# Specification and Negotiation in Incomplete Contracts*

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**Abstract**

This paper investigates the strategic interaction between the buyer and sellers in procurement auctions of incomplete contracts. We introduce a structural model where the buyer chooses the initial design of a contract endogenously, and use it to analyze the auctions of highway procurement contracts by The California Department of Transportation (CalTrans). We find that the contract winners have substantial bargaining power in the negotiation following the auctions. On average, the holdup on the buyer due to the contractual incompleteness is 20.4% of the engineering estimate of the project costs. Counterfactual cost-plus contracts would reduce the buyer's surplus in over 71.7% of the projects in the data, with an average reduction of $382,074.

**Keywords:** Contract specification, post-auction negotiation, incomplete contracts
**JEL:** D44, D82, C14, C57, H57, L13, L74, R42

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# 1 Introduction

Government procurement contracts are often subject to negotiation and modification *after* their assignment through standard low-price auctions. The initial contract specification, or design, might be revised, and an additional transfer be negotiated between the buyer and the auction winner. Examples include the auctions of procurement contracts by The California Department of Transportation (CalTrans) in Bajari, Houghton, and Tadelis (2014) and The Texas Department of Transportation (DOT) in De Silva, Dunne, Kosmopoulou, and Lamarche (2015). Awareness of such incompleteness impacts how the sellers (contractors) compete in the auction and interact with the buyer (government). It also affects the answer to policy questions such as comparing the buyer surplus under alternative forms of contracts.

In this paper, we investigate several empirical and theoretical questions related to the strategic incentives of the buyer and the contractors under incomplete contracts in the highway procurement auctions held by CalTrans. The questions we investigate are partly motivated by several stylized facts in the data. First, a non-negligible proportion of the contracts are not revised after the procurement auction. Second, among the revised contracts, the negotiated transfers vary significantly even conditional on the size of the revision (as measured by the difference in costs). Third, while the characteristics of the contracts (e.g., the job type and the capacity of contractors involved) have no obvious effect on the magnitude of the revision, they do have a significant impact on the negotiated transfers.

These patterns naturally lead to the following questions. What factors determine the revision of a contract after the procurement auction, and the transfer to the contractor in the negotiation after the auction? How do those factors depend on the bargaining power of the buyer and the features of the contract? Is it possible to use data on the contractors' bids and the negotiated transfers to infer the social surplus generated by the contract and infer the costs for contractors? How to quantify the holdup on the buyer due to ex ante uncertainty about the new design? How would a buyer's surplus change under an alternative (counterfactual) form of cost-plus contracts?

We investigate these questions using a structural model with several distinctive new features. First, our model endogenizes the buyer's choice of the initial contract specification prior to the auction. Taking into account the strategic incentive in this initial choice of the contract design is important for measuring the markup of contractors and their holdup on the buyer. Second, we rationalize the revision of contracts by new, feasible designs which are drawn after the auction, and which yield positive net social surplus. We also model the negotiated transfer as a Nash Bargaining solution. Contract characteristics affect the negotiated outcome through their impact on the buyer's bargaining power. Third, we maintain a flexible information structure whereby neither the buyer nor the contractors are required to have perfect foresight or rational expectations of the revised contract or transfers.[1]

[1]The information structure in our model differs from the existing empirical literature. For instance, Bajari, Houghton, and Tadelis (2014) and De Silva, Dunne, Kosmopoulou, and Lamarche (2016) assumed

**Identification of Model Elements** The structural elements in our model include the distribution of contractors' costs, the bargaining power of the contractors in the negotiation after the auctions, the surplus and costs due to the contract revision, and the buyer's prior belief about the new, feasible contract design to be drawn after the auction. Several sources of variation in the data allow us to recover these elements using the equilibrium implication of the model. These sources include the joint variation of the initial and revised contract specification, the distribution of the bids and the payment in the procurement auctions (based on the initial designs), and how the negotiated transfers vary with contract specification as well as the characteristics of contractors.

Our identification strategy takes several steps. First, we recover the buyer's belief about the new feasible contract design from the *joint* distribution of the initial and revised contract specification. This step uses the equilibrium implications that the buyer's belief is consistent, and that the buyer's choice of the initial design is monotone in its private signal, which allows for a scale normalization of the signal.

Next, we extend the argument in Guerre, Perrigne, and Vuong (2000) and use the distribution of bids in the procurement auction to recover the contractors' *adjusted* costs based on the initial contract specification in the auction.[2] These costs include an *endogenous* downward adjustment made by the contractors to take into account their holdup on the buyer (that is, the expected share of the net surplus from the revision to be conceded to the contractor in negotiation). These adjusted costs reflect how the contractors compete strategically in the auctions, knowing that additional gains are possible in subsequent negotiation.

We then identify the model elements that determine the holdup, which include the net surplus from the contract revision and the bargaining power of contractors. To do so, we utilize the joint variation in the initial design and the negotiated transfers in data. In equilibrium, the buyer chooses an optimal initial design of the contract by equating its marginal effect on the social surplus with that on ex ante costs, which consist of the expected auction payment and the holdup. We show that the marginal effect of the initial contract design on both elements in ex ante costs can be recovered from the data. First, the marginal effect on the expected auction payment is directly identifiable using the bidding data. Second, the interpretation of the negotiated transfer as a Nash bargaining solution allows us to link the marginal effect of the contract design on the transfers observed in the data to the marginal effect on the holdup. Combining these structural links, we recover the marginal effect of the contract design on social surplus.

Finally, Nash Bargaining implies that the negotiated transfers in the data are weighted

that the contractors have rational expectation about the actual quantities to be used in the new design conditional on the contract being revised. Jung, Kosmopoulou, Lamarche, and Sicotte (2014) assumed that contractors form expectations about the future adjustments on each item based on the historical probability of revision and negotiation.

[2]The seminal work in Guerre, Perrigne, and Vuong (2000) shows how to invert the seller strategies using bids from a standard first-price auction in Bayesian Nash equilibrium (BNE). In comparison, our contribution is to show that with contractual incompleteness, a similar first-order condition exists in a monotone Perfect Bayesian equilibrium (PBE), with sellers' strategies based on their *adjusted* costs which account for their ex ante hold-up on the buyer due to contract revisions.

sums of the incremental surplus and costs due to contract revision. The weights depend on the relative bargaining power of the contractor and the buyer. Thus, we use the observed marginal impact of contract specification on negotiated transfers to back out the bargaining power and the revision costs. This step uses the social surplus identified earlier, and requires flexible shape restrictions on the costs for revision.

With these model elements recovered, the contractor's holdup on the buyer in the adjustment costs is identified. It follows that the bidding strategy of contractors and the distribution of their costs are both identified. Using these primitives, we compare the buyer surplus in our model with that under a counterfactual cost-plus contract.

**Preview of Empirical Findings** We use our model to analyze the auctions of highway procurement contracts by the California Department of Transportation (CalTrans). We find that the auction winners have significant bargaining power against the buyer, which depends on the intensity of competition as well as the contractor characteristics such as utilization rates, defined as ratios of contractors' backlog over their capacity. Our estimates also indicate that there is an increasing return in the social surplus from highway construction, and that the net surplus from the contract revision is nonlinear in the size of revision. These results are consistent with the reduced-form patterns in the data that motivate our structural approach.

The average markup in a contractor's quote is around 11%. These markups vary with contract characteristics such as the job type and the utilization rate of competitors, and decrease sharply with the intensity of competition in auctions. Besides, our estimates suggest that auction winners enjoy cost advantage over competitors, which allows them to win with high markups. In addition, we find that ignoring how contractors respond strategically to the uncertainty about contract revision would lead to an over-estimation of their markups in the quotes. On average, the markups are over-estimated about 26% without taking into account the contractual incompleteness.

Our estimates suggest that incomplete contracts lead to sizable holdups on the buyer (on average 20.4% of the engineering estimate, which is CalTrans initial estimate of the project cost). The holdup as a percentage of the engineering estimate is higher for contracts involving major jobs or more bidders. We show that counterfactual cost-plus contracts would yield lower buyer surplus for 71.7% of the projects in the data, with an average reduction of $\$382,074$. This indicates that the buyer's gains in ex ante surplus under fixed-price contracts mostly outweigh the cost of holdup due to incomplete contracts.

**Relation to Existing Literature** Models of incomplete contracts have been used to study employment relation (Simon (1951), Klein, Crawford, and Alchian (1978)), ownership and the property rights of firms (Williamson (1985), Grossman and Hart (1986), Hart and Moore (1990)), and international trade (Spencer (2005)). Our work in this paper is related to Tirole (2009), which models contractual incompleteness as a consequence of the buyer's optimal choice of cognitive effort. We extend Tirole (2009) to an environment where the initial contract price is determined via competitive bidding.

This article contributes to a growing empirical literature on bargaining games in various contexts. This includes the bilateral negotiation between hospitals and managed-care organizations in Gowrisankaran, Nevo, and Town (2014), the employer-insurer and hospital-insurer negotiation over premiums and reimbursements in Ho and Lee (2017), and the price negotiation between suppliers and buyers in the health industry in Grennan (2013).

Our work is closely related to the literature on post-auction bargaining (e.g., Elyakime, Laffont, Loisel, and Vuong (1997) and Larsen (2014)) and on incomplete contracts (e.g., Crocker and Reynolds (1993), Bajari, McMillan, and Tadelis (2009), Bajari, Houghton, and Tadelis (2014), Lewis and Bajari (2014) and De Silva, Dunne, Kosmopoulou, and Lamarche (2016)). Compared with these other papers, our structural model has distinctive new features about the cause and the information structure of the incomplete contracts. In addition, we address new empirical questions such as measuring the contractor holdup on the buyer and a quantitative comparison of buyer surplus under different contract formats. Our model requires an original argument for identification that has not been used in the other contexts.

Our paper also contributes to a broader literature on the identification of structural models of auctions and contracts (e.g., D'Haultfoeuille and Février (2007), Aryal, Perrigne, and Quang (2012), Perrigne and Vuong (2011) and Perrigne and Vuong (2012)). These papers build their argument around the mapping between the unobserved types of agents and the features of the contracts in the data (e.g., the price and the package offered in the contracts). In comparison, we consider a model where the classical arguments based on the contractor incentives alone are insufficient for identifying the full model. Hence we construct a new argument that capitalizes on the buyer's rationality and the Nash Bargaining interpretation of negotiated transfers for identifying the model.

While we use the same source of data as Bajari, Houghton, and Tadelis (2014), there are major qualitative differences between our model and that in Bajari, Houghton, and Tadelis (2014). Bajari, Houghton, and Tadelis (2014) maintained that the buyer's choice of the initial contract specification is exogenous and that the contractors have perfect foresight about the negotiated transfers as well as the new feasible design, both of which are assumed exogenous.[3] The model in Bajari, Houghton, and Tadelis (2014) does not rationalize why both sides of a contract would adopt an incompleteness pact that allows them to switch to a new design in the future. Nor does it link the holdup on the buyer to the ex ante uncertainty about the new design. In comparison, we relax these assumptions and model a game of sequential moves with private information that rationalizes incomplete contracts and answers new empirical and policy questions mentioned above.

