | \(n/a\)        | \(-\lambda_2\) |
| 1       | 0                                | \(-\lambda_1\) | \(-\lambda_1 - 4\lambda_2\) | \(-\lambda_1 - \lambda_2\) | \(n/a\) |

Clearly, the policymaker has no incentive to deviate in case iii).

In summary, the policymaker has no incentive to deviate, if the conditions of the lemma hold. The uninformative equilibrium exists.

**Lemma 4:** By symmetry, we only show the result for \(\phi^* = -1, 0\). Consider the strategy profile in which a) the policymaker always signals the state of nature \(s = \theta\) and chooses the instituted policy \(y^*\) that maximizes (1) given \(\hat{y}\) and \(\theta\) (to be specified in each case below); b) the media firm 1 never investigates (e.g., \(e_1 = 0\)), reports \(r_1\) and prices \(p_{i1}\), where

\[
r_1(s, \phi^*) = s\quad\text{and}\quad p_{i1} = \begin{cases} v_i, & \text{if } \phi_i = r_1; \\ 0, & \text{otherwise.} \end{cases}
\]

C) the media firm 2 never investigates (e.g., \(e_2 = 0\)), reports \(r_2\) and prices \(p_{i2}\) where

\[
r_2(s, \phi^*) = \begin{cases} \phi^*, & \text{if } s \neq \phi^*; \\ -1, & \text{if } s = \phi^* = 0; \\ 0, & \text{if } s = \phi^* \text{ and } \phi^* \neq 0. \end{cases}\quad\text{and}\quad p_{i2} = \begin{cases} v_i, & \text{if } \phi_i = r_2; \\ 0, & \text{otherwise.} \end{cases}
\]

The media and the public’s posterior beliefs about the state of nature are

\[
\phi^* = -1
\]
We first show that the media firm 1 has no incentive to deviate, then show that the media firm 2 does not want to deviate. Finally, we show that the policymaker has no incentive to deviate.

**Media firm 1:** According to the tie-breaking rule, the media firm 1’s pricing strategy is clearly optimal as the media firm 1 will make a negative profit if it further undercuts the media firm 2. The media firm 1 does not investigate as it learns the state of nature $\theta$ from the policymaker’s signal $s$. It remains to show that the media firm 1’s report is optimal given the media firm 2 and the policymaker’s strategies. We denote $\hat{r}_1$ as the media firm 1’s deviating report.

**Condition i) and ii):** When $\phi^* = -1$, we have

| signal $s$ | investigation outcome $z$ | media duopoly belief $E[\theta|s, z]$ | report $(r_1, r_2)$ | public posterior belief $E[\theta|r_1, r_2]$ |
|------------|----------------------------|---------------------------------|---------------------|----------------------------------|
| $-1$       | $\emptyset$               | $-1$                            | $(-1, 0)$           | $-1$                             |
| $0$        | $\emptyset$               | $0$                             | $(0, -1)$           | $0$                              |
| $1$        | $\emptyset$               | $1$                             | $(1, -1)$           | $1$                              |
| all signals| $-1$                      | $-1$                            | other reports $\frac{1}{3} \times (-1) + \frac{1}{3} \times 0 + \frac{1}{3} \times 1 = 0$ | |
| all signals| $0$                       | $0$                             | other reports $\frac{1}{3} \times (-1) + \frac{1}{3} \times 0 + \frac{1}{3} \times 1 = 0$ | |
| all signals| $1$                       | $1$                             |                     |                                  |
Clearly, the media firm 1 has no incentive to deviate to another report if condition i) or ii) holds.

*Condition iii):* When \( \phi^* = 0 \), we have

| signal \( s \) | posterior belief \( E[\theta|s] \) | payoff of \( r_1 \) | payoff of \( \hat{r}_1 = -1 \) | payoff of \( \hat{r}_1 = 0 \) | payoff of \( \hat{r}_1 = 1 \) |
|---|---|---|---|---|---|
| -1 | -1 | \( \frac{1-\alpha}{2} \bar{v} \) | \( -\gamma \) | \( \frac{1-\alpha}{2} \bar{v} - 4\gamma \) | 
| 0 | 0 | \( \alpha \bar{v} \) | \( -\gamma \) | \( \frac{1-\alpha}{2} \bar{v} - \gamma \) | 
| 1 | 1 | \( \frac{1-\alpha}{2} \bar{v} \) | \( \frac{1-\alpha}{2} \bar{v} - 4\gamma \) | \( -\gamma \) | \( \frac{1-\alpha}{2} \bar{v} - \gamma \) |

Clearly, the media firm 1 has no incentive to deviate to another report if condition iii) holds.

In summary, the media firm 1 has no incentive to deviate.

**Media firm 2:** According to the tie-breaking rule, the media firm 2’s pricing strategy is clearly optimal as the media firm 2 will make a negative profit if it further undercuts the media firm 1. The media firm 2 does not investigate as it learns the state of nature \( \theta \) from the policymaker’s signal \( s \). It remains to show that the media firm 2’s report is optimal given the media firm 1 and the policymaker’s strategies. We denote \( \hat{r}_2 \) as the media firm 2’s deviating report.

*Condition i) and ii):* When \( \phi^* = -1 \), we have

| signal \( s \) | posterior belief \( E[\theta|s] \) | payoff of \( r_2 \) | payoff of \( \hat{r}_2 = -1 \) | payoff of \( \hat{r}_2 = 0 \) | payoff of \( \hat{r}_2 = 1 \) |
|---|---|---|---|---|---|
| -1 | -1 | \( \frac{1-\alpha}{2} \bar{v} - \gamma \) | 0 | \( \frac{1-\alpha}{2} \bar{v} - 4\gamma \) | 
| 0 | 0 | \( \alpha \bar{v} - \gamma \) | \( \frac{1-\alpha}{2} \bar{v} - \gamma \) | 
| 1 | 1 | \( \alpha \bar{v} - 4\gamma \) | \( \frac{1-\alpha}{2} \bar{v} - \gamma \) | 

By (C4), we have

\[
\alpha \bar{v} - 4\gamma > \frac{1-\alpha}{2} \bar{v} - \gamma. \tag{A.2}
\]

Therefore, the media firm 2 has no incentive to deviate to another report if condition i) or ii) holds.

*Condition iii):* When \( \phi^* = 0 \), we have
Clearly, the media firm 2 has no incentive to deviate to another report if condition iii) holds. In summary, the media firm 2 has no incentive to deviate.

Policymaker: By the definition of $y^*$ in the underlying strategy, the policymaker does not want to implement alternative policy. Thus, it suffices to derive the instituted policy $y^*$ for every combination of $\hat{y}$ and $\theta$, which is displayed in the following table.

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>$\hat{y} = -1$</th>
<th>$\hat{y} = 0$</th>
<th>$\hat{y} = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{y} = -1$</td>
<td>$y^* = -1$</td>
<td>$y^* = 0$</td>
<td>$y^* = 1$</td>
</tr>
<tr>
<td>$\hat{y} = 0$</td>
<td>$y^* = 0$</td>
<td>$y^* = 0$</td>
<td>$y^* = 1$</td>
</tr>
<tr>
<td>$\hat{y} = 1$</td>
<td>$y^* = \begin{cases} -1, &amp; \text{if } 0 &lt; \lambda_1 &lt; \frac{1}{3}; \ 0, &amp; \text{otherwise.} \end{cases}$</td>
<td>$y^* = 0$</td>
<td>$y^* = 1$</td>
</tr>
</tbody>
</table>

Then we need to derive the instituted policy $y^*$ for every state of nature $\theta$ under the prescribed strategy, and show that the policymaker does not want to deviate at the signaling stage. We denote $\hat{s} \in \{-1, 0, 1\} \setminus \{s\}$ as the policymaker’s deviation at the signaling stage.

**Condition i):** When $\phi^* = -1$ and $\max\{0, \frac{3\alpha - 2}{3\alpha - 1}\} < \eta < 1$, we have

i-a) $\phi^* = -1$ and $\left\{ \begin{array}{l} \frac{1}{3} < \alpha \leq \frac{2}{3} \quad \text{and} \quad \frac{2-3\alpha}{3-3\alpha} < \eta < \frac{3\alpha}{3\alpha+1}; \\ \frac{2}{3} < \alpha < 1 \quad \text{and} \quad \frac{3\alpha-2}{3\alpha-1} < \eta < \frac{3\alpha}{3\alpha+1}. \end{array} \right.$

i-b) $\phi^* = -1$ and $\frac{1}{3} < \alpha < \frac{2}{3}$ and $0 < \eta < \frac{2-3\alpha}{3-3\alpha}$;

i-c) $\phi^* = -1$ and $\frac{1}{3} < \alpha < 1$ and $\frac{3\alpha}{3\alpha+1} < \eta < 1$.