**Roadmap** In Section 2 we describe our model and define its equilibrium. In Section 3 we discuss the identification of the model elements using the bid and transfer data. Section 4 explains the institutional background about the auctions of highway procurement contracts by CalTrans, describes the data, and summarizes stylized facts that motivate

[3]See the second paragraph in Section II.A and equation (1) in Section II.B in Bajari, Houghton, and Tadelis (2014).

our structural model. Section 5 describes our parametric estimation procedure. Section 6 reports estimates for the structural parameters as well as the contractor markups and the hold-up on the buyer. In Section 7 we compare the buyer surplus under counterfactual cost-plus contracts with that in the data. Section 8 concludes. Proofs, figures and tables are collected in the appendix.

# 2 The Model and Equilibrium

Our model of procurement auctions with incomplete contracts accommodates any heterogeneity on the contract level, which we suppress in the notation in order to simplify exposition.

**Buyer and Seller Decisions.** A buyer announces an initial specification (design) for a procurement contract $X \in \mathcal{X}$, where the support $\mathcal{X}$ is convex and compact in the real line $\mathbb{R}$. There are $N$ sellers (contractors) who are interested in competing for the contract. Once informed of $X$, each contractor $i$ draws a private cost $C_i$ for completing the contract with the design $X$, and quotes a price $P_i \in \mathbb{R}_+$. The distribution of $C_i$ conditional on the initial design is stochastically increasing in $X$. The buyer awards the contract to the seller who quotes the lowest price. For any initial design $X$, the costs of contractors are drawn independently from a continuous distribution $F_{C|X}$ with support $\mathcal{C} \subset \mathbb{R}_+$ which may also depend on $X$. In what follows we denote random variables by upper-case letters and their realized values by lower-case letters.

**Contract Incompleteness.** The contract is incomplete in that the buyer and the auction winner agree that the initial design may be replaced by a new design $X^* \in \mathcal{X}$ following the auction. The modification takes place *after* a winner is chosen to execute the contract with the initial design. The new design $X^*$ is unknown ex ante to the buyer and all contractors. Thus it is considered stochastic when the buyer announces the initial design and the contractors quote their prices. The private cost $C_i$ for implementing the contract conditional on the initial design $X$ is independent from $X^*$.

The buyer and the auction winner make a joint decision on whether to adopt the new design or not based on the following rule. Let $\pi : \mathcal{X} \rightarrow \mathbb{R}$ be the social surplus collected by the buyer. For example, suppose the procurement contract is about constructing a tollway. Then $\pi(X)$ is the present value of the stream of revenues to be collected by the buyer from that tollway. The incremental surplus under the new design is $\phi(X, X^*) \equiv \pi(X^*) - \pi(X)$. Let $a : \mathcal{X} \times \mathcal{X} \rightarrow \mathbb{R}$ be the incremental cost for changing the design from $X$ to $X^*$. These include costs for additional construction or logistic tasks. We maintain that such incremental costs are non-separable in $X$ and $X^*$ so that the marginal costs in general depend on $(X, X^*)$. Assume both $\pi$ and $a$ are bounded and continuously differentiable over their domains. Upon seeing the new design $X^*$ after the auction, the buyer and the auction winner agree to adopt it if and only if it yields a positive net incremental surplus relative to the initial design $X$. That is, $s(X, X^*) \equiv \phi(X, X^*) - a(X, X^*) > 0$ . Otherwise, the initial design $X$ is implemented. The final realized total costs for implementing a project with the new design is the sum of the auction payment based on the initial design plus the ex post, realized revision cost $a(x, x^*)$.

**Post-auction Negotiation.** If $X^*$ is adopted, the buyer and the winner negotiate transfers *in addition to* the payment determined in the auction. The contractor covers the incremental costs upfront as they arise in construction.[4] The incremental surplus is collected by the buyer. Both parties take these into account as they negotiate the transfers. We maintain that the post-auction transfer is determined via a Nash Bargaining solution as follows. Let $y(X, X^*) \in \mathbb{R}$ denote the negotiated transfer from the buyer to the auction winner; and let $\gamma \in (0, 1)$ be a constant parameter reflecting the bargaining power of the winner.[5] To simplify notation we suppress the input argument $(X, X^*)$ in $y, \phi, a, s$. The contract price from the auction enters the buyer's and the contractor's disagreement value in Nash Bargaining. After the negotiated transfers are made, the auction winner obtains a share of the net incremental surplus that is proportional to its bargaining power. That is,

$$y - a = \gamma s \Leftrightarrow y = \gamma\phi + (1 - \gamma)a \tag{1}$$

if a new design is adopted (See Appendix A for details in deriving the Nash Bargaining solution). On the other hand, there is no negotiated transfer, or $y = 0$, if no new design is adopted.

**Information Structure.** Prior to announcing the initial design, the buyer observes a signal $\widetilde{X} \in \widetilde{\mathcal{X}}$, where the support $\widetilde{\mathcal{X}}$ is a compact interval in $\mathbb{R}$, and in general is not the same as $\mathcal{X}$. The private signal $\widetilde{X}$ is correlated with $X^*$. In what follows, we let $F_{R'|R}$ and $f_{R'|R}$ denote the conditional distribution and the conditional density of $R'$ given $R$ respectively, and write $F_{R'|R=r}$ and $f_{R'|R=r}$ if there is need to be specific with the value conditioned on. The model elements $\gamma, \pi, a, F_{C|X}$ and $F_{X^*|\widetilde{X}}$ are common knowledge among the buyer and contractors. Assume $F_{X^*|\tilde{X}=\tilde{x}}$ is absolutely continuous and increasing over $\mathcal{X}$ for all $\tilde{x}$.

To reiterate, $\widetilde{X}$ and $C_i$ are private information for the buyer and the contractor $i$ respectively. At the beginning of the auction, the buyer announces $X$ to maximize its ex ante payoff, based on its signal $\widetilde{X}$ and taking into account the strategic incentives of contractors under incomplete contracts. A contractor is informed of $X$, draws its private cost $C_i$ from $F_{C|X}$ and quotes a price to maximize its ex ante profit, which also takes into account the negotiated transfer ex ante. We maintain that the number of contractors in an auction $N$ is common knowledge among the buyer and contractors in the bidding stage.

**The Equilibrium.** A contractor $i$'s pure strategy is a mapping from his information

[4]In some contracts, the buyer, instead of the contractor, covers the incremental costs for contract revision upfront. In this case, the negotiated transfer from the buyer to the contractor equals the contractor's share of the net incremental surplus $\phi - a$, which must be proportional to the latter's bargaining power in a Nash Bargaining solution. Our identification and estimation method is readily extended to these contracts.

[5]The bargaining power of the buyer against the auction winner depends on the contract and the seller's characteristics in general. In the model and identification sections, we focus on homogeneous auctions to simplify the exposition. However, our results is generalizable conditional on observed auction and contractors heterogeneity. In the application we estimate a model with heterogeneous auctions.

$(C_i, X)$ to a price he quotes; a buyer's pure strategy is a mapping from a signal $\widetilde{X}$ to an initial design $X$. Let $s_+ \equiv \max\{s, 0\}$ denote the *realized* net incremental surplus. In a symmetric pure-strategy Perfect Bayesian Equilibrium (*psPBE*), the buyer follows a pure strategy $\alpha^*$, and each contractor follows a pure strategy $\beta^*$ and holds a belief about the new design $\lambda^*(X^*|X) : \mathcal{X} \times \mathcal{X} \longrightarrow [0, 1]$ such that: (a) For all $(x, c_i)$,

$$\beta^*(c_i, x) = \arg\max_{b \in \mathbb{R}_+} \Pr\left( \min_{j \neq i} \beta^*(C_j, X) \geq b \middle| X = x \right) [b - c_i + \delta(x; \lambda^*)] \tag{2}$$

where $\delta(x; \lambda^*) \equiv E_{\lambda^*}[\gamma s_+(x, X^*)|X = x]$ is a contractor's expected share of the net incremental surplus according to his belief $\lambda^*(\cdot|x)$; (b) the belief $\lambda^*$ about $X^*$ given $X$ is consistent with $F_{X^*|\widetilde{X}}$ and $\alpha^*$ for all $x$ on the support of $\alpha^*(\tilde{X})$; and (c) $\alpha^*$ is the buyer's best response when all contractors follow $\beta^*$:

$$\alpha^*(\tilde{x}) = \arg\max_{x \in \mathcal{X}} \{\pi(x) - \varphi(x; \beta^*) + \mu(x, \tilde{x})\}, \tag{3}$$

where $\mu(x, \tilde{x}) \equiv E[(1 - \gamma)s_+(x, X^*)|\widetilde{X} = \tilde{x}]$ and $\varphi(x; \beta^*)$ is the buyer's expected payment in the auction with design $x$ when all contractors follow the strategy $\beta^*$. (We provide the closed form for $\varphi$ in Appendix B.) In addition, the contractor's belief off the equilibrium support of $\alpha^*(\tilde{X})$ is such that any deviation from the equilibrium path is not payoff-improving for the buyer.

As (3) shows, the buyer's choice of the initial design $X$ involves a key tradeoff. First, the initial design affects the expected payment in the auction, with a marginal effect determined by the dependence between the design $X$ and the contractor's cost. Furthermore, the initial design affects the buyer's expected share of the net incremental surplus due to contract revision. However, the sign of this marginal effect is ambiguous because it depends on the form of the incremental surplus function and the joint distribution of the initial and the new design.

The term $\mu(x, \tilde{x})$ in (3) is the buyer's ex ante share of the net incremental surplus due to contract revision, and is assumed continuous in the buyer's private signal. The expectation in $\mu(x, \tilde{x})$ integrates out $X^*$ with respect to its distribution conditional on $\widetilde{X} = \tilde{x}$, and does not depend on the contractor belief $\lambda^*$. We refer to $\delta(x; \lambda^*)$ is a contractor's *holdup* on the buyer under the initial design in equilibrium. In *symmetric monotone psPBE*, $\alpha^*$ is increasing over $\widetilde{\mathcal{X}}$, and $\beta^*$ is increasing over $\mathcal{C}$ for any $x \in \mathcal{X}$. The consistency of contractor beliefs in (b) means $F_{X^*|X=x} = F_{X^*|\widetilde{X}=\alpha^{*-1}(x)}$ for all $x$ on the equilibrium support of $\alpha^*(\widetilde{X})$.

We provide a heuristic overview of our argument for the existence of symmetric monotone psPBE, which takes several steps and is presented formally in Appendix B. First, for any given initial design $X$ and belief $\lambda$, a contractor's optimization problem is similar to that in a standard lowest-price procurement auction, except that its private cost is adjusted downward into $C_i - \delta(x; \lambda)$ because of its holdup on the buyer. Using auction theory, we characterize the bidder strategy and the expected auction price in a bidding equilibrium as functions of $X$ and $\lambda$. Next, note the first-order condition that characterizes the buyer's rationality takes the form of an ordinary differential equation (ODE). We argue that there exists a strictly monotone solution to this ODE under appropriate conditions based on the Picard's Existence Theorem. These conditions include the Lipschitz continuity of the functional form of ODE in the initial design, and the stochastic increasingness of $F_{C|X}$ in the initial design. Finally we characterize contractor beliefs outside the equilibrium support of the initial contract design. Such beliefs are ultra-pessimistic in that they assign no probability mass to new designs with positive net surplus. We show that such beliefs rationalize the buyer's choice of initial designs on the equilibrium path.