Suppose condition i-a) holds, consider the stage at which the policymaker chooses the instituted policy. The policymaker’s instituted policy $y^*$ under the prescribed strategy is
It remains to show that the policymaker does not want to deviate at the signaling stage. The policymaker’s payoffs of the underlying reporting strategy $s$ and the deviating strategy $\hat{s}$ are

| $\theta$ | public posterior belief $E[\theta| r]$ | payoff of $s$ | payoff of $\hat{s} = -1$ | payoff of $\hat{s} = 0$ | payoff of $\hat{s} = 1$ |
|---------|---------------------------------|-------------|-----------------|----------------|-----------------|
| $-1$    | $-1$                           | 0           | $-\lambda_1 - \lambda_2$ | $-\lambda_1 - 4\lambda_2$ |
| $0$     | $0$                            | 0           | $-\lambda_1 - \lambda_2$ | $n/a$            | $-\lambda_1$       |
| $1$     | $1$                            | $-\lambda_1$| $-\lambda_1 - 4\lambda_2$| $-\lambda_1 - \lambda_2$| $n/a$            |

Clearly, the policymaker has no incentive to deviate if condition i-a) holds. Suppose condition i-b) holds, consider the stage at which the policymaker chooses the instituted policy. The policymaker’s instituted policy $y^*$ under the prescribed strategy is

| $\theta$ | public posterior belief $E[\theta| r]$ | payoff of $s$ | payoff of $\hat{s} = -1$ | payoff of $\hat{s} = 0$ | payoff of $\hat{s} = 1$ |
|---------|---------------------------------|-------------|-----------------|----------------|-----------------|
| $-1$    | $-1$                           | 0           | $-\lambda_1$   | $n/a$      | $-\lambda_1 - \lambda_2$ |
| $0$     | $0$                            | 0           | $-\lambda_2$   | $n/a$      | $-\lambda_2$    |
| $1$     | $1$                            | $-\lambda_1$| $-\lambda_1 - 4\lambda_2$| $-\lambda_1 - \lambda_2$| $n/a$            |

It remains to show that the policymaker does not want to deviate at the signaling stage. The policymaker’s payoffs of the underlying reporting strategy $s$ and the deviating strategy $\hat{s}$ are
Clearly, the policymaker has no incentive to deviate if condition i-b) holds.

Suppose condition i-c) holds, consider the stage at which the policymaker chooses the instituted policy. The policymaker’s instituted policy $y^*$ under the prescribed strategy is

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>report $(r_1, r_2)$</th>
<th>effective public opinion $\hat{y}$</th>
<th>instituted policy $y^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>$(-1, 0)$</td>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>0</td>
<td>$(0, -1)$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>$(1, -1)$</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

It remains to show that the policymaker does not want to deviate at the signaling stage. The policymaker’s payoffs of the underlying reporting strategy $s$ and the deviating strategy $\hat{s}$ are

| $\theta$ | public posterior belief $E[\theta| r]$ | payoff of $s$ | payoff of $\hat{s} = -1$ | payoff of $\hat{s} = 0$ | payoff of $\hat{s} = 1$ |
|----------|--------------------------------------|----------------|-------------------------|------------------------|------------------------|
| -1       | -1                                   | 0              | n/a                     | $-\lambda_1 - \lambda_2$ | $-\min\{1 + \lambda_1, 4\lambda_1\} - 4\lambda_2$ |
| 0        | 0                                    | 0              | $-\lambda_1 - \lambda_2$ | n/a                    | $-\lambda_1 - \lambda_2$ |
| 1        | 1                                    | 0              | $-\min\{1 + \lambda_1, 4\lambda_1\} - 4\lambda_2$ | $-\lambda_1 - \lambda_2$ | n/a                    |

Clearly, the policymaker has no incentive to deviate if condition i-c) holds.

**Condition ii):** When $\phi^* = -1$ and $0 < \eta < \max\{0, \frac{3\alpha - 2}{3\alpha + 1}\}$, we have

$$\phi^* = -1 \text{ and } \frac{2}{3} < \alpha < 1 \text{ and } 0 < \eta < \frac{3\alpha - 2}{3\alpha + 1}$$

Suppose condition ii) holds, consider the stage at which the policymaker chooses the instituted policy. The policymaker’s instituted policy $y^*$ under the prescribed strategy is

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>report $(r_1, r_2)$</th>
<th>effective public opinion $\hat{y}$</th>
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<tr>
<td>-1</td>
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<tr>
<td>0</td>
<td>$(0, -1)$</td>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>1</td>
<td>$(1, -1)$</td>
<td>-1</td>
<td>$\begin{cases} -1, &amp; \text{if } 0 &lt; \lambda_1 &lt; \frac{1}{3}; \ 0, &amp; \text{otherwise.} \end{cases}$</td>
</tr>
</tbody>
</table>

It remains to show that the policymaker does not want to deviate at the signaling stage. The
policymaker’s payoffs of the underlying reporting strategy $s$ and the deviating strategy $\hat{s}$ are

| $\theta$ | public posterior belief $E[\theta|r]$ | payoff of $s$ | payoff of $\hat{s} = -1$ | payoff of $\hat{s} = 0$ | payoff of $\hat{s} = 1$ |
|----------|-------------------------------------|---------------|---------------------------|------------------------|------------------------|
| -1       | -1                                  | 0             | n/a                       | $-\lambda_2$           | $-4\lambda_2$          |
| 0        | 0                                   | $-\lambda_1$  | $-\lambda_1 - \lambda_2$ | n/a                    | $-\lambda_1 - \lambda_2$ |
| 1        | 1                                   | $-\min\{1 + \lambda_1, 4\lambda_1\}$ | $-\min\{1 + \lambda_1, 4\lambda_1\} - 4\lambda_2$ | $-\min\{1 + \lambda_1, 4\lambda_1\} - \lambda_2$ | n/a |

Clearly, the policymaker has no incentive to deviate if condition ii) holds.

*Condition iii)*: When $\phi^* = 0$, we have

- iii-a) $\phi^* = 0$ and $\frac{1}{3} < \alpha < 1$ and $\frac{1}{2} < \eta < 1$;
- iii-b) $\phi^* = 0$ and $\frac{1}{3} < \alpha < 1$ and $0 < \eta < \frac{1}{2}$.

Suppose condition iii-a) holds, consider the stage at which the policymaker chooses the instituted policy. The policymaker’s instituted policy $y^*$ under the prescribed strategy is

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>report $(r_1, r_2)$</th>
<th>effective public opinion $\hat{y}$</th>
<th>instituted policy $y^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>$(r_1, r_2)$</td>
<td>0</td>
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</tr>
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<td>0</td>
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</tr>
<tr>
<td>1</td>
<td>$(1, 0)$</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

It remains to show that the policymaker does not want to deviate at the signaling stage. The policymaker’s payoffs of the underlying reporting strategy $s$ and the deviating strategy $\hat{s}$ are

| $\theta$ | public posterior belief $E[\theta|r]$ | payoff of $s$ | payoff of $\hat{s} = -1$ | payoff of $\hat{s} = 0$ | payoff of $\hat{s} = 1$ |
|----------|-------------------------------------|---------------|---------------------------|------------------------|------------------------|
| -1       | -1                                  | 0             | n/a                       | $-\lambda_1 - \lambda_2$ | $-\min\{1 + \lambda_1, 4\lambda_1\} - 4\lambda_2$ |
| 0        | 0                                   | 0             | $-\lambda_1 - \lambda_2$ | n/a                    | $-\lambda_1 - \lambda_2$ |
| 1        | 1                                   | 0             | $-\min\{1 + \lambda_1, 4\lambda_1\} - 4\lambda_2$ | $-\lambda_1 - \lambda_2$ | n/a |

Clearly, the policymaker has no incentive to deviate if condition iii-a) holds.

Suppose condition iii-b) holds, consider the stage at which the policymaker chooses the instituted policy. The policymaker’s instituted policy $y^*$ under the prescribed strategy is
\[ \begin{array}{|c|c|c|c|} \hline \theta & \text{report} (r_1, r_2) & \text{effective public opinion} \hat{y} & \text{instituted policy} y^* \\
\hline -1 & (-1, 0) & 0 & 0 \\
0 & (0, -1) & 0 & 0 \\
1 & (1, 0) & 0 & 0 \\
\hline \end{array} \]

It remains to show that the policymaker does not want to deviate at the signaling stage. The policymaker’s payoffs of the underlying reporting strategy \( s \) and the deviating strategy \( \hat{s} \) are

\[
\begin{array}{|c|c|c|c|c|} \hline \theta \text{ public posterior belief } E[\theta|r] & \text{payoff of } s & \text{payoff of } \hat{s} = -1 & \text{payoff of } \hat{s} = 0 & \text{payoff of } \hat{s} = 1 \\
\hline -1 & -1 & -\lambda_1 & \text{n/a} & -\lambda_1 - \lambda_2 & -\lambda_1 - 4\lambda_2 \\
0 & 0 & 0 & -\lambda_2 & \text{n/a} & -\lambda_2 \\
1 & 1 & -\lambda_1 & -\lambda_1 - 4\lambda_2 & -\lambda_1 - \lambda_2 & \text{n/a} \\
\hline \end{array}
\]

Clearly, the policymaker has no incentive to deviate if condition iii-b) holds.