**Further Remarks.** In this model, the buyer has a noisy signal $\widetilde{X}$ about the new design $X^*$ before the auction, whereas the contractors have no private signals and update their beliefs about the new design given the buyer's choice of the initial design $X$. Such a specification of information structure is largely motivated by practical considerations. In most cases, the gross social surplus from public projects accrues directly to the buyer, who may well have some rough idea about which contract designs result in higher social surplus. This is in part reflected in the buyer's private signal about $X^*$. It is also worth noting that there is no explicit information asymmetry between the buyer and the contractors in equilibrium. This is because in a psPBE the contractor's belief about $X^*$ is consistent via Bayesian updating conditional on any $X$ on the equilibrium support of the initial design. Thus in equilibrium the contractors could correctly recover the buyer's signal $\widetilde{X}$ by inverting $\alpha^*$ on the equilibrium support of $X$. With $F_{X^*|\widetilde{X}}$ assumed common knowledge for the buyer and the contractors, this means a contractor is no less informed about the feasible new design $X^*$ relative to the buyer on the equilibrium path.

Our model simplifies several aspects in the negotiation that follows the procurement auction. First, the Nash Bargaining solution used in the negotiation posits the net incremental surplus is known to both the buyer and the auction winner. Hence our model consists of an incomplete information aspect (during the initial low-price procurement auction) and a complete information aspect (in the negotiation after the auction). This is a limitation in our approach of modeling, and it leads to a loss of generality in the information structure. An alternative setup of bargaining with incomplete information would be more robust. Nevertheless, as Myerson (1984) noted, the generalization of the Nash Bargaining solution with private information is complicated. The identification of the model elements in that case remains an open question.[6]

We introduce this working assumption above as a first-order approximation of the actual information structure, in order to keep the identification and estimation of the model tractable. We believe that the complete-information Nash Bargaining solution provides a reasonable approximation of the buyer-seller negotiation in our specific application because the new design $X^*$ and the negotiation take place after the auction. At that point,

[6] In a related but different environment, Elyakime, Laffont, Loisel, and Vuong (1997) takes a similar approach and uses complete-information Nash bargaining as an approximation of the acutal information structure for the sake of model tractability. Larsen (2014) estimated a bargaining game with two-sided incomplete information that follows ascending auctions, using insights from the implications of a Bayesian Nash equilibrium. The equilibrium characterization under the auction format he considers is qualitatively different; and his approach does not apply in our setting of lowest-price procurement auctions given the data available.

both the auction winner and the buyer may well be informed about the actual costs of contract revision through implementation and monitoring. In contrast, a contractor's initial cost $C_i$ is drawn before the auction and hence remains the contractor's private information in the bidding stage. Moreover, in a symmetric monotone psPBE, the contractors can invert the buyer's strategy $\alpha^*$ to back out $\tilde{x}$ from the initial design $x$ in a symmetric monotone psPBE. Likewise the buyer could invert the auction winner's bidding strategy in equilibrium and recover the contractor's private costs from the price it quotes while negotiating after the auction. These equilibrium implications provide further justification for our use of the complete-information Nash Bargaining solution in the negotiation.

Another working assumption in our model is that the incremental costs for revising contracts depend on the features of the project (such as the job type) but not on the identity of the auction winner. Without this simplification, there would be an additional source of ex ante uncertainty regarding contractor costs, thus making the equilibrium characterization and the identification of model elements intractable. We believe this simplifying assumption is a reasonable first-order approximation of the relation between the contract features and the negotiated outcome in the data. Descriptive analyses of our data suggest that the variation in the negotiated transfers in the data are mostly explained by the observed heterogeneity on the contract level, rather than individual characteristics of the auction winner. Furthermore, despite the simplification, the negotiated transfer depends on contract heterogeneity through the net surplus due to contract revision and through the contractor's bargaining power in negotiation.

In our model, the specification of a contract is summarized by a scalar variable that could be interpreted as the buyer's engineer estimates for the total cost. This simplifies the practice of specifying contracts in terms of itemized quantities. Such a simplification allows us to construct a structural model with two desirable features. On the one hand, the dimension of unknown parameters is low enough to warrant the robust identification of model elements; on the other hand the model is rich enough to capture the strategic interaction between the buyer and contractors given the uncertainty about the new design and given the order of sequential actions. An alternative model with multi-dimensional contracts is more general and potentially closer to the actual practice, but would increase the dimension of unknown parameters (which include the joint distribution of item-specific costs). The equilibrium characterization of that multi-dimensional model would be qualitatively different and more complicated; and its robust identification remains a challenging, open question.

# 3 Identification

The data report the auction payment $V$, the initial contract designs $X$, as well as the adopted new designs $X^*$ and negotiated transfers $Y$ if the new design is adopted. We explain how to use the joint distribution of these variables to recover the model parameters $\pi, a, \gamma, F_{C|X}$ and $F_{X^*,\tilde{X}}$. To do so, we use the implications of buyer and contractor rationality in equilibrium as well as the Nash Bargaining solution. Our identification method applies conditional on contract heterogeneity observed in data as well as the number of auction participants $N$. While presenting our method in this section, we suppress both variables in the notation for simplicity.

- Buyer and contractor strategies

The model is identified up to a monotone transformation of the buyer's signal $\widetilde{X}$ at best. (See the online appendix of this paper for a formal statement and proof.) Thus without loss of generality we normalize the distribution of $\widetilde{X}$ to a standard uniform over $[0,1]$.[7] Let $D = 1$ if the new design $X^*$ is adopted with a negotiated transfer, and 0 otherwise. This dummy variable is reported in the data.

The buyer's strategy $\alpha^*(\cdot)$ is identified because its monotonicity implies that $\alpha^*(\tau) = x_\tau$ for all $\tau \in [0,1]$, where $x_\tau$ is the $\tau$-th quantile of the initial designs reported in the data. Besides, the monotonicity of $\alpha^*$ implies

$$F_{X^*|s>0,\widetilde{X}=\tau}(x^*) \equiv \Pr\{X^* \leq x^*|\widetilde{X} = \tau, s > 0\} = \Pr\{X^* \leq x^*|X = x_\tau, D = 1\}. \quad (4)$$

for all $x^*$ and $\tau \in (0,1)$. Hence the distribution of the new design conditional on the contract revision and the buyer's signal is identified.

- Adjusted costs for contractors

Contractors bid strategically in the procurement auction, knowing that additional gains are possible due to contract revision. Thus we can use a standard argument in Guerre, Perrigne, and Vuong (2000) to recover the (endogenous) adjusted costs for contractors in the auction based on the initial design $X$. Such costs include a downward adjustment from actual costs $C_i$, which take into account the contractors' holdup on the buyer. That is, the adjusted cost for a contractor $i$ under an initial design $x$ is $\widetilde{C}_i \equiv C_i - \delta(x;\lambda^*)$, where $\delta(x;\lambda^*)$ is the ex ante share of net surplus due to contract revision in equilibrium.

To do so, first note that for a given pair of the initial design $x$ and equilibrium belief $\lambda^*$, the contractor's hold-up on the buyer $\delta(x;\lambda^*)$ is constant. Thus we can define a *hypothetical* low-price procurement auction without incompleteness, where contractors' costs are i.i.d. draws from the distribution of adjusted costs $\widetilde{C}_i$ given $x$ and $\lambda^*$. Let $\tilde{\beta}(\cdot, x)$ denote the contractor strategy in symmetric monotone psPBE in such a procurement auction with no incompleteness. That is,

$$\tilde{\beta}(\widetilde{c}_i, x) = \arg\max_{b\in\mathbb{R}_+} \Pr\left\{ \min_{j\neq i} \tilde{\beta}(\widetilde{c}_j, X) \geq b \,\middle|\, X = x \right\} (b - \tilde{c}_i) \text{ for all } x. \quad (5)$$

The solution to (5) is related to the equilibrium strategy $\beta^*$ in the original game in (2) through the equality $\tilde{\beta}(\tilde{c}_i, x) = \beta^*(\tilde{c}_i + \delta(x;\lambda^*), x)$ for all $x$. Therefore we can use a

[7]Tirole (2009) considered a model of incomplete contracts where both parties exert cognitive effort to learn about the appropriate design and how to draft the contract accordingly. The pre-contractual decision is to choose the cost/effort level of information acquisition. The identification of the information structure, including its scale, is possible in that case if the choices of information acquisition costs and the signal for appropriate design are reported in data.

standard argument from Guerre, Perrigne, and Vuong (2000) to invert $\tilde{\beta}$ and recover the contractors' adjusted costs $\tilde{c}_i$ from the distribution of quoted prices in the auction. (See Lemma C1 in Appendix C.)

- Social surplus

We recover the social surplus $\pi$ using the buyer's rationality in equilibrium, and the link between negotiated transfers and $\gamma, a, \pi$ in Nash Bargaining. This approach is robust to the contractor's belief off the equilibrium support of $\alpha^*(\widetilde{\mathcal{X}})$. We maintain that for all $\tilde{x} \in \widetilde{\mathcal{X}}$, the buyer's solution to (3) admits a unique solution in the interior of $\mathcal{X}$.[8]

In a symmetric monotone psPBE, a buyer with private signal $\tilde{x}$ chooses an initial design $x$ to maximize ex ante payoff $\pi(x) + \mu(x, \tilde{x}) - \varphi(x; \beta^*)$, where $\varphi(x; \beta^*)$ is the buyer's expected payment under the design $x$ when contractors follow the strategy $\beta^*$. The first-order condition for an interior solution is:

$$\varphi'(\alpha^*(\tilde{x}); \beta^*) = \pi'(\alpha^*(\tilde{x})) + \frac{\partial}{\partial x} \left[ \int_{\{s(x,t)>0\}} (1-\gamma) s(x,t) dF_{X^*|\widetilde{X}=\tilde{x}}(t) \right]_{x=\alpha^*(\tilde{x})}. \tag{6}$$

That is, the optimal choice of initial designs must strike a balance between a buyer's marginal cost in expected auction payment and its marginal benefit in social surplus under the initial design plus its ex ante share of net surplus from revision.

Recall that the negotiated transfer in Nash Bargaining is $y = \gamma\phi + (1-\gamma)a$. Substituting this equality into the first-order condition (6) and applying the Leibniz rule, we show that the marginal effect of initial designs on the social surplus is:

$$\pi'(x) = [1 - p^*(x)]^{-1} \left( \varphi'(x; \beta^*) + \int_{\{s(x,t)>0\}} y_1(x,t) dF_{X^*|X=x}(t) \right) \tag{7}$$

for all $x$ on the equilibrium support of $X$, where $p^*(x) \equiv \Pr\{D = 1 | X = x\}$ is the probability that the contract is revised to the new design, and $y_1$ denotes the partial derivative of $y$ with respect to its first argument. (See Appendix C for a formal statement of conditions and a proof of this result.) The right-hand side of (7) consists of identifiable quantities only. Both $y(x,t)$ and $\varphi(x; \beta^*)$ are directly identified in the data for all $x$ on the equilibrium support of $X$ and new design $t$ such that $s(x,t) > 0$; and the distribution of $X^*$ conditional on $X = x$ is also identified over the set of new designs $t$ with $s(x,t) > 0$. Thus $\pi'$ is identified over the equilibrium support of $X$.

At best the social surplus $\pi$ is identified up to location normalization. This is because the equilibrium strategies and the negotiated transfers only depend on the the derivative $\pi'(\cdot)$ and the difference in social surplus between the initial and new designs. Without loss of generality, set $\pi(\underline{x}) = \pi_0$ for some constant $\pi_0$, with $\underline{x}$ being the infimum of the equilibrium support of $X$. Then we can recover the surplus function as $\pi(x) = \pi_0 + \int_{\underline{x}}^{x} \pi'(z) dz$ for all $x$ on the equilibrium support.