In summary, the policymaker has no incentive to deviate if any of the three conditions of the lemma holds. The fully separating equilibrium exists.

**Lemma 5:** By symmetry, we only show the result for \( \phi^* = -1 \). Consider the strategy profile in which a) the policymaker signals \( s \) and chooses the instituted policy \( y^* \) that maximizes (1) given \( \hat{y} \) and \( \theta \) (to be specified in each case below), where

\[
s(\theta) = \begin{cases} 
\phi^*, & \text{if } \theta = \phi^*; \\
-\phi^*, & \text{otherwise}.
\end{cases}
\]

b) the media firm 1 exerts effort \( e_1 \) to investigate, reports \( r_1 \) and prices \( p_{i1} \), where

\[
e_1(s, \phi^*) = \begin{cases} 
0, & \text{if } s = \phi^*; \\
1, & \text{otherwise}.
\end{cases}
\]

and

\[
r_1(s, \phi^*) = \begin{cases} 
\phi^*, & \text{if } s = \phi^*; \\
-\phi^*, & \text{if } s = 0 \text{ and } z_1 = -\phi^*; \\
-\phi^*, & \text{if } s = -\phi^* \text{ and } z_1 = -\phi^* \text{ or } \emptyset; \\
0, & \text{otherwise}.
\end{cases}
\]

\[
p_{i1} = \begin{cases} 
v_i, & \text{if } \phi_i = r_1; \\
0, & \text{otherwise}.
\end{cases}
\]
c) the media firm 2 never investigates (e.g., \( e_2 = 0 \)), reports \( r_2 \) and prices \( p_{i2} \) where

\[
r_2(s, \phi^*) = \begin{cases} 0, & \text{if } s = \phi^*; \\ \phi^*, & \text{otherwise.} \end{cases}
\]

and

\[
p_{i2} = \begin{cases} v_i, & \text{if } \phi_i = r_2; \\ 0, & \text{otherwise.} \end{cases}
\]

The media and the public’s posterior beliefs about the state of nature are

| signal \( s \) | investigation outcome \( z \) | media duopoly belief \( E[\theta | s, z] \) | report \( (r_1, r_2) \) | public posterior belief \( E[\theta | r_1, r_2] \) |
|----------------|--------------------------|-------------------------------|------------------|---------------------|
| −1             | \( \emptyset \)          | −1                            | (−1, 0)          | −1                  |
| 0              | \( \emptyset \)          | \( \frac{1}{2} \times (-1) + \frac{1}{3} \times 0 + \frac{1}{3} \times 1 = 0 \) | (0, −1)          | 0                   |
| 1              | \( \emptyset \)          | \( \frac{1}{2} \times 0 + \frac{1}{2} \times 1 = \frac{1}{2} \)         | (1, −1)          | \( \frac{1-\rho}{2-\rho} \times 0 + \frac{1-\rho}{2-\rho} \times 1 = \frac{1}{2-\rho} \) |
| all signals    | −1                       | −1                            | other reports    | \( \frac{1}{2} \times (-1) + \frac{1}{3} \times 0 + \frac{1}{3} \times 1 = 0 \) |
| all signals    | 0                        | 0                             |                  |                     |
| all signals    | 1                        | 1                             |                  |                     |

We first show that the media firm 1 has no incentive to deviate, then show that the media firm 2 does not want to deviate. Finally, we show that the policymaker has no incentive to deviate.

When \( \phi^* = -1 \) and \( \max\{0, \frac{3\alpha-2}{3\alpha}\} < \eta < \max\{0, \frac{3\alpha-2}{3\alpha-1}\} \), we have

\[
\phi^* = -1 \quad \text{and} \quad \frac{2}{3} < \alpha < 1 \quad \text{and} \quad \frac{3\alpha - 2}{3\alpha} < \eta < \frac{3\alpha - 2}{3\alpha - 1}.
\]

Media firm 1: According to the tie-breaking rule, the media firm 1’s pricing strategy is clearly optimal as the media firm 1 will make a negative profit if it further undercuts the media firm 2. When the signal \( s \) is −1, we have
Clearly, the media firm 1 has no incentive to deviate upon observing the signal \( s = 0 \). When the signal \( s \) is 0, we have

| effort \( e_1 \) | investigation outcome \( z_1 \) | posterior belief \( E[\theta|s, z] \) | expected payoff \( \bar{r}_1 \) |
|----------------|-----------------|-----------------|-----------------|
| 0              | \( \emptyset \)  | \( -1 \)        | \( \alpha \bar{\nu} \) |
| 1              | \( \emptyset \)  | \( -1 \)        | \( \alpha \bar{\nu} - c \) |
| 1              | \( -1 \)         | \( -1 \)        | \( \alpha \bar{\nu} - c \) |

\[
\text{expected payoff of } \hat{r}_1 = -1 \quad \text{of } \hat{r}_1 = 0 \quad \text{of } \hat{r}_1 = 1
\]

<table>
<thead>
<tr>
<th></th>
<th>n/a</th>
<th>( -\gamma )</th>
<th>( \frac{1-\alpha}{2} \bar{\nu} - 4\gamma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>n/a</td>
<td>( -\gamma - c )</td>
<td>( \frac{1-\alpha}{2} \bar{\nu} - 4\gamma - c )</td>
<td></td>
</tr>
<tr>
<td>n/a</td>
<td>( -\gamma - c )</td>
<td>( \frac{1-\alpha}{2} \bar{\nu} - 4\gamma - c )</td>
<td></td>
</tr>
</tbody>
</table>

Clearly, the media firm 1’s report is optimal given the signal \( s = 0 \) and its effort \( e_1 \in \{0, 1\} \). It remains to show that the media firm 1 chooses investigation (e.g., \( e_1 = 1 \)) upon observing the signal \( s = 0 \). The media firm 1’s expected payoff of conducting investigation is

\[
\rho \left( \frac{1}{3} (1 - \alpha) \bar{\nu} - \gamma - c + \frac{1}{3} (1 - \alpha) \bar{\nu} - c + \frac{1}{3} (1 - \alpha) \bar{\nu} - c \right) + (1 - \rho) \left( \frac{1}{2} \bar{\nu} - \frac{2}{3} \gamma - c \right) = \frac{1-\alpha}{2} \bar{\nu} - \frac{2}{3} \rho \gamma - c. \quad (A.3)
\]

By (C3), we have

\[
\frac{1-\alpha}{2} \bar{\nu} - \frac{2}{3} \rho \gamma - c > \frac{1-\alpha}{2} \bar{\nu} - \frac{2}{3} \gamma.
\]
Therefore, the media firm 1 chooses investigation (e.g., \( e_1 = 1 \)) upon observing the signal \( s = 0 \).

When the signal \( s \) is 1, we have

| effort \( e_1 = 0 \) | posterior belief \( E[\theta | s, z_1] \) | expected payoff of \( r_1 \) of \( \hat{r}_1 = -1 \) | expected payoff of \( \hat{r}_1 = 0 \) | expected payoff of \( \hat{r}_1 = 1 \) |
|----------------------|--------------------------------------|-----------------|-----------------|-----------------|
| 0 \( \frac{1}{2} \times 0 + \frac{1}{2} \times 1 = \frac{1}{2} \) | \( \frac{1 - \alpha}{2} \bar{v} - \frac{1}{2} \gamma \) | \( -\frac{\gamma}{2} \) | \( \frac{1 - \alpha}{2} \bar{v} - \frac{1}{2} \gamma \) | n/a |
| 0 \( \frac{1}{2} \times 0 + \frac{1}{2} \times 1 = \frac{1}{2} \) | 0 | \( -\gamma \) | n/a | n/a |
| 1 \( \frac{1}{2} \times 0 + \frac{1}{2} \times 1 = \frac{1}{2} \) | \( \frac{1 - \alpha}{2} \bar{v} - \gamma \) | \( -4\gamma \) | \( \frac{1 - \alpha}{2} \bar{v} - \gamma \) | n/a |

| effort \( e_1 = 1 \) | posterior belief \( E[\theta | s, z_1] \) | expected payoff of \( r_1 \) of \( \hat{r}_1 = -1 \) | expected payoff of \( \hat{r}_1 = 0 \) | expected payoff of \( \hat{r}_1 = 1 \) |
|----------------------|--------------------------------------|-----------------|-----------------|-----------------|
| 0 \( \frac{1}{2} \times 0 + \frac{1}{2} \times 1 = \frac{1}{2} \) | \( \frac{1 - \alpha}{2} \bar{v} - \frac{1}{2} \gamma \) | \( -\frac{\gamma}{2} \) | \( \frac{1 - \alpha}{2} \bar{v} - \frac{1}{2} \gamma \) | n/a |
| 0 \( \frac{1}{2} \times 0 + \frac{1}{2} \times 1 = \frac{1}{2} \) | 0 | \( -\gamma \) | n/a | n/a |
| 1 \( \frac{1}{2} \times 0 + \frac{1}{2} \times 1 = \frac{1}{2} \) | \( \frac{1 - \alpha}{2} \bar{v} - \gamma \) | \( -4\gamma \) | \( \frac{1 - \alpha}{2} \bar{v} - \gamma \) | n/a |