[8]If the solution to (3) is on the boundary of $\mathcal{X}$ for some $\tilde{x} \in \widetilde{\mathcal{X}}$, then our results hold with $\mathcal{X}_e$ defined as $\{x \in \mathcal{X}_I : x = \alpha^*(\tilde{x}) \text{ for some } \tilde{x} \in [0,1]\}$ where $\mathcal{X}_I$ is the interior of $\mathcal{X}$.

- Bargaining power and costs for contract revision

Recall that the negotiated transfer $y$ is related to the bargaining power $\gamma$ and the costs for contract revision $a$ as follows

$$y(x, x^*) = \gamma[\pi(x^*) - \pi(x)] + (1 - \gamma)a(x, x^*). \tag{8}$$

With the (difference in) social surplus already identified, we use the joint variation in $y, x, x^*$ to back out the remaining parameters $\gamma$ and $a$. To do so, we maintain the following condition on revision costs over the equilibrium support of $X$.

**(A1)** *There exist $x, \xi, x^*, \xi^* \in \mathcal{X}$ such that $s(x, x^*) > 0$, $s(\xi, \xi^*) > 0$, $a(x, x^*) = a(\xi, \xi^*)$ and $\pi(x^*) - \pi(x) \neq \pi(\xi^*) - \pi(\xi)$.*

Under this condition there exist two pairs of initial and new designs that lead to the same costs for revisions but different incremental surplus. To locate such pairs, it is not necessary to know the functional forms of $a$ and $\pi$; shape restrictions on these functions are sufficient. For example, suppose $a$ is a $k$-th order polynomial of the difference $x^* - x$ while $\pi$ is a $k'$-th order polynomial in $x$ with $k, k' \geq 2$. Then (A1) holds with any two pairs on the equilibrium support of designs such that $x^* - x = \xi^* - \xi$ and $x \neq \xi$.

For $(x, x^*)$ and $(z, z^*)$ that satisfy (A1), $y(x, x^*) - y(\xi, \xi^*) = \gamma[\pi(x^*) - \pi(x) - (\pi(\xi^*) - \pi(\xi))]$, where the differences in social surplus on the right-hand side already identified in the previous step. Thus the bargaining power parameter $\gamma$ is identified. With $\gamma, \pi$ recovered, the revision cost $a$ is also identified from (8) for all $x$ and $x^*$ on the equilibrium support of $X$ with $s(x, x^*) > 0$.

- Holdup on the buyer

To quantify the contractor's holdup on the buyer and the distribution of costs for contractors, we need to extend the identification of $\pi$ over the set of designs

$$\mathcal{X}_s \equiv \mathcal{X}_e \cup \{x^* : \exists x \in \mathcal{X}_e \text{ s.t. } s(x, x^*) > 0\},$$

where $\mathcal{X}_e \equiv \{x : \alpha^*(\tilde{x}) = x \text{ for some } \tilde{x} \in [0, 1]\}$ denotes the equilibrium support of the initial design $\alpha^*(\widetilde{\mathcal{X}})$. We also need to recover the revision costs $a$ over the set of pairs $\{(x, x^*) : x \in \mathcal{X}_e \text{ and } s(x, x^*) > 0\}$. To do so, we use the following condition:

**(A2)** There exists a real-valued, differentiable function $\tilde{a}$ such that $a(x, x^*) = \tilde{a}(x^* - x)$ for all $x, x^* \in \mathcal{X}$.

Under this condition, $a_1(x, x^*) + a_2(x, x^*) = 0$, where $a_j$ is the partial derivative of $a$ with respect to its $j$-th argument. This provides a useful link between the marginal effect of designs on observed transfers and that on the social surplus. That is,

$$\tfrac{\partial}{\partial t} y(t, x^*)|_{t=x} + \tfrac{\partial}{\partial t} y(x, t)|_{t=x^*} = \gamma\pi'(x^*) - \gamma\pi'(x)$$

for all $(x, x^*)$ such that $x \in \mathcal{X}_e$ and $s(x, x^*) > 0$. This equality allows us to recover the marginal effect $\pi'(x^*)$ outside the equilibrium support of initial design (i.e., at $x^* \in$

$\mathcal{X}_s \backslash \mathcal{X}_e$). Thus we can recover the cost of contract revisions and the contractor's holdup on the buyer, using this and the knowledge of other elements identified earlier, such as the equilibrium belief and bargaining power. Furthermore, with knowledge of the contractor holdup we can recover the distribution of actual costs from that of the adjusted costs identified earlier. (See the corollary in Appendix C for details.)

- Alternative conditions for identifying revision costs

There are alternative restrictions on $a$ and $\pi$ that are sufficient for identifying the bargaining power. For example, suppose that for some $x_0$ and $x_0^*$ the level of the adjustment cost is known: $a(x_0, x_0^*) = a_0$ where $s(x_0, x_0^*) > 0$. Let $y_0$ and $\phi_0$ denote the observed transfer $y(x_0, x_0^*)$ and incremental surplus $\phi(x_0, x_0^*)$ respectively, where $\phi(x, x') \equiv \pi(x') - \pi(x)$. Then knowledge of $(x_0, x_0^*, a_0)$ allows us to recover $\gamma = (y_0 - a_0)/(\phi_0 - a_0)$.

Another alternative condition is when the incremental cost is homogenous of degree one while the incremental surplus has non-constant (diminishing or increasing) returns to scale. In this case, consider any two pairs with positive net incremental surplus $(x_1, x_1^*)$ and $(x_2, x_2^*) \equiv (tx_1, tx_1^*)$ with a known constant $t > 0$. Let $y_k, \phi_k, a_k$ be shorthands for the functions $y, \phi, a$ evaluated at $(x_k, x_k^*)$ for $k = 1, 2$. Because the incremental cost $a$ is homogeneous of degree one, we have $a_2 = ta_1$ and

$$(y_2 - \gamma\phi_2)/(y_1 - \gamma\phi_1) = t,$$

which implies $\gamma$ is identified as $(y_2 - ty_1)/(\phi_2 - t\phi_1)$. (The non-constant returns to scale in the incremental surplus $\phi$ ensures the denominator is non-zero.)

Finally, if the model assumes a parametric form of $\pi$ and $a$, then we do not need the exclusion and shape restrictions in (A1)-(A2) to point identify the bargaining power and recover $\pi$ and $a$ off the equilibrium support.

- Discussion: exogenous participation

So far we have maintained that the number of bidders $N$ is known to the contractors as well as the buyer during the auctions. Athey and Haile (2007) argued (in Section 6.3.3.) that in some procurement auctions the contractors may in fact know which of their competitors have the capability to compete for a given contract or which firms have been invited to bid. Bajari, Houghton, and Tadelis (2014) maintained this assumption in their analysis of the highway procurement auctions by CalTrans.

In other contexts, the actual number of auction participants $N$ is not public information to the parties involved in the auction. Nevertheless our identification strategy remains valid in such cases as long as the variation in $N$ is exogenous. (Athey and Haile (2007) provides an example of how such exogenous variation arises in a model where bidders' entry decisions are related to costly signal acquisition.) With the actual distribution of $N$ being common knowledge among the buyer and the contractors, the existence of symmetric monotone psPBE $\{\alpha^*, \beta^*\}$ follows from an argument similar to the proof in Appendix B. The only necessary change is that the ex ante return for the contractors and the ex ante payment by the buyer need to integrate out the number of bidders using the commonly known distribution of $N$.

As for identification of the model when $N$ is not known to the bidders, the results on identifying the distribution of the new design conditional on contract revision and the buyer's signal is built on the monotonicity of the buyer's strategy. Thus the results remain valid under any symmetric monotone psPBE in a model with bidders' uncertainty about participation. Also, the result about recovering the adjusted costs of contractors holds when the contractors and the buyer are uncertain about the participation, provided the data report the prices quoted by all contractors in an auction. In this case, Athey and Haile (2007) showed the markup in the inverse bidding strategy needs to incorporate the uncertainty about participation. This is done by integrating out $N$ using the actual distribution of the number of bidders, which is common knowledge among all parties and is directly recoverable from the data (see the equations (6.23) and (6.24) in Athey and Haile (2007)).[9] The identification of $\pi, a$ and $\gamma$ then follows from an argument similar to that presented above.

It remains an open question how to identify a model where decisions to participate in auctions are selective in the sense that the distribution of costs for active participants differs from the unconditional cost distribution in the population (e.g., if entry is based on a preliminary signal that is correlated with private costs to be drawn in the bidding stage). We leave this topic for future research.

# 4 CalTrans Auctions: Background and Data

The California Department of Transportation (CalTrans) is a government department in the State of California that is responsible for the planning, construction, and maintenance of public transportation facilities such as highway, bridge, and rails. It awards highway construction projects to contractors through lowest-price procurement auctions. Before each auction, CalTrans announces the initial design of the contract in the form of an engineer estimate $X$ for total project costs. These engineer estimates are reported as the dot product of the quantity for each category of inputs and their per-unit prices. Once informed of engineer estimates, the contractors draw private costs for completing the project and quote their prices. The contractor who quotes the lowest price is awarded the contract. In most cases, CalTrans and the auction winner end up with an agreement to adopt a new specification $X^*$ after the auction, and additional transfers in the form of adjustment, deduction or payment for extra work are made through negotiation.[10]

Some institutional facts indicate that CalTrans chooses an initial specification to maximize its ex ante payoff. For example, CalTrans' Highway Design Manual (2015) states that it is necessary to take into account uncertainty about the costs and welfare in the

[9]Furthermore, Song (2006) showed that identification of inverse bidding strategy in first-price procurement auctions, where neither the bidders nor the researcher observe the set of actual participants, is also possible even if only the winning bid $V$ and the second lowest bid are reported in the data.

[10]Unlike in Bajari, Houghton, and Tadelis (2014), we measure the changes of input quantities through the difference between the initial and the actual specification.

design (specification) to be implemented. In practice, engineers who prepare the estimates do anticipate project uncertainty, such as changes proposed by the winning contractor (Project Risk Management Handbook: A Scalable Approach, CalTrans, 2012) and stochastic quotes from contractors (CalTrans Estimating Practices).

- Summary statistics

We use the same source of data as Bajari, Houghton, and Tadelis (2014). The data include 5,908 bids submitted by contractors in 1,306 procurement auctions by CalTrans between 1995 and 2000. Over 90% of the contracts in the data receive 2 to 9 quotes from contractors. For each contract, the data report the initial specification (engineer estimates), the actual specification adopted (calculated using the Blue Book prices published in the Contract Cost Data Book (CCDB) and the actual quantities of items used for the project), and the negotiated transfer after the auction.[11] The data record zero transfer if no new specification is adopted after the auction. The data also report the bids submitted by all contractors in each auction, and cost-related characteristics for the contractors. These include the distance between the contractor's location and the work site for the project ($dis$), a dummy variable that equals one if the contractor is a "fringe" competitor ($fri$), and the contractor's utilization rate defined as the ratio of its backlog over its capacity ($uti$).[12]

In addition, we classify the contracts into two types based on the nature of the work according to the project description. Type-one contracts ($job = 1$) involve major construction or rebuilding (e.g., replace bridge or widen highway, etc). Type-zero contracts ($job = 0$) only require relatively minor or decorative tasks (e.g., realign curves, install traffic signals or other accessories, etc).