Clearly, the media firm 1’s report is optimal given the signal \( s = 1 \) and its effort \( e_1 \in \{0, 1\} \). It remains to show that the media firm 1 chooses investigation (e.g., \( e_1 = 1 \)) upon observing the signal \( s = 1 \). The media firm 1’s expected payoff of conducting investigation is

\[
\rho \left( \frac{1}{2} \left( \frac{1 - \alpha}{2} \bar{v} - c \right) + \frac{1}{2} \left( \frac{1 - \alpha}{2} \bar{v} - c \right) \right) + (1 - \rho) \left( \frac{1 - \alpha}{2} \bar{v} - \frac{1}{2} \gamma - c \right) = \frac{1 - \alpha}{2} \bar{v} - \frac{1 - \rho}{2} \gamma - c. \tag{A.4}
\]

By (C3), we have

\[
\frac{1 - \alpha}{2} \bar{v} - \frac{1 - \rho}{2} \gamma - c > \frac{1 - \alpha}{2} \bar{v} - \frac{1}{2} \gamma.
\]

Therefore, the media firm 1 chooses investigation (e.g., \( e_1 = 1 \)) upon observing the signal \( s = 1 \). In summary, the media firm 1 has no incentive to deviate.

**Media firm 2:** According to the tie-breaking rule, the media firm 2’s pricing strategy is clearly optimal as the media firm 2 will make a negative profit if it further undercuts the media firm 1. When the signal \( s \) is \(-1\), we have
Clearly, by (C4), the media firm 2 has no incentive to deviate upon observing the signal $s = -1$.

When the signal $s$ is 0 or 1, the media firm 2’s expected payoff of the underlying strategy is $\alpha \overline{v} - \frac{5}{2} \gamma$. The payoff of any deviation that the media firm 2 reports other than $-1$ is bounded above by $\frac{1 - \alpha}{2} \overline{v}$. By (C4), we have

$$\alpha \overline{v} - \frac{5}{2} \gamma > \frac{1 - \alpha}{2} \overline{v}.$$

Therefore, the media firm 2 has no incentive to deviate upon observing the signal $s = 0$ or 1.

In summary, the media firm 2 has no incentive to deviate.

**Policymaker:** By the definition of $y^*$ in the underlying strategy, the policymaker does not want to implement alternative policy. Thus, it suffices to derive the instituted policy $y^*$ for every combination of $\hat{y}$ and $\theta$, which is displayed in the following table.

<table>
<thead>
<tr>
<th>$\hat{y}$</th>
<th>$\theta = -1$</th>
<th>$\theta = 0$</th>
<th>$\theta = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{y} = -1$</td>
<td>$y^* = -1$</td>
<td>$y^* = -1$</td>
<td>$y^* = { -1, \text{ if } 0 &lt; \lambda_1 &lt; \frac{1}{3}; 0, \text{ otherwise.} }$</td>
</tr>
<tr>
<td>$\hat{y} = 0$</td>
<td>$y^* = 0$</td>
<td>$y^* = 0$</td>
<td>$y^* = 1$</td>
</tr>
<tr>
<td>$\hat{y} = 1$</td>
<td>$y^* = { 1, \text{ if } 0 &lt; \lambda_1 &lt; \frac{1}{3}; 0, \text{ otherwise.} }$</td>
<td>$y^* = 0$</td>
<td>$y^* = 1$</td>
</tr>
</tbody>
</table>
Then we need to derive the instituted policy $y^*$ for every state of nature $\theta$ under the prescribed strategy, and show that the policymaker does not want to deviate at the signaling stage. We denote $\hat{s} \in \{-1,0,1\} \setminus \{s\}$ as the policymaker’s deviation at the signaling stage.

Suppose the condition of this lemma holds, consider the stage at which the policymaker chooses the instituted policy. The policymaker’s instituted policy $y^*$ under the prescribed strategy is

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>report $(r_1, r_2)$</th>
<th>effective public opinion $\hat{y}$</th>
<th>instituted policy $y^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>(-1, 0)</td>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>0</td>
<td>(0, -1), if $z_1 = 0$; (1, -1), otherwise.</td>
<td>-1, if $(r_1, r_2) = (0,-1)$; 0, if $(r_1, r_2) = (1, -1)$.</td>
<td>-1, if $\hat{y} = -1$; 0, if $\hat{y} = 0$.</td>
</tr>
<tr>
<td>1</td>
<td>(1, -1)</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

It remains to show that the policymaker does not want to deviate at the signaling stage. The policymaker’s payoffs of the underlying reporting strategy $s$ and the deviating strategy $\hat{s}$ are

| $\theta$ | public posterior belief $E[\theta|r]$ | payoff of $s$ |
|----------|--------------------------------------|---------------|
| -1       | -1                                   | 0             |
| 0        | $\frac{1-\rho}{2-\rho} \times 0 + \frac{1}{2-\rho} \times 1 = \frac{1}{2-\rho}$ | $\rho \times (-\lambda_1) + (1 - \rho) \times 0 - \lambda_2 = -\rho \lambda_1 - \lambda_2$ |
| 1        | $\frac{1-\rho}{2-\rho} \times 0 + \frac{1}{2-\rho} \times 1 = \frac{1}{2-\rho}$ | $-\lambda_1$ |

<table>
<thead>
<tr>
<th>payoff of $\hat{s} = -1$</th>
<th>payoff of $\hat{s} = 0$</th>
<th>payoff of $\hat{s} = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>n/a</td>
<td>$-\lambda_2$</td>
<td>$\rho \times 0 + (1 - \rho) \times (-\lambda_1) - 4\lambda_2 = -(1 - \rho)\lambda_1 - 4\lambda_2$</td>
</tr>
<tr>
<td>$-\lambda_1 - \lambda_2$</td>
<td>$-\lambda_1$</td>
<td>n/a</td>
</tr>
<tr>
<td>$-\min{1 + \lambda_1, 4\lambda_1} - 4\lambda_2$</td>
<td>$-\min{1 + \lambda_1, 4\lambda_1} - \lambda_2$</td>
<td>n/a</td>
</tr>
</tbody>
</table>

By (C2), we have

$$-\rho \lambda_1 - \lambda_2 > -\lambda_1.$$ 

Clearly, the policymaker has no incentive to deviate. In summary, the semi-separating equilibrium exists.
Proofs of Propositions

Proposition 1: The first part of the proposition immediately follows from corollary 1, and the second part of the proposition immediately follows from corollary 2.

Proposition 2: We shows that under (C1) - (C4), the proposition is true if the public policy preference is biased towards left or right (e.g., $\phi^* = -1, 1$), the public moderately cares about the truth (e.g., $\max\{0, \frac{3\alpha-2}{3\alpha+1}\} < \eta < \max\{0, \frac{3\alpha-2}{3\alpha-1}\}$), and the policymaker’s constituency concern is not trivial (e.g., $\frac{1}{3} < \lambda_1 < 1$). By symmetry, we only show the case in which $\phi^* = -1$.

When $\phi^* = -1$, $\max\{0, \frac{3\alpha-2}{3\alpha+1}\} < \eta < \max\{0, \frac{3\alpha-2}{3\alpha-1}\}$, and $\frac{1}{3} < \lambda_1 < 1$, we have

$$\phi^* = -1 \text{ and } \frac{2}{3} < \alpha < 1 \text{ and } \frac{3\alpha-2}{3\alpha+1} < \eta < \frac{3\alpha-2}{3\alpha-1} \text{ and } \frac{1}{3} < \lambda_1 < 1.$$  \hspace{1cm} (A.5)

By lemma 3, in equilibrium, the public’s posterior belief about the state of nature $\theta$ is the same as their prior belief, so the overall consumer policy surplus in (6) is

$$-y^2 + \frac{1}{6} y(6 - 18\alpha - 6\eta(1 - 3\alpha)) - \frac{1}{6} (3 + 3\alpha + (1 - 3\alpha)\eta), \quad y \in \{-1, 0, 1\}. \hspace{1cm} (A.6)$$

When (A.5) is true, the policy $y = -1$ solves (6) (i.e., $y = -1$ maximizes (A.6)). Then the effective public opinion $\hat{y}$ is $-1$ for all state of nature $\theta \in \{-1, 0, 1\}$.