[Insert Table 1 here]

Table 1 presents the summary statistics for our data. The average transfer due to contract revisions after the auction is about $258,000 per contract, which is about 9.67% of the average size of the final specification implemented after the auction. Among all contracts, 94.10% (1,229 cases) reported non-zero transfers negotiated after the adoption of a new design following the auction while 5.90% (77 cases) report zero transfer. Type-one (major construction) jobs account for 37.9% of all contracts with an average transfer of $280,775; Type-zero contracts have an average transfer of $243,657. For each auction, we use the proportion of fringe contractors competing in the auction ($afri$) and the average utilization rate and distance across contractors ($auti$ and $adis$) as contract-level characteristics. For each contractor in an auction, we also record the minimum distance to the job site ($rdis$) and the minimum utilization rate among its competitors ($ruti$). Table 1 also reports the summary statistics of these contract- and contractor-level variables.

[11] Following Bajari, Houghton, and Tadelis (2014), we calculate the total negotiated transfers after the auctions by adding up transfers under three categories in the data: "adjustment", "deduction" and "extra work".

[12] A fringe competitor is defined as a contractor that has won less than one percent of the value of contracts awarded in the data.

- Non-structural evidence and motivating facts

We start with some descriptive analyses of how the transfers negotiated after the auction are related to contract specification and characteristics. As expected, the size of contract revision, $(x^* - x)$, which is the difference between the initial specification $x$ and the final specification $x^*$, has a substantial impact on the transfer negotiated after the auction. Figure 1 plots the empirical distributions of transfers conditional on small-, medium- and large-sized revisions respectively, where the size categories are intervals defined by the lower, middle and upper terciles of $x^* - x$. For each pair of transfer distributions in Figure 1, the Kolmogorov-Smirnov test rejects the null of identical distribution at the 1% significance level.

[Insert Figure 1 here]

The data reveal several distinctive patterns that motivate our structural model in Section 2. First, there is substantial variation in negotiated transfers even after controlling for the size of contract revision.

[Insert Figure 2 here]

Figure 2 plots the empirical distribution of transfers conditioning on contract characteristics such as job type, the proportion of fringe competitors ($afri$), and the average utilization rate among competitors ($auti$). The proportion of fringe competitors and the average utilization rate are discretized into intervals defined by their lower, middle and upper terciles. For each pair plotted in Figure 2, we report the $p$-value from a Kolmogorov-Smirnov test in the caption. In six cases, the test rejects the null of identical distribution at 10% significance level. These suggest that the negotiated transfers vary significantly across contracts with different characteristics (such as the job type $afri$ and $auit$), even after controlling for the size of contract revision.

[Insert Table 2 here]

Second, there is strong evidence that the size of contract revision affects the transfers nonlinearly and through its interaction with contract characteristics. This is evident from Figure 2, which shows the impact of job type and average utilization on transfers differ across various sizes of contract revisions. For instance, the distribution of transfers for major projects first-order stochastically dominates the distribution for minor jobs for small-size revisions, while such dominance does not appear for large-size revisions.

Table 2 further quantifies the effect of various factors on the transfers by regressing the latter on the contract characteristics and the size of contract revision $(x^* - x)$. The quadratic term of the size of contract revision and its interaction with contract characteristics are significant across all three model specifications. The type of work ($job$) has a significant positive effect on the observed transfer, and the effect becomes less pronounced as the size of the revision increases. The number of bidders, the proportion of fringe bidders in an auction, and the average utilization rate of participants ($nbid, afri, auti$) all affect the transfer through their interaction with the initial and the new specification. These results indicate that the size of specification change and contract characteristics affect the transfers significantly and nonlinearly. In addition, the impacts of the characteristics on the transfer depend on the size of specification change.

[Insert Table 3 here]

Third, there is little correlation between the contract characteristics and the size of specification change *per se*. In Table 3 we regress the size of specification changes on the contract characteristics *nbid*, *job*, $afri$, *auti* and *adis*. We find that these characteristics are jointly insignificant at the 5% level across all three specifications. Combining the results from Table 2 and 3, we conclude that contract characteristics affect the negotiated transfers through some channel *other than* a direct impact on the size of contract revision. This further supports the setup of our model in Section 2, where contract characteristics affect the transfers through the contractor's bargaining power $\gamma$, but not via any direct impact on the size of specification change.

The model in Section 2 is motivated in part by these stylized facts in the CalTrans highway procurement data. It allows us to disentangle the roles of various factors determining the contract revision, the negotiated transfers as well as the auction payment.

# 5 Econometric Implementation

We estimate a parametric model which accommodates the heterogeneity in the contracts and sellers in the data. Our results in Section 3 are limited to the benchmark case with no unobservable structural errors that are known to the buyer and sellers. Nonparametric identification of an econometric model which incorporates these structural errors is an open question. In this section we adopt a parametric approach by exploiting the restrictions on the functional forms of the model elements in order to recover the structural parameters. Identification of the parametric model is discussed in the online appendix.

- Specification of surplus, revision costs and bargaining power

For simplicity we drop the indices of a contract $j$ and a contractor $i$ in the notation. The bargaining power of a contractor in the negotiation after the auction is:

$$\gamma(w;\varrho) = \exp(w'\varrho)/[1+\exp(w'\varrho)], \tag{9}$$

where $w \equiv [job, nbid, afri, auti, adis]$ is the vector of contract characteristics, with *nbid* being the number of contractors who participate in the auction. The vector $w$ consists of contract/auction characteristics known to CalTrans and contractors at the time of their decisions.

The specification in (9) is motivated by and consistent with the reduced-form evidence in Section 4 that the contract/auction characteristics (such as job type and the intensity of competition) affect transfers through some channel other than the size of contract revisions. Under this specification, the bargaining power as perceived by CalTrans and contractors in the auctions does not depend on ex post individual characteristics of the auction winner, whose identity is unknown before the auction is concluded.[13]

The transfer $y$ due to Nash Bargaining is

$$Y = D\left[\gamma\phi + (1-\gamma)a + \varepsilon\right], \tag{10}$$

where

$$D = 1\{\phi - a + \eta > 0\} \tag{11}$$

is the dummy for contract revision, and $(\varepsilon, \eta)$ are bivariate normal with zero mean, standard deviation $(1, \sigma)$, and a correlation coefficient $\rho$. The change in social surplus due to contract revision is $\pi(x^*, job; \theta) - \pi(x, job; \theta)$, where

$$\pi(x, job; \theta) = \theta_1 x + \theta_2(x \times job) + \theta_3 x^2; \tag{12}$$

and the costs for revising the contract is

$$a(x, x^*, job; \theta) \equiv \theta_0 + \theta_4(x^* - x) + \theta_5(x^* - x) \times job + \theta_6(x^* - x)^2, \tag{13}$$

where $\theta_0$ is the fixed cost. The errors $(\varepsilon, \eta)$ capture measurement noises, or any idiosyncratic factors that affect the negotiated transfer or the joint decision to revise the contract, but that are orthogonal to the initial and new specification (e.g., additional compensation for delay in payment). Under (12) the social surplus depends on the job type and specification, but not the characteristics of the auction winner. The costs for revising the contract $a(\cdot)$ include the adaptation costs in Bajari, Houghton, and Tadelis (2014).

The specifications in (12) and (13) are supported by the reduced form patterns in Table 2 (as discussed in Section 4). In addition to the sources of variation we use for nonparametric identification in Section 3, parametric restrictions in this section also provide further identifying power to recover model elements. We discuss the identification of the parametric model in the online appendix.

- Two-step estimation of structural parameters

We estimate $(\theta, \varrho, \rho, \sigma)$ via two steps. In the first step, we estimate some components in $\theta$ by applying the probit procedure to (13) with a vector of explanatory variables $[x^* - x$, $(x^* - x) \times job$, $x^{*2} - x^2$, $(x^* - x)^2]$. This returns estimates for $(\hat{\theta}_1 - \hat{\theta}_4, \hat{\theta}_2 - \hat{\theta}_5, \hat{\theta}_3, \hat{\theta}_6) \equiv \hat{\theta}_{-1,2}$. The second step is to estimate the remaining parameters $\tau \equiv (\theta_1, \theta_2, \varrho, \rho, \sigma)$ using an extremum estimator:

$$\hat{\tau} = \arg\max_{\tau} \left\{\mathcal{L}_J(\tau) - \mathcal{M}_J(\tau)\right\}, \tag{14}$$

[13]We run a regression of negotiated transfers on the winners' characteristics (fringe, utilization and distance), the specification change $x^* - x$, and the contract characteristics $job$, $nbid$ , $afri$, $auti$ and $adis$. The results suggest that the auction winner's characteristics do not affect the transfer significantly across different model specifications.

where $\mathcal{L}_J$ is the log-likelihood for the transfers when there is contract revision, and $\mathcal{M}_J$ is based on a set of moments derived from the first-order condition due to CalTrans' optimization in equilibrium as shown in (6).[14] That is,

$$\mathcal{L}_J(\tau) \equiv J^{-1} \sum\nolimits_j \log \Phi\left( \frac{\hat{I}_{s,j} + \frac{\rho}{\sigma}\left(y_j - \hat{I}_{o,j}\right)}{\sqrt{1-\rho^2}} \right) + \log \Phi' \left( \frac{y_j - \hat{I}_{o,j}}{\sigma} \right) - \log \sigma - \log \Phi(\hat{I}_{s,j})$$

where $j$ is an index for contracts, $\Phi$ and $\Phi'$ denote standard normal cdf and pdf, and

$$\hat{I}_{o,j} \equiv \gamma_j \hat{\phi}_j + (1-\gamma_j)\hat{a}_j \text{ and } \hat{I}_{s,j} \equiv \hat{\phi}_j - \hat{a}_j$$

with $\gamma_j \equiv \gamma(w_j; \varrho)$, $\hat{\phi}_j \equiv \pi(x_j^*, w_j; \tau, \hat{\theta}_{-1,2}) - \pi(x_j, w_j; \tau, \hat{\theta}_{-1,2})$ and $\hat{a}_j \equiv a(x_j, x_j^*, w_j; \tau, \hat{\theta}_{-1,2})$; and

$$\mathcal{M}_J \equiv J^{-1} \sum\nolimits_j \left[\hat{\varphi}_1(x_j, w_j) - \hat{\pi}_1(x_j, job_j; \tau) - \hat{\mu}_1(x_j, \tilde{x}_j, job_j; \tau)\right]^2$$

where $\hat{\varphi}_1$, $\hat{\pi}_1$, $\hat{\mu}_1$ are estimates for the marginal effect of $x$ on $\varphi(x,w)$, $\pi(x, job)$ and $\mu(x, \tilde{x}, job)$ (which is defined as $E[(1-\gamma(w))s_+(x, X^*, job)|\tilde{X} = \tilde{x}, job]$); and $\tilde{x}_j \equiv r$ if $x_j$ is the $r$-th quantile of the initial specification in the sample. (Recall that $\tilde{X}$ is normalized to standard uniform.) To calculate $\hat{\varphi}_1$, first estimate a regression model

$$\varphi(x, w; \kappa) \equiv E(Y|x, w) = \tilde{w}'\kappa \tag{15}$$

where $\tilde{w} \equiv [1, x, x^2, afri, auti, adis, nbid, job, x \times afri, x \times auti, x \times adis, x \times nbid, x \times job]$. (Note this *ex ante* payment does not depend on $x^*$ which is unknown in the auction stage.) Then calculate $\hat{\varphi}_1$ by plugging in OLS estimates in (15).[15] To calculate $\hat{\mu}_1$, note that under our specification,

$$\mu_1(x, \tilde{x}, job; \tau) = E[(1-\gamma(w; \varrho)|job] \times E\left[ \Phi(s(X^*, x, job)) \frac{\partial s(X^*, x, job)}{\partial x} \middle| \tilde{x}, job \right], \tag{16}$$

where the first expectation is with respect to the identity of the auction winner, and the second with respect to $X^*$ conditional on $\tilde{x}$. For each trial value $\tau$, construct $\hat{\mu}_1(x_j, \tilde{x}_j, job_j; \tau)$ by plugging $\hat{\theta}_{-1,2}$ into $s$ and $\partial s / \partial x$ and taking the conditional sample average to estimate the first term in (16), and then estimating the second term using simulated observations of $x^*$ drawn from the estimated distribution of $X^*$ given $\widetilde{X} = \tilde{x}_j$.[16][17]

[14] An alternative to our estimation method would be to derive the first-order conditions of the log-likelihood (the score equations) for the negotiated transfers, and stack them with the first-order condition from the buyer's optimization problem in a GMM estimator. This approach is algebraically demanding because the score function of the log-likelihood of negotiated transfers is cumbersome (mostly due to our parametrization of the bargaining power).