By (A.6)), in equilibrium, the implemented policy $y^*$ and the policy outcome are

| $\theta$ | public posterior belief $E[\theta|r]$ | effective public opinion $\hat{y}$ | implemented policy $y^*$ | policy outcome $PS(y^*, \theta)$ | optimal policy outcome $PS^*(\theta)$ |
|-----|-------------------------------|-----------------------------|-------------------|-----------------|----------------|
| $-1$ | 0                              | $-1$                        | $-1$              | $-\frac{5}{2}(1 - \alpha)(1 - \eta)$ | $-\frac{5}{2}(1 - \alpha)(1 - \eta)$ |
| 0    | 0                              | $-1$                        | $-1$              | $-\frac{1}{4}(5 - 5\alpha(1 - \eta) - 3\eta)$ | $-\frac{1}{4}(5 - 5\alpha(1 - \eta) - 3\eta)$ |
| 1    | 0                              | $-1$                        | 0                 | $-\frac{1}{2}(1 + \alpha(1 - \eta) + \eta)$ | $-\frac{1}{2}(1 + \alpha(1 - \eta) + \eta)$ |

Clearly, in equilibrium, the media monopoly leads to the optimal policy outcome.

Proposition 3: i) We show that the media monopoly leads to a better policy outcome than the state-owned media does if the follow conditions hold

$$\phi^* = -1, 1 \text{ and } \frac{2}{3} < \alpha < 1 \text{ and } \frac{3\alpha-2}{3\alpha+1} < \eta < \frac{3\alpha-2}{3\alpha-1} \text{ and } \frac{1}{3} < \lambda_1 < 1.$$  \hspace{1cm} (A.7)
In this case, the public moderately cares about the truth as \( \max\{0, \frac{3\alpha-2}{3\alpha} \} < \eta < \max\{0, \frac{3\alpha-2}{3\alpha-1} \} \).

By corollary 2, the state-owned media leads to a suboptimal policy outcome if the public policy preference is biased towards left or right (e.g., \( \phi^* = -1, 1 \)), and the public moderately cares about the truth (e.g., \( \max\{0, \frac{3\alpha-2}{3\alpha} \} < \eta < \max\{0, \frac{3\alpha-2}{3\alpha-1} \} \)). Then the same result is true if (A.7) holds.

On the other hand, by proposition 2, the media monopoly leads to the optimal policy outcome if the public policy preference is biased towards left or right (e.g., \( \phi^* = -1, 1 \)), and the public moderately cares about the truth (e.g., \( \max\{0, \frac{3\alpha-2}{3\alpha} \} < \eta < \max\{0, \frac{3\alpha-2}{3\alpha-1} \} \)), and the policymaker’s constituency concern is not trivial (e.g., \( 1 < \lambda_1 < 1 \)). Then the same result is true if (A.7) holds.

The first part of the proposition immediately follows.

ii) We show that the media monopoly leads to a worse policy outcome than the state-owned media does if the follow conditions hold

\[
\phi^* = -1, 1 \text{ and } \max\left\{ \frac{3\alpha-2}{3\alpha-1} : \frac{3\alpha-2}{3\alpha-3} \right\} < \eta < 1. \tag{A.8}
\]

In this case, the public strongly cares about the truth as \( \max\{\frac{3\alpha-2}{3\alpha-1}, \frac{3\alpha-2}{3\alpha-3} \} < \eta < 1 \).

By corollary 3, the media monopoly leads to a suboptimal policy outcome if the public policy preference is biased towards left or right (e.g., \( \phi^* = -1, 1 \)), and the public strongly cares about the truth (e.g., \( \max\{0, \frac{3\alpha-2}{3\alpha} \} < \eta < 1 \)). Then the same result is true if (A.8) holds.

On the other hand, by corollary 1, the state-owned media leads to the optimal policy outcome if the public policy preference is biased towards left or right (e.g., \( \phi^* = -1, 1 \)), and the public strongly cares about the truth (e.g., \( \max\{0, \frac{3\alpha-2}{3\alpha} \} < \eta < 1 \)). When \( \frac{1}{3} < \alpha < 1 \), we have

\[
\max\{0, \frac{3\alpha-2}{3\alpha-1} \} \leq \max\left\{ \frac{3\alpha-2}{3\alpha-1}, \frac{3\alpha-2}{3\alpha-3} \right\}.
\]

Then the same result is true if (A.8) holds.

The second part of the proposition immediately follows.

iii) We show that the media monopoly leads to the same optimal policy outcome as the state-owned media does if the follow conditions hold

\[
\phi^* = -1, 1 \text{ and } 0 < \eta < \max\{0, \frac{3\alpha-2}{3\alpha+1} \} \text{ and } 0 < \lambda_1 < \frac{1}{3}. \tag{A.9}
\]

By corollary 1, the state-owned media leads to the optimal policy outcome if the public policy preference is biased towards left or right (e.g., \( \phi^* = -1, 1 \)), the public weakly cares about the truth (e.g., \( 0 < \eta < \max\{0, \frac{3\alpha-2}{3\alpha+1} \} \)), and the policymaker’s constituency concern is trivial (e.g.,
0 < \lambda_1 < \frac{1}{3})$. Then the same result is true if (A.9) holds.

By corollary 4, the media monopoly leads to the optimal policy outcome if the public policy preference is biased towards left or right (e.g., $\phi^* = -1, 1$), the public weakly cares about the truth (e.g., $0 < \eta < \max\{\frac{3\alpha-2}{3\alpha+1}, \frac{3\alpha-2}{3\alpha-1}\}$), and the policymaker's constituency concern is trivial (e.g., $0 < \lambda_1 < \frac{1}{3}$). When $\frac{1}{3} < \alpha < 1$, we have

$$\max\{0, \frac{3\alpha-2}{3\alpha+1}\} \leq \max\{\frac{3\alpha-2}{3\alpha+1}, \frac{3\alpha-2}{3\alpha-3}\}.$$  

Then the same result is true if (A.9) holds.

The third part of the proposition immediately follows.

**Proposition 4:** By symmetry, we only show the result for $\phi^* = -1, 0$. We break our analysis into four cases listed as follows.

Case i)  
$$\begin{array}{l}
\phi^* = -1 \text{ and } \max\{0, \frac{3\alpha-2}{3\alpha+1}\} < \eta < 1; \\
\phi^* = 0; \\
0 < \lambda_1 < \frac{1}{3}; \\
\end{array}$$

Case ii)  
$$\begin{array}{l}
\phi^* = -1 \text{ and } 0 < \eta < \max\{0, \frac{3\alpha-2}{3\alpha+1}\} \text{ and } \frac{1}{3} < \lambda_1 < 1; \\
\end{array}$$

Case iii)  
$$\begin{array}{l}
\phi^* = -1 \text{ and } \max\{0, \frac{3\alpha-2}{3\alpha+1}\} < \eta < \max\{0, \frac{3\alpha-2}{3\alpha-1}\}; \\
\end{array}$$

Case iv)  
$$\begin{array}{l}
\phi^* = -1 \text{ and } \max\{0, \frac{3\alpha-2}{3\alpha+1}\} < \eta < \max\{0, \frac{3\alpha-2}{3\alpha}\}. \\
\end{array}$$

For each case, we show that if the policy outcome comparison is feasible, then the proposition is true.

In case i), by corollary 1 and 5, in equilibrium, the public obtains the optimal policy outcome in both the state-owned media market and the duopoly media market. The proposition is true in this case.

In case ii), by lemma 1 and corollary 1, in equilibrium, the policy outcome in the state-owned media market is
\[ \theta \quad PS(-1, \theta) \quad PS(0, \theta) \quad PS(1, \theta) \]

<table>
<thead>
<tr>
<th>(\theta)</th>
<th>(PS(-1, \theta))</th>
<th>(PS(0, \theta))</th>
<th>(PS(1, \theta))</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>(-\frac{5}{2}(1 - \alpha)(1 - \eta))</td>
<td>(-\frac{1}{2}(1 + \alpha(1 - \eta) + \eta))</td>
<td>(-\frac{1}{2}(1 + 7\alpha(1 - \eta) + 7\eta))</td>
</tr>
<tr>
<td>0</td>
<td>(-\frac{1}{2}(5 - 5\alpha(1 - \eta) - 3\eta))</td>
<td>(-\frac{1}{2}(1 + \alpha)(1 - \eta))</td>
<td>(-\frac{1}{2}(1 + 7\alpha(1 - \eta) + \eta))</td>
</tr>
<tr>
<td>1</td>
<td>(-\frac{1}{2}(5 - 5\alpha(1 - \eta) + 3\eta))</td>
<td>(-\frac{1}{2}(1 + \alpha(1 - \eta) + \eta))</td>
<td>(-\frac{1}{2}(1 + 7\alpha)(1 - \eta))</td>
</tr>
</tbody>
</table>

\(PS^*(\theta)\) | optimal policy | implemented policy \(y^*\) |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(-\frac{5}{2}(1 - \alpha)(1 - \eta))</td>
<td>-1</td>
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<tr>
<td>(-\frac{1}{2}(5 - 5\alpha(1 - \eta) - 3\eta))</td>
<td>-1</td>
<td>-1</td>
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<tr>
<td>(-\frac{1}{2}(5 - 5\alpha(1 - \eta) + 3\eta))</td>
<td>-1</td>
<td>0</td>
</tr>
</tbody>
</table>

On the other hand, by lemma 4 and corollary 5, in equilibrium, the policy outcome in the duopoly media market is

\[ \theta \quad PS(-1, \theta) \quad PS(0, \theta) \quad PS(1, \theta) \]

<table>
<thead>
<tr>
<th>(\theta)</th>
<th>(PS(-1, \theta))</th>
<th>(PS(0, \theta))</th>
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<tbody>
<tr>
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<td>(-\frac{5}{2}(1 - \alpha)(1 - \eta))</td>
<td>(-\frac{1}{2}(1 + \alpha(1 - \eta) + \eta))</td>
<td>(-\frac{1}{2}(1 + 7\alpha(1 - \eta) + 7\eta))</td>
</tr>
<tr>
<td>0</td>
<td>(-\frac{1}{2}(5 - 5\alpha(1 - \eta) - 3\eta))</td>
<td>(-\frac{1}{2}(1 + \alpha)(1 - \eta))</td>
<td>(-\frac{1}{2}(1 + 7\alpha(1 - \eta) + \eta))</td>
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<tr>
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<td>(-\frac{1}{2}(5 - 5\alpha(1 - \eta) + 3\eta))</td>
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</table>

\(PS^*(\theta)\) | optimal policy | implemented policy \(y^*\) |
<table>
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</tr>
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</tr>
</tbody>
</table>

Clearly, in equilibrium, the public obtains the same suboptimal policy outcome in both the state-owned media market and the duopoly media market. The proposition is true in this case.

In case iii), by corollary 2 and 6, in equilibrium, the public obtains a better / the same policy outcome in the duopoly media market than in the state-owned media market. The proposition is true in this case.

In case iv), we show that there is no equilibrium in the state-owned media market, so the policy outcome comparison is not feasible. Suppose the contrary is true, then the equilibrium is one of
the following: fully separating equilibrium, semi-separating equilibrium or pooling equilibrium.

**Fully separating equilibrium:** We follow the steps in lemma 1 and compute the policymaker’s payoffs of the underlying reporting strategy \( s \) (i.e. truth-telling) and the deviating strategy \( \hat{s} \).

| \( \theta \) | public posterior belief \( E[\theta|r] \) | effective public opinion \( \hat{y} \) | payoff of \( s \) |
|-----------|-----------------|-----------------|-----------------|
| -1        | -1              | -1              | 0               |
| 0         | 0               | -1              | -\( \lambda_1 \) |
| 1         | 1               | 0               | -\( \lambda_1 \) |

<table>
<thead>
<tr>
<th>payoff of ( \hat{s} = -1 )</th>
<th>payoff of ( \hat{s} = 0 )</th>
<th>payoff of ( \hat{s} = 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>n/a</td>
<td>-( \lambda_2 )</td>
<td>-( \lambda_1 - 4\lambda_2 )</td>
</tr>
<tr>
<td>-( \lambda_1 - \lambda_2 )</td>
<td>n/a</td>
<td>-( \lambda_2 )</td>
</tr>
<tr>
<td>-( \min{4\lambda_1, 1 + \lambda_1} - 4\lambda_2 )</td>
<td>n/a</td>
<td>-( \lambda_2 )</td>
</tr>
</tbody>
</table>

Since \( \lambda_2 < (1 - \rho)\lambda_1 < \lambda_1 \), the policymaker wants to deviate when \( \theta = 0 \). The fully separating equilibrium does not exist.

**Semi-Separating equilibrium:** There are three possible signaling strategies \( s^{(-1,0)} \), \( s^{(-1,1)} \) and \( s^{(0,1)} \) for the semi-separating equilibrium, where

\[
s^{(-1,0)} = \begin{cases} s_1^{(-1,0)}, & \text{if } \theta = -1, 0; \\ s_2^{(-1,0)} \neq s_1^{(-1,0)}, & \text{if } \theta = 1. \end{cases}
\]

and

\[
s^{(-1,1)} = \begin{cases} s_1^{(-1,1)}, & \text{if } \theta = -1, 1; \\ s_2^{(-1,1)} \neq s_1^{(-1,1)}, & \text{if } \theta = 0. \end{cases}
\]

and

\[
s^{(0,1)} = \begin{cases} s_1^{(0,1)}, & \text{if } \theta = 0, 1; \\ s_2^{(0,1)} \neq s_1^{(0,1)}, & \text{if } \theta = -1. \end{cases}
\]

For the signaling strategy \( s^{(-1,0)} \), the policymaker wants to deviate to \( s_2^{(-1,0)} \) when \( \theta = 0 \). In fact, such deviation increases the policymaker’s utility by at least \( \lambda_1 - \lambda_2 \). The benefit \( \lambda_1 \) comes from the fact that the effective public opinion \( \hat{y} \) is -1 if the policymaker signals \( s_1^{(-1,0)} \), and is 0 if the policymaker deviates to \( s_2^{(-1,0)} \). The cost \( \lambda_2 \) comes from the fact that such deviation incurs a reputation cost at most \( \lambda_2 \).

For the signaling strategy \( s^{(-1,1)} \), we denote \( s_3^{(-1,1)} \in \{-1, 0, 1\} \setminus \{s_1^{(-1,1)}, s_2^{(-1,1)}\} \). Then the policymaker wants to deviate to \( s_3^{(-1,1)} \) when \( \theta = s_3^{(-1,1)} \). In fact, such deviation increases the
policymaker’s utility by at least \( \lambda_2 \) as the effective public opinion \( \hat{y} \) is \(-1\) for all signals \( s_1^{(-1,1)}, s_2^{(-1,1)} \) and \( s_3^{(-1,1)} \).

For the signaling strategy \( s^{(0,1)} \), we denote \( s_3^{(0,1)} \in \{-1,0,1\} \setminus \{s_1^{(0,1)}, s_2^{(0,1)}\} \). Then the policymaker wants to deviate to \( s_3^{(0,1)} \) when \( \theta = s_3^{(0,1)} \). In fact, such deviation increases the policymaker’s utility by at least \( \lambda_2 \) as the effective public opinion \( \hat{y} \) is \(-1\) for all signals \( -1, 0 \) and \( 1 \). Therefore, the pooling equilibrium does not exist.

Thus, the semi-separating equilibrium does not exist.

**Pooling equilibrium:** We denote the pooling signal as \( s_{pool} \in \{-1,0,1\} \). Then the policymaker wants to deviate to truth telling when \( \theta \in \{-1,0,1\} \setminus \{s_{pool}\} \). In fact, such deviation increases the policymaker’s utility by at least \( \lambda_2 \) as the effective public opinion \( \hat{y} \) is \(-1\) for all signals \(-1, 0 \) and \( 1 \). Therefore, the pooling equilibrium does not exist.

In summary, in case iv), there is no equilibrium in the state-owned media market, so the policy outcome comparison is not feasible. Then the proposition is true in this case.

**Proposition 5:** It suffices to show that the proposed ranking exists if (C1) - (C4) are true. Suppose (C1) - (C4) are true. By the second part of proposition 3, the media monopoly leads to a worse policy outcome than the state-owned media does if (A.8) holds. Then the public has the following ranking if (A.8) holds, regardless of \( 0 < \lambda_1 < 1 \).

State-Owned Media Ownership \( \succeq \) Monopoly Media Ownership.

On the other hand, by proposition 4, the public has the following ranking if (A.8) holds, regardless of \( 0 < \lambda_1 < 1 \).

Duopoly Media Ownership \( \succeq \) State-Owned Media Ownership.

In summary, if (A.8) holds, regardless of \( 0 < \lambda_1 < 1 \), the public will rank the market structures of media ownership as follows

Duopoly Media Ownership \( \succeq \) State-Owned Media Ownership \( \succeq \) Monopoly Media Ownership.

### C Proofs of Corollaries

**Corollary 1:** By symmetry, we only show the result for \( \phi^* = -1, 0 \). By lemma 1, there is a
fully separating equilibrium if any of the three conditions of the corollary holds. We show that in equilibrium, the policy outcome will be optimal if any of the three conditions of the corollary holds.