[15] That is, $\hat{\varphi}_1(x, w)$ is $\hat{\kappa}_1 + 2\hat{\kappa}_2 x + \hat{\kappa}_8 afri + \hat{\kappa}_9 auti + \hat{\kappa}_{10} adis + \hat{\kappa}_{11} nbid + \hat{\kappa}_{12} job$ where $\hat{\kappa}_1, \hat{\kappa}_2, \hat{\kappa}_8, \hat{\kappa}_9, \hat{\kappa}_{10}, \hat{\kappa}_{11}, \hat{\kappa}_{12}$ are regression estimates for the coefficients in front of $x$, $x^2$ and the last five product terms of $\tilde{w}$ in (15).

[16] Recall that we normalize the distribution of $\tilde{X}$ to $U(0,1)$. The monotone strategy in equilibrium implies that $F_{X^*|\tilde{X}=\alpha^{*-1}(x)}$ equals $F_{X^*|X=x}$. The density of $X^*$ given $X$ in auctions with transfers is approximated well in data by a normal distribution. Thus we specify the relation between $X^*$ and $X$ as $(X^* - X)/X \sim N\left(\nu, \tilde{\sigma}^2\right)$ for auctions with transfers, and then use a maximum likelihood formula to estimate $\nu, \tilde{\sigma}$. The second expectation is then calculated for each $\tau$ using simulated draws of $x^*$ based on these estimates.

[17] In our specification $F_{X^*|X=x}$ does not depend on $w$. This simplification is in part motivated by

This two-step estimator is consistent for $(\theta, \varrho, \rho, \sigma)$. First, $\theta_{-1,2}$ is consistently estimated via probit in the first step by a standard argument. Furthermore, under our parametrization, the true $\tau$ is a unique maximizer of the probability limit of $\mathcal{L}_J$. (See the online appendix for details.) Next, note $\mathcal{M}_J$ is a sample analog for

$$E\left\{\left[\varphi_1(x,w) - \pi_1(x, job; \tau) - \mu_1(x, \tilde{x}, job; \tau)\right]^2\right\}, \tag{17}$$

where the *ex ante* payment $\varphi(x, w)$ depends on the contract characteristics $w$ and is directly identifiable in data. Buyers' optimization in equilibrium implies that the true parameter $\tau$ is the unique minimizer of (17) under the standard conditions in nonlinear least squares.[18] Hence the true parameter $\tau$ is a unique maximizer of the probability limit of $\mathcal{L}_J - \mathcal{M}_J$ in the second step.

Because the estimand in (14) is a smooth function of the sample analogs, the two-step extremum estimator is asymptotically normal under regularity conditions. (If we were to follow the asymptotic plug-in approach for inference, then the asymptotic variance of $\hat{\tau}$ would need to include a term that accounts for the first-stage estimation error from the probit procedure.) In practice, we use the bootstrap procedure to construct the standard error. To implement the maximization routine in the second step, we pick an initial value for $\tau$ by estimating a two-stage MLE that maximizes $\mathcal{L}_J$ alone (which produces a consistent estimator for $\tau$).

- Estimating contractor costs and holdup

Recall that $\delta(x; \lambda^*) = E_{\lambda^*}[\gamma s_+(x, X^*)|X = x]$ and $y - a = \gamma s$ whenever $s > 0$. Let $q(x, job)$ denote the probability that a contract is revised with a negotiated transfer ($D = 1$) conditional on information available to contractors in the auction. Let $f_{P|Z}(\cdot|z_{j,k})$ and $F_{P|Z}(\cdot|z_{j,k})$ denote the density and the distribution of bids conditional on bidder $k$'s characteristics in project $j$. In equilibrium,

$$c_{j,i} = p_{j,i} - \left(\sum_{k \neq i} \frac{f_{P|Z}(p_{j,i}|z_{j,k})}{1 - F_{P|Z}(p_{j,i}|z_{j,k})}\right)^{-1} + E\left[Y - a(x_j, X^*, job_j)|x_j, z_{j,i}, D_j = 1\right] q(x_j, job_j), \tag{18}$$

where $z_{j,i} \equiv [fri_{j,i}, uti_{j,i}, dis_{j,i}, job_j, ruti_{j,i}, rdis_{j,i}]$ and $ruti_{j,i}, rdis_{j,i}$ are minimum distance and utilization rate among the competitors competing against $i$. Let $\Delta_{j,i}$ denote the conditional expectation of $Y - a$ in (18). It conditions only on $x_j$ and $z_{j,i}$ because the contractor's ex ante share of the net surplus depends on contractor characteristics known prior to the auction.

We adopt a logit specification for the probability for contractual incompleteness $q$:

$$q(x_j, job_j; \vartheta) = \exp(v'\vartheta)/[1 + \exp(v'\vartheta)] \tag{19}$$

some pattern we find in a reduced-form analysis: Regressions of $x^*$ and $x^* - x$ respectively on $x$ and $w$ suggest that the vector of contract characteristics is jointly insignificant from a statistical perspective. These results are not included in this paper due to space constraints, but are available upon request.

[18] That is, for any $\tau' \neq \tau$, $\pi_1(x, job; \tau) + \mu_1(x, \tilde{x}, job; \tau) \neq \pi_1(x, job; \tau') + \mu_1(x, \tilde{x}, job; \tau')$ at least for some $\tilde{x}$, $w$ and $x = \alpha^*(\tilde{x})$.

where $v \equiv [1,\ x_j,\ job_j,\ x_j \times job_j,\ x_j^2]$. Note (19) differs from (11) in that the former specifies the *ex ante* probability of contract revisions, which does not condition on ex post information such as $X^*$. Denote the maximum likelihood estimator in this step by $\hat{\vartheta}$.

We model contractors' bidding strategies (normalized by engineer estimates) via the following regression:

$$\frac{p_{j,i}}{x_j} = g(z_{j,i};\nu) + e_{j,i}, \tag{20}$$

where $g(\cdot)$ is linear in $\nu$, and $e_{j,i}$ is independent from $z_{j,i}$. As in Bajari, Houghton, and Tadelis (2014), the specification conditions on the number of contractors in an auction (which is suppressed in notation), and allows for heterogeneity in the structural error via the contract size $x_j$.[19] We adopt a pooled-OLS to estimate the parameter in $g(.;\nu)$ conditional on the number of contracts. We then use this estimate, denote $\hat{\nu}$, to estimate the distribution of $e_{j,i}$. To see how, note that for a given number of contractors, $F_{P|Z}$ equals:

$$F_{P|Z}(p_{j,i}|z_{j,i}) = \Pr\{g(z_{j,i};\nu) + e_{j,i} \leq p_{j,i}/x_j\} = F_e(p_{j,i}/x_j - g(z_{j,i};\nu)),$$

where $F_e(\cdot)$ is the distribution of $e_{j,i}$ for contracts with the same number of bidders as contract $j$ (we suppress $n$ in $F_{P|Z}$, $F_e$ to simplify notation). The conditional density of bids is:

$$f_{P|Z}(p_{j,i}|z_{j,i}) \equiv \tfrac{\partial}{\partial p}F_e(p_{j,i}/x_j - g(z_{j,i};\nu)) = f_e(p_{j,i}/x_j - g(z_{j,i};\nu))/x_j.$$

We estimate $F_{P|Z}$ and $f_{P|Z}$ by plugging in $\hat{\nu}$, and the empirical distribution and kernel density of $\hat{e}_{j,i}$ to the right-hand sides. Let $\hat{F}_{P|Z}$ and $\hat{f}_{P|Z}$ denote these estimates.

The expectation of $Y - a$ is estimated as

$$\widehat{\Delta}_{j,i} = \widehat{E}\left(Y_j|x_j, z_{j,i}, d_j = 1\right) - S^{-1}\sum\nolimits_{s=1}^{S} a(x_j, x^*_{j,s}, job_j; \hat{\theta}), \tag{21}$$

where the first term (the conditional expected transfer) in $\widehat{\Delta}_{j,i}$ is estimated using a single-index specification and Ichimura's semiparametric least squares in (Ichimura (1993)); the second term in $\widehat{\Delta}_{j,i}$ is a simulation-based estimate for the ex ante revision costs $a$ conditional on contractual incompleteness, initial specification and contract characteristics. Specifically, $x^*_{j,s}$ are independent draws from the estimated density of $x^*$ given $x_j$ and $d_j = 1$ (which is estimated in the previous step, see footnote 16 for details).

We replace $f_{P|Z}$, $F_{P|Z}$, $\Delta_{j,i}$ and $q(x_j, job_j)$ by their estimates $\hat{f}_{P|Z}$, $\widehat{F}_{P|Z}$, $\widehat{\Delta}_{j,i}$ and $\hat{q}_j \equiv q(x_j, job_j; \hat{\vartheta})$ to obtain the estimate of costs for bidder $i$ in contract $j$:

$$\hat{c}_{j,i} = p_{j,i} - \left(\sum\nolimits_{k=1,k\neq i}^{n_j} \frac{\hat{f}_{P|Z}(p_{j,i}|z_{j,k})}{1 - \widehat{F}_{P|Z}(p_{j,i}|z_{j,k})}\right)^{-1} + \widehat{\Delta}_{j,i}\hat{q}_j. \tag{22}$$

[19] Note that the inverse bidding strategy for contractors is identified nonparametrically due to (18). Hence in principle the reduced-form linear specification in (20) is not necessary for estimating bidding strategies in large samples. We implement this regression form to estimate bidding strategies mostly due to data constraints.

A contractor's holdup on the buyer is estimated as $\widehat{\Delta}_{j,i}\hat{q}_j$. A contractor's markup is defined as the difference between its ex ante payoff, estimated by $p_{j,i} + \widehat{\Delta}_{j,i}\hat{q}_j$, and its initial costs in the auction $\hat{c}_{j,i}$. Using (22), we estimate the markup by $\hat{\varpi}_{j,i} \equiv \left(\sum_{k=1,k\neq i}^{n_j} f_{P|Z}(p_{j,i}|z_{j,k})/[1 - F_{P|Z}(p_{j,i}|z_{j,k})]\right)^{-1}$.