**Condition i):** When \( \phi^* = -1 \) and \( \max\{0, \frac{3\alpha-2}{3\alpha-1}\} < \eta < 1 \), we have

- i-a) \( \phi^* = -1 \) and \( \left\{ \frac{1}{3} < \alpha \leq \frac{2}{3} \text{ and } \frac{3\alpha-2}{3\alpha-3} < \eta < \frac{3\alpha}{3\alpha+1} \right\} \);  
- i-b) \( \phi^* = -1 \) and \( \frac{1}{3} < \alpha < \frac{2}{3} \) and \( 0 < \eta < \frac{3\alpha-2}{3\alpha-3} \);  
- i-c) \( \phi^* = -1 \) and \( \frac{1}{3} < \alpha < 1 \) and \( \frac{3\alpha}{3\alpha+1} < \eta < 1 \).

Suppose condition i-a) holds, we have

\[
\begin{align*}
\theta & \quad PS(-1, \theta) & \quad PS(0, \theta) & \quad PS(1, \theta) \\
-1 & \quad -\frac{5}{2}(1-\alpha)(1-\eta) & \quad -\frac{1}{2}(1+\alpha(1-\eta)+\eta) & \quad -\frac{1}{2}(1+7\alpha(1-\eta)+7\eta) \\
0 & \quad -\frac{1}{2}(5-5\alpha(1-\eta)-3\eta) & \quad -\frac{1}{2}(1+\alpha)(1-\eta) & \quad -\frac{1}{2}(1+7\alpha(1-\eta)+\eta) \\
1 & \quad -\frac{1}{2}(5-5\alpha(1-\eta)+3\eta) & \quad -\frac{1}{2}(1+\alpha(1-\eta)+\eta) & \quad -\frac{1}{2}(1+7\alpha)(1-\eta)
\end{align*}
\]

\[
\begin{align*}
PS^*(\theta) & \quad \text{optimal policy} & \quad \text{implemented policy } y^* \\
-\frac{5}{2}(1-\alpha)(1-\eta) & \quad -1 & \quad -1 \\
-\frac{1}{2}(1+\alpha)(1-\eta) & \quad 0 & \quad 0 \\
-\frac{1}{2}(1+\alpha(1-\eta)+\eta) & \quad 0 & \quad 0
\end{align*}
\]

Clearly, the policy outcome will be optimal in the state-owned media market if condition i-a) holds.

Suppose condition i-b) holds, we have
\[ \theta \quad PS(-1, \theta) \quad PS(0, \theta) \quad PS(1, \theta) \]

<table>
<thead>
<tr>
<th>( \theta )</th>
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<tr>
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<tr>
<td>0</td>
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\[ PS^*(\theta) \]

<table>
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</tr>
</thead>
<tbody>
<tr>
<td>(-\frac{1}{2}(1 + \alpha(1 - \eta) + \eta))</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(-\frac{1}{2}(1 + \alpha)(1 - \eta))</td>
<td>0</td>
<td>0</td>
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<td>(-\frac{1}{2}(1 + \alpha(1 - \eta) + \eta))</td>
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<td>0</td>
</tr>
</tbody>
</table>

Clearly, the policy outcome will be optimal in the state-owned media market if condition i-b) holds.

Suppose condition i-c) holds, we have

\[ \theta \quad PS(-1, \theta) \quad PS(0, \theta) \quad PS(1, \theta) \]

<table>
<thead>
<tr>
<th>( \theta )</th>
<th>( PS(-1, \theta) )</th>
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<tr>
<td>-1</td>
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\[ PS^*(\theta) \]

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<td>0</td>
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<tr>
<td>(-\frac{1}{2}(1 + 7\alpha)(1 - \eta))</td>
<td>1</td>
<td>1</td>
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</tbody>
</table>

Clearly, the policy outcome will be optimal in the state-owned media market if condition i-c) holds.

**Condition ii):** When \( \phi^* = -1 \), \( 0 < \eta < \max\{0, \frac{3\alpha - 2}{3\alpha + 1}\} \) and \( 0 < \lambda_1 < \frac{1}{3} \), we have
\[ \phi^* = -1 \text{ and } \frac{2}{3} < \alpha < 1 \text{ and } 0 < \eta < \frac{3\alpha - 2}{3\alpha + 1} \text{ and } 0 < \lambda_1 < \frac{1}{3} \]

Suppose condition ii) holds, we have

<table>
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<td>-1</td>
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<td>(-\frac{1}{2}(5 - 5\alpha(1 - \eta) + 3\eta))</td>
<td>-1</td>
<td>-1</td>
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</tbody>
</table>

Clearly, the policy outcome will be optimal in the state-owned media market if condition ii) holds.

**Condition iii):** When \( \phi^* = 0 \), we have

iii-a) \( \phi^* = 0 \text{ and } \frac{1}{3} < \alpha < 1 \text{ and } \frac{1}{2} < \eta < 1; \)

iii-b) \( \phi^* = 0 \text{ and } \frac{1}{3} < \alpha < 1 \text{ and } 0 < \eta < \frac{1}{2}. \)

Suppose condition iii-a) holds, we have
Clearly, the policy outcome will be optimal in the state-owned media market if condition iii-a) holds.

Suppose condition iii-b) holds, we have

<table>
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<td>1</td>
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<td>$-(1 - \alpha(1 - \eta))$</td>
<td>$-(2 - \alpha)(1 - \eta)$</td>
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</table>

Clearly, the policy outcome will be optimal in the state-owned media market if condition iii-b) holds.

In summary, the policy outcome will be optimal in the state-owned media market if any of the three conditions of the corollary holds.

**Corollary 2:** By symmetry, we only show the result for $\phi^* = -1$. By lemma 2, there is a
semi-separating equilibrium if the conditions of the corollary hold. We show that in equilibrium, the policy outcome may be suboptimal if the conditions of the corollary hold.

When \( \phi^* = -1 \) and \( \max\{0, \frac{3\alpha - 2}{3\alpha - 1}\} < \eta < \max\{0, \frac{3\alpha - 2}{3\alpha - 3}\} \), we have

\[
\phi^* = -1 \text{ and } \frac{2}{3} < \alpha < 1 \text{ and } \frac{3\alpha - 2}{3\alpha} < \eta < \frac{3\alpha - 2}{3\alpha - 1}
\]

Suppose this condition holds, we have

<table>
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\( PS^*(\theta) \)

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</table>

Clearly, the policy outcome may be suboptimal in the state-owned media market if the conditions of the corollary hold.

**Corollary 3:** By symmetry, we only show the result for \( \phi^* = -1, 0 \). By lemma 3, in equilibrium, the policymaker always tells the truth whereas the media monopoly offers a pooling report. We show that in equilibrium, the policy outcome may be suboptimal if any of the two conditions of the corollary holds.

*Condition i)*: When \( \phi^* = -1 \) and \( \max\{\frac{3\alpha - 2}{3\alpha - 1}, \frac{3\alpha - 2}{3\alpha - 3}\} < \eta < 1 \), we have

i-a) \( \phi^* = -1 \text{ and } \begin{cases} \frac{1}{3} < \alpha \leq \frac{2}{3} \text{ and } \frac{2-3\alpha}{3-3\alpha} < \eta < \frac{3\alpha}{3\alpha+1} \\ \frac{2}{3} < \alpha < 1 \text{ and } \frac{3\alpha-2}{3\alpha-1} < \eta < \frac{3\alpha}{3\alpha+1} \end{cases} \); 

i-b) \( \phi^* = -1 \text{ and } \frac{1}{3} < \alpha < 1 \text{ and } \frac{3\alpha}{3\alpha+1} < \eta < 1 \).

Suppose condition i-a) holds, we have
<table>
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Clearly, the policy outcome may be suboptimal in the monopoly media market if condition i-a) holds.

Suppose condition i-b) holds, we have

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Clearly, the policy outcome may be suboptimal in the monopoly media market if condition i-b) holds.

*Condition ii)*: When \( \phi^* = 0 \) and \( \frac{1}{2} < \eta < 1 \), we have
Clearly, the policy outcome may be suboptimal in the monopoly media market if condition ii) holds.

In summary, the policy outcome may be suboptimal in the monopoly media market if any of the two conditions of the corollary holds.

**Corollary 4:** By symmetry, we only show the result for \( \phi^* = -1, 0 \). By lemma 3, in equilibrium, the policymaker always tells the truth whereas the media monopoly offers a pooling report. We show that in equilibrium, the policy outcome will be optimal if any of the two conditions of the corollary holds.