# 6 Results

- Determinants of auction payment

We start with a descriptive analysis of how the buyer's initial payment in the procurement auction is related to the contract characteristics. As explained in the next paragraph, we find that the initial contract design has a positive nonlinear effect on auction payment, especially via interaction with contract characteristics. This corroborates several key aspects in our model: that the contractor costs are stochastically increasing in the contract design, that the contractors adopt monotone strategies, and that the contract characteristics affect the auction payment through contractors' cost adjustment based on its bargaining power and ex ante holdup on the buyer.

[Insert Table 4 here]

Table 4 reports OLS estimates from regressing the auction payment on contract characteristics in (15). In each specification, the initial contract design (engineer estimates) has a significant positive marginal effect on the auction payment. There is also evidence in the third specifications that the effect diminishes as the engineer estimates increase. Another pattern consistent across all specifications is that the contract/auction-level characteristics, which include the job type, the proportion of fringe competitors ($afri$) as well as the average distance and utilization rate of competitors ($auti$) and ($adis$), are all significant via their interaction with engineer estimates. This is consistent with the notion that auction characteristics affect contractors' adjusted costs through the bargaining power of contractors and ex ante holdup on the buyer. Also, it is worth mentioning that a higher average utilization rate among the contractors tends to lower the auction payment. This indicates that there is an economy of scale in the costs for contractors that work on multiple contracts simultaneously.

- Estimates of contractor bargaining power

Our estimates indicate that the contractor tends to retain a substantial share of the ex ante net surplus due to contract revision, and that on average the bargaining power depends on the intensity of competition as well as the utilization rates of alternative contractors.

[Insert Table 5 here]

The upper panel in Table 5 reports the average bargaining power of the contractor in a negotiation following contract revisions, i.e., the mean of $\gamma(w_j; \hat{\varrho})$ across contracts, conditional on the job type ($job = 1$ for major work and 0 otherwise). In both cases, the contractor has significant bargaining power against CalTrans. For example, in each of the first two specifications, the contractor's bargaining power is significantly higher than 50%, regardless of the job type. In the third and most comprehensive specification the average bargaining power is 68.9% for major jobs and 79.3% for minor jobs.

The lower panel in the table reports the estimates for the mean marginal effect of contract and contractor characteristics on bargaining power. According to the first specification, on average the bargaining power for a contractor is about 6% higher if it had to defeat an additional competitor in the auction. This might be because a contractor's competitive advantage (such as cost or logistic efficiency) leads to more leverage in the negotiation with CalTrans. The average utilization rate is important in explaining the bargaining power in the third specification. Our estimates show that if the average utilization rate in an auction is increased by 10%, then the bargaining power of the contractor increases by 4.4%. This conforms with the intuition that a contractor has larger bargaining power against CalTrans when its competitors are on average more occupied or committed to other projects.

- Social surplus and revision costs

Our estimates indicate that there is an increasing return in the social surplus for highway construction, and that the net incremental surplus from revisions is nonlinear in the size of revisions.

[Insert Table 6 here]

Table 6 reports parameter estimates in the social surplus and incremental costs for contract revision. The coefficient for the squared engineer estimates is significantly positive, suggesting some economy of scale in social surplus as the contract size increases. We reject the null hypothesis that $s$ is linear in $(x^* - x)$ (coefficients for $x^2$ in $\pi$ and coefficients for $(x^* - x)^2$ in $a$ are jointly zero) with a $p$-value less than 0.001 in the most comprehensive specification. In addition, a Wald test for the joint significance of $x$ and $x^*$ in $s$ also yields a $p$-value less than 0.001.

To quantify the effect of job types on the negotiated transfer, we use the estimates in Table 6 to calculate the average difference between the truncated mean of transfer (conditional on contract revision) for major contracts ($job = 1$) and minor contracts respectively. This measures the ex ante marginal effect of the job type on the truncated means, after integrating out other contract characteristics and the initial and final specification. For the most general specification, this estimated difference is $\$52,000$ with a bootstrap standard error of $\$20,000$. The tests under the other two nested specifications also report statistical significance of $job$ at the 1% level.

[Insert Table 7 here]

Table 7 estimates a logit model for contract revision in (19). Our estimates illustrate that the engineer estimates are a significant determinant for contractual revision whereas job types are not. This conforms with the observation in Tirole (2009) that the holdup problem occurs under the incomplete contracts where the probability of incompleteness is determined by the endogenous choice of the buyer.

- Contractor markups

Table 8 reports the estimates for the ratio between markups and contractor costs $\hat{\varpi}_{j,i}/\hat{c}_{j,i}$. We find that the markups are substantial (with a sample average of 10.6%), and are slightly lower for contracts with substantial work ($job = 1$). Our estimates also reveal how markups depend on other features of the auctions and contractors.

[Insert Table 8 here]

The markup ratios are lower in auctions involving more competitors, which supports our assumption that contractors are aware of the number of competitors in auctions. Besides, the auction winners tend to bid with higher markups than the others. One possible interpretation is that the cost for the auction winner tends to be much lower than contractors, thus allowing it to quote competitive prices even with a high markup. This might also explain why our estimates for markup ratios are slightly higher for major competitors in the industry (non-fringe contractors).

About half of the contractors competing in the auctions are not committed to other projects simultaneously (zero utilization rates). The markup ratios for these contractors are on average lower than those with positive utilization rates. Our estimates suggest that the contractors with moderate or high utilization rates have greater markup ratios than those with lower utilization rates. On the other hand, the distance between the contractor's location and the job sites has no substantial effect on the estimated markups.

[Insert Table 9 here]

Ignoring the strategic adjustment of the contractors in the auctions would over-estimate the markups in their equilibrium bids. We now quantify the size of over-estimation in the markups if the effect of holdups from incomplete contracts are ignored, and bids are interpreted as generated from equilibrium in standard lowest-price procurement auctions. Table 9 reports estimates for markup ratios under these assumptions. The overall the average markup ratio is estimated to be 13.3%, which is 25.5% higher than the estimates in Table 8. We present the estimates at different quantiles in Table 9. The markups are over-estimated up to 26.4% (at the 25th quantiles of overall estimates) if contract incompleteness is not accounted for. The over-estimation of markups is prevalent across all contractor characteristics and the features of contracts.

- Holdups on CalTrans

We use the parameter estimates above to calculate the ratio between the contractor's holdup on CalTrans and the engineer estimates based on the initial contract design in each auction. The average holdup ratio is 20.4% across contracts with revisions in the data. Figure 3 illustrates histograms of these estimated ratios conditional on the type of contracts (major or minor) and contractor participation. The histograms show that these ratios tend to be higher for contracts involving major jobs. The average ratio for major projects is 21.6%; that for minor projects is 19.5%. The difference is statistically significant at 1% level based on a two-sample Kolmogorov-Smirnov test. The estimates also indicate that the ratios are higher for contracts involving more active bidders.

[Insert Figures 3 here]

To see how contract characteristics impact the holdups, we regress the estimated holdup ratios on *auti*, $afri$, *nbid* and $job$ conditioning on bidder participation and report their marginal effects on holdups in Table 10. Across all specifications, the proportion of fringe competitors in an auction ($afri$) has a significantly positive effect on the holdup ratio. For example, the holdup ratio is about 13.4%-14.4% higher in the auctions that only involve fringe contractors than those only involving major (non-fringe) contractors when the number of bidders $n \in \{3, 4, 5\}$.

[Insert Table 10 here]

The impact of the job type of contracts ($job$) on holdup depends on the intensity of competition. For auctions with moderate competition ($3 \leq n \leq 8$), which accounts for more than 76% of contracts in the data, the job type has a significantly positive impact on holdup ratios. This is consistent with our structural estimates above. Table 7 suggests that the contract type has no significant effect on the ex ante probability for contract revision; Table 6 indicates that the net surplus from revisions is higher for contracts that involve major jobs (the average marginal effect of job type on revision surplus is statistically positive at 5% level). Taken together, those estimates explain why the holdup ratios are higher in contracts that require more substantial work.

# 7 Cost-plus versus Fixed-price Contracts

In practice, a common alternative to the "fixed-price" contract analyzed above is a "cost-plus" contract.[20] Cost-plus contracts do not specify any fixed payment, but reimburse contractors for actual costs incurred plus a profit margin negotiated *ex ante* (either in a lump-sum or pro-rata form). Cost-plus contracts are popular in the U.S. defense industry, where the total value of cost-plus contracts is $78 billion for the fiscal year 2007.[21]

[20]The term "fixed-price" refers to the fact that, prior to the negotiation of transfers due to contract revisions, the payment to the contractor determined in the auction is fixed.

[21]Source: "Defense Industrial Initiatives Current Issues: Cost-Plus Contracts", by Joachim Hofbauer and Greg Sanders, Center for Strategic and International Studies, 2008

Use estimates from Section 6, we calculate the surplus for buyers under cost-plus contracts with a negotiated lump-sum profit margin, and compare it with those estimated for fixed-price contracts in the data.

- Competition under cost-plus contracts

We consider a counterfactual context where CalTrans and the contractors have common knowledge about ex ante uncertainty of new design $X^*$. Let $j$ denote the index for projects/contracts reported in the data. A seller/contractor $i$ competes for a project $j$ by quoting a lump-sum profit margin in a cost-plus contract $\zeta_{j,i}$; CalTrans awards the contract to the seller that leads to the highest ex ante surplus for the government. Note that for the government, its expected cost for the project varies with the identity of the contractor it selects.

For each project $j$ and contractor/seller $i$, let $\tilde{z}_{j,i} \equiv (job_j, uti_{j,i}, fri_{j,i}, dis_{j,i})$ denote the project and contractor characteristics in $z_{j,i}$ that affect private costs. The buyer's ex ante surplus from awarding the contract to $i$ is

$$\int \left[\pi(t, job_j) - E(C^*_{j,i}|X^*_j = t, \tilde{z}_{j,i})\right] dF_{X^*_j|\tilde{z}_{j,i}}(t) - \zeta_{j,i} \tag{23}$$

where $\pi$ is the social surplus, $F_{X^*_j|\tilde{z}_{j,i}}$ the ex ante distribution of the new design conditional on $\tilde{z}_{j,i}$ and $C^*_{j,i}$ the costs for the new design $X^*_j$. The buyer chooses a contractor to maximize ex ante surplus in (23). Contractors compete by quoting profit margins in cost-plus contracts. A contractor $i$'s profit from quoting $\zeta_{j,i}$ is:

$$\begin{aligned} & \zeta_{j,i} \times 1\{E[\pi(X^*_j, job_j) - C^*_{j,i}|\tilde{z}_{j,i}] - \zeta_{j,i} \geq E[\pi(X^*_j, job_j) - C^*_{j,i'}|\tilde{z}_{j,i'}] - \zeta_{j,i'}\ \forall i' \neq i\} \\ = \quad & \zeta_{j,i} \times 1\{\zeta_{j,i} \leq \zeta_{j,i'} + E(C^*_{j,i'}|\tilde{z}_{j,i'}) - E(C^*_{j,i}|\tilde{z}_{j,i})\ \forall i' \neq i\}, \end{aligned}$$

where $1\{\cdot\}$ is the indicator function. The distribution of $X^*_j$ and $C^*_{j,i}$ are common knowledge among contractors. This is a simultaneous game of complete information between contractors. We calculate the buyer surplus under a Nash Equilibrium in which the winner is $i = \arg\min_k E(C^*_{j,k}|\tilde{z}_{j,k})$ with a quote $\zeta_{j,i} \equiv E(C^*_{j,i'}|\tilde{z}_{j,i'}) - E(C^*_{j,i}|\tilde{z}_{j,i})$ where $i' = \arg\min_{k \neq i} E(C^*_{j,k}|\tilde{z}_{j,k})$.[22]

- Buyer surplus under both types of contracts

Under cost-plus contracts, the surplus for the buyer is

$$E[\pi(X^*_j, job_j) - C^*_{j,i} - \zeta_{j,i}|w_j], \tag{24}$$

where $i$ denotes the winner who has the lowest costs for design $X^*$ *ex ante*, and $w_j$ consists of contract characteristics ($auti_j$, $afri_j$, $adis_j$, $nbid_j$, $job_j$).