*Condition i):* When \( \phi^* = -1 \), \( \max\{\frac{3\alpha-2}{3\alpha+1}, \frac{3\alpha-2}{3\alpha-3}\} < \eta < 1 \) and \( 0 < \lambda_1 < \frac{1}{3} \), we have

i-a) \( \phi^* = -1 \) and \( \frac{2}{3} < \alpha < 1 \) and \( 0 < \eta < \frac{3\alpha-2}{3\alpha+1} \) and \( 0 < \lambda_1 < \frac{1}{3} \);

i-b) \( \phi^* = -1 \) and \( \frac{1}{3} < \alpha < \frac{2}{3} \) and \( 0 < \eta < \frac{2-3\alpha}{3-3\alpha} \) and \( 0 < \lambda_1 < \frac{1}{3} \).

Suppose condition i-a) holds, we have
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<td>$-\frac{1}{2} (1 + \alpha(1 - \eta) + \eta)$</td>
<td>$-\frac{1}{2} (1 + 7\alpha(1 - \eta) + \eta)$</td>
</tr>
<tr>
<td>1</td>
<td>$-\frac{1}{2} (5 - 5\alpha(1 - \eta) + 3\eta)$</td>
<td>$-\frac{1}{2} (1 + \alpha(1 - \eta) + \eta)$</td>
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$PS^*(\theta)$ | optimal policy | implemented policy $y^*$ |
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<td>-1</td>
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Clearly, the policy outcome will be optimal in the monopoly media market if condition i-a) holds.

Suppose condition i-b) holds, we have

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<tr>
<td>1</td>
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$PS^*(\theta)$ | optimal policy | implemented policy $y^*$ |
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</tr>
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Clearly, the policy outcome will be optimal in the monopoly media market if condition i-b) holds.

**Condition ii):** When $\phi^* = 0$ and $0 < \eta < \frac{1}{2}$, we have
Clearly, the policy outcome will be optimal in the monopoly media market if condition ii) holds.

In summary, the policy outcome will be optimal in the monopoly media market if any of the two conditions of the corollary holds.

**Corollary 5:** By symmetry, we only show the result for $\phi^* = -1, 0$. By lemma 4, there exists an informative equilibrium if any of the three conditions of the corollary holds. We show that in equilibrium, the policy outcome will be optimal if any of the three conditions of the corollary holds.

**Condition i):** When $\phi^* = -1$ and $\max\{0, \frac{3\alpha-2}{3\alpha-1}\} < \eta < 1$, we have

- i-a) $\phi^* = -1$ and $\left\{ \begin{aligned} \frac{1}{3} &< \alpha \leq \frac{2}{3} \quad \text{and} \quad \frac{3\alpha-2}{3\alpha-3} < \eta < \frac{3\alpha}{3\alpha+1} \\ \frac{2}{3} &< \alpha < 1 \quad \text{and} \quad \frac{3\alpha-2}{3\alpha-1} < \eta < \frac{3\alpha}{3\alpha+1} \end{aligned} \right.$;
- i-b) $\phi^* = -1$ and $\frac{1}{3} < \alpha < \frac{2}{3}$ and $0 < \eta < \frac{3\alpha-2}{3\alpha-3}$;
- i-c) $\phi^* = -1$ and $\frac{1}{3} < \alpha < 1$ and $\frac{3\alpha}{3\alpha+1} < \eta < 1$.

Suppose condition i-a) holds, we have

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<th>$\theta$</th>
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<tr>
<td>$-1$</td>
<td>$-(2 - \alpha)(1 - \eta)$</td>
<td>$-(1 - \alpha(1 - \eta))$</td>
<td>$-(2 - \alpha + (2 + \alpha)\eta)$</td>
</tr>
<tr>
<td>$0$</td>
<td>$-(2 - \alpha(1 - \eta) - \eta)$</td>
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Clearly, the policy outcome will be optimal in the duopoly media market if condition i-a) holds.

Suppose condition i-b) holds, we have

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Clearly, the policy outcome will be optimal in the duopoly media market if condition i-b) holds.

Suppose condition i-c) holds, we have
Clearly, the policy outcome will be optimal in the duopoly media market if condition i-c) holds.

*Condition ii)*: When \( \phi^* = -1 \), \( 0 < \eta < \max\{0, \frac{3\alpha - 2}{3\alpha + 1}\} \) and \( 0 < \lambda_1 < \frac{1}{3} \), we have

\[
\phi^* = -1 \text{ and } \frac{2}{3} < \alpha < 1 \text{ and } 0 < \eta < \frac{3\alpha - 2}{3\alpha + 1} \text{ and } 0 < \lambda_1 < \frac{1}{3}
\]

Suppose condition ii) holds, we have
Condition iii): When $\phi^* = 0$, we have

iii-a) $\phi^* = 0$ and $\frac{1}{3} < \alpha < 1$ and $\frac{1}{2} < \eta < 1$;

iii-b) $\phi^* = 0$ and $\frac{1}{3} < \alpha < 1$ and $0 < \eta < \frac{1}{2}$.

Suppose condition iii-a) holds, we have

\[
\begin{array}{c|ccc}
\theta & PS(-1, \theta) & PS(0, \theta) & PS(1, \theta) \\
\hline
-1 & -(2 - \alpha)(1 - \eta) & -(1 - \alpha(1 - \eta)) & -(2 - \alpha + (2 + \alpha)\eta) \\
0 & -(2 - \alpha(1 - \eta) - \eta) & -(1 - \alpha(1 - \eta) - \eta) & -(2 - \alpha(1 - \eta) - \eta) \\
1 & -(2 - \alpha + (2 + \alpha)\eta) & -(1 - \alpha(1 - \eta)) & -(2 - \alpha)(1 - \eta) \\
\end{array}
\]

$PS^*(\theta)$ optimal policy implemented policy $y^*$

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<td>0</td>
</tr>
<tr>
<td>$-(2 - \alpha)(1 - \eta)$</td>
<td>1</td>
<td>1</td>
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Clearly, the policy outcome will be optimal in the duopoly media market if condition iii-a) holds.

Suppose condition iii-b) holds, we have

\[
\begin{array}{c|ccc}
\theta & PS(-1, \theta) & PS(0, \theta) & PS(1, \theta) \\
\hline
-1 & -(2 - \alpha)(1 - \eta) & -(1 - \alpha(1 - \eta)) & -(2 - \alpha + (2 + \alpha)\eta) \\
0 & -(2 - \alpha(1 - \eta) - \eta) & -(1 - \alpha(1 - \eta) - \eta) & -(2 - \alpha(1 - \eta) - \eta) \\
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$PS^*(\theta)$ optimal policy implemented policy $y^*$

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Clearly, the policy outcome will be optimal in the duopoly media market if condition iii-b) holds.

In summary, the policy outcome will be optimal in the duopoly media market if any of the three conditions of the corollary holds.

**Corollary 6:** By symmetry, we only show the result for $\phi^* = -1$. By lemma 5, there is a semi-separating equilibrium if the conditions of the corollary hold. We show that in equilibrium, the policy outcome may be suboptimal if the conditions of the corollary hold.

When $\phi^* = -1$ and $\max\{0, \frac{3\alpha - 2}{3\alpha}\} < \eta < \max\{0, \frac{3\alpha - 2}{3\alpha - 1}\}$, we have

$$\phi^* = -1 \text{ and } \frac{2}{3} < \alpha < 1 \text{ and } \frac{3\alpha - 2}{3\alpha} < \eta < \frac{3\alpha - 2}{3\alpha - 1}$$

Suppose this condition holds, we have

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<td>-1</td>
<td>$\begin{cases} -1, &amp; \text{if } z_1 = 0; \ 0, &amp; \text{if } z_1 = \emptyset. \end{cases}$</td>
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Clearly, the policy outcome may be suboptimal in the duopoly media market if the conditions of the corollary hold.

**Corollary 7:** It suffices to show that the proposed ranking exists if (C1) - (C4) and the inequality $\frac{1}{3} < \lambda_1 < 1$ are true. Consider the case in which

$$\phi^* = -1 \text{ and } \frac{2}{3} < \alpha < 1 \text{ and } \frac{3\alpha - 2}{3\alpha} < \eta < \frac{3\alpha - 2}{3\alpha - 1} \text{ and } \frac{1}{3} < \lambda_1 < 1.$$  \hspace{1cm} (A.10)
By proposition 2, the policy outcome will be optimal in the monopoly media market if \((A.10)\) holds. Moreover, by corollary 6, the policy outcome may be suboptimal in the duopoly media market if \((A.10)\) holds. Thus, when \((A.10)\) holds, we have the following ranking

\[
\text{Monopoly Media Ownership} \succeq \text{Duopoly Media Ownership.}
\]

On the other hand, comparing corollary 2 to corollary 6, we know that the public obtains a better / the same policy outcome in the duopoly media market than in the state-owned media market if \((A.10)\) holds. Thus, when \((A.10)\) holds, we have the following ranking

\[
\text{Duopoly Media Ownership} \succeq \text{State-Owned Media Ownership.}
\]

The corollary immediately follows.