[22] In case of ties, we use a tie-breaking rule whereby the contract is awarded to the bidder with the lowest expected costs.

Buyer surplus under fixed-price contracts is

$$E[\pi(X_j, job_j) - \varphi(X_j, W_j; \lambda^*) + \mu(X_j, \tilde{X}_j, W_j)|w_j], \tag{25}$$

where $\mu$ is the buyer's expected share of net surplus due to the new design after conceding the holdup to the contractor, and $\varphi$ is the expected auction price under initial specification $X_j$ and contract characteristics. For a meaningful comparison, we maintain in our analysis that the distribution of $X_j^*$ given $w_j$ in counterfactual cost-plus contracts is the same as that in the sample.

The ranking of buyer surplus across these two types of contracts is ambiguous because of the trade-offs involved. First, under cost-plus contracts the auction winner is the one with the lowest *ex ante* costs under the design $X^*$ while under a fixed-price contract the auction winner is the one with the lowest *ex post* costs under the initial design $X$, *after adjusting for the holdup*. Second, under cost-plus contracts the buyer collects the expected *gross* surplus $E[\pi(X_j^*, Job_j)|w_j]$; whereas under fixed-price contracts the buyer concedes a proportion of the net surplus to the contractor, thus collecting an expected gross surplus of $E[\pi(\alpha^*(\tilde{X}_j), job_j) + \mu(\alpha^*(\tilde{X}_j), \tilde{X}_j, W_j)|w_j]$. Finally, the buyer's expected payment under cost-plus contracts is $E[C_{j,i}^* + \zeta_{j,i}|w_j]$, or ex ante second lowest costs under the new design. In contrast, the buyer's expected payment under fixed-price contracts is $E[\varphi(X_j, W_j; \lambda^*)|w_j]$, where $\varphi$ consists of the lowest costs under $X_j$ and a markup that is adjusted based on contractor expected share of net surplus. All in all, measuring the difference in buyer surplus under these contracts is an open empirical question because of these tradeoffs.

- Comparison of buyer surplus

Our results of counterfactual analysis show that the buyer surplus under a cost-plus contract is lower than that under the fixed-price contract for 71.7% of the projects reported in the data. The average reduction in buyer surplus under the cost-plus contracts is $382,074, which is 13.6% of average engineer estimates. This implies that, for most of the projects in the data, the buyer's gains in the expected surplus under a fixed-price contract are greater than the increase in its expected payment.

[Insert Figures 4 here]

Figure 4 reports the histograms of estimated differences in buyer surplus (that under cost-plus minus that under fixed-price) in the auctions conditional on the job type and on the number of bidders. While the job types do not seem to have any significant impact on the difference in buyer surplus, a greater number of bidders tends to make the distribution of the difference slightly positively skewed. To understand what is driving this pattern, recall the number of bidders affects a buyer's expected payment under both forms of contracts through contractor strategies and the lowest expected costs. It also affects buyers' ex ante surplus under the fixed-price contract through the holdup. The figure indicates that these trade-offs eventually work in favor of the buyer under fixed-price contracts in most cases.

[Insert Table 11 here]

We regress the estimated differences in buyer surplus on contract characteristics, i.e., $(auti, afri, job, adis)$, and report their average marginal effects in Table 11. First, across all specification and level of bidder participation, the type of jobs have significantly negative impact on the difference in buyer surplus. That is, from a buyer's perspective, a fixed-price contract is ex ante more desirable relative to a cost-plus contract when the jobs involve major tasks ($job = 1$). For example, under the third specification, under moderate competition ($n \in \{3, 4, 5\}$), the difference between buyer surplus is reduced by $\$237,000$ (about 8.4% of average engineer estimates) when the contract involves major jobs.

Second, cost-plus contracts compare more favorably with fixed-price contracts when there is a higher proportion of fringe competitors in the auction. With moderate bidder participation ($n \in \{3, 4, 5\}$), the difference in surplus would increase by $\$549,000$ if the competition is between fringe contractors alone. This effect is explained by the difference in contractor costs and bargaining power between fringe and non-fringe competitors.

Third, the average marginal effect of average utilization rates is positive, implying that the advantage of fixed-price contracts on buyer's surplus over cost-plus contracts diminishes if contractors are more occupied. This can be partially interpreted by utilization's positive impacts on contractors' bargaining power presented in Table 5: Higher bargaining power of the contractor due to higher average utilization rates benefits contractors in post-auction bargaining but hurts the buyer. In addition, more occupied contractors may have less incentives to exert efforts and lower costs in fixed-price contracts and this negatively affects the advantage of fixed-price over cost-plus contracts.

Our analysis of buyer surplus provides a benchmark comparison between fixed-price and cost-plus contracts in that it abstracts away from any disincentive for cost-saving under cost-plus contracts. If the contractor costs under the cost-plus format is stochastically higher than fixed-price contracts, the difference in buyer surplus would be even lower than what we estimate. Hence our benchmark comparison can be interpreted as an upper bound on the difference in buyer surplus if there is cost-saving under fixed-price contracts. Our estimates show this upper bound is negative for most cases in the data.

# 8 Concluding Remarks

This paper studies procurement auctions with incomplete contracts. We introduce a model that endogenizes a buyer's initial specification of the contract, and maintains a flexible assumption on the information structure. The model rationalizes the decision to revise a contract and the transfers negotiated as Nash Bargaining outcome between the buyer and the auction winner. We show that the model components are nonparametrically identified from the contract prices and negotiated transfers, and use the model to analyze CalTrans auctions of highway procurement contracts. Our estimates shed lights on how contractors respond strategically to the presence of contractual incompleteness, and what determines the size of the holdup in highway procurement projects. We find that fixed-price contracts mostly yield higher surplus for the buyer over cost-plus contracts when there is uncertainty about the final design. A direction for future research is to use the method we propose to evaluate the impact of incomplete contracts on the efficiency of mechanisms, e.g., whether revenue equivalence still holds once incompleteness occurs. While our method is introduced in the context of procurement auctions, it might be extended to other formats of pre-contractual competition.

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# Appendix

## A Nash Bargaining in Post-auction Negotiation

Let $v$ denote the auction price and $c$ denote the auction winner's *ex post* cost for implementing the contract under the initial design $x$. The disagreement value (a.k.a. reservation value) for the auction winner is

$$d_c \equiv v - c$$

while the disagreement value for the buyer is

$$d_b \equiv \pi - v$$

with $\pi$ being the social surplus under the design $x$. With a new feasible design $x^*$, the ex post total net social surplus to be shared among the buyer and the auction winner is

$$u_0 \equiv \pi^* - a - c$$

where $\pi^*$ is the social surplus under $x^*$ and $a$ is the *incremental* costs for delivering the contract under $x^*$ (in addition to the cost $c$ for delivering the contract under $x$). With $\gamma$ denoting the bargaining power of the contractor, the Nash Bargaining solution is characterized by

$$\max_{u_c, u_b} (u_c - d_c)^\gamma (u_b - d_b)^{1-\gamma} \text{ s.t. } u_b + u_c \leq u_0.$$

By a standard argument,

$$u_c \equiv \gamma(u_0 - d_b) + (1-\gamma)d_c = \gamma s + v - c$$

and

$$u_b \equiv u_0 - u_c = (1-\gamma)s + \pi - v$$

where $s \equiv \pi^* - \pi - a$ is the net incremental surplus. As stated in the text, we maintain that the contractor covers the incremental costs $a$ as they arise in construction while the incremental surplus $\pi^* - \pi$ is eventually accrued to the buyer. Then the negotiated transfer $y$ needs to satisfy:

$$d_c + y - a = u_c \text{ and } d_p + \pi^* - \pi - y = u_b$$

which is equivalent to

$$y - a = \gamma s \text{ and } \pi^* - \pi - y = (1-\gamma)s.$$

This proves that the negotiated transfer is as characterized in Section 2.

## B Existence of Symmetric Monotone psPBE

Our first step is to link the contractor beliefs to their bidding strategies in the auction stage. Denote a generic contractor belief about the new design conditional on the initial announcement by $\lambda : \mathcal{X} \times \mathcal{X} \to [0,1]$. Let $\delta(x;\lambda) \equiv E_\lambda[\gamma s_+(x, X^*)|X = x]$, and the notation $E_\lambda(.)$ is a reminder that $X^*$ is integrated out with respect to the belief $\lambda(X^*|X)$. Let $G(\cdot \mid x;\lambda)$ denote the distribution of contractors' adjusted costs $\widetilde{C}_i(\lambda) \equiv C_i - \delta(x;\lambda)$ conditional on $X = x$ and $\lambda$. That is,

$$G(t \mid x;\lambda) \equiv \Pr\{C_i - \delta(x;\lambda) \le t \mid X = x\}.$$

A standard argument such as that in Krishna (2009) show that a symmetric monotone pure-strategy Bayesian Nash equilibrium exist in a procurement auction where contractors' costs are drawn from $G(\cdot \mid X;\lambda)$. More specifically, let $\widetilde{G} \equiv 1 - G$ so that $\widetilde{G}(c \mid x;\lambda)^{N-1} \equiv \Pr\{\min_{j\neq i} \widetilde{C}_j(\lambda) \ge c \mid x;\lambda\}$ and $1 - \widetilde{G}(c \mid x;\lambda)^{N-1} \equiv \Pr\{\min_{j\neq i} \widetilde{C}_j \le c \mid x;\lambda\}$. For all $i = 1, 2, \cdots, N$, $c \in \mathcal{C}$ and $x \in \mathcal{X}$,

$$\beta_i(c, x;\lambda) = \frac{\int_c^{\bar{c}} s d[1 - \widetilde{G}(s \mid x;\lambda)^{N-1}]}{\widetilde{G}(c \mid x;\lambda)^{N-1}}, \tag{B.1}$$

where $\bar{c}$ denotes the supremum of $\mathcal{C}$.

Next, we link contractor beliefs to the buyer's expected payment and the buyer's choice of initial design. It follows from (B.1) that the buyer's expected payment when contractors hold belief $\lambda$ is

$$\begin{aligned}\varphi_\lambda(x) &\equiv \varphi(x;\beta(., x;\lambda)) = N \int_0^{\bar{c}} \beta_i(c, x;\lambda) \Pr\left\{\min_{j\neq i} \widetilde{C}_j \ge c \middle| X = x;\lambda\right\} dG(c \mid x;\lambda) \\ &= N \int_0^{\bar{c}} sd\left[1 - \widetilde{G}(s \mid x;\lambda)^{N-1}\right] - N \int_0^{\bar{c}} s\widetilde{G}(s \mid x;\lambda) d\left[1 - \widetilde{G}(s \mid x;\lambda)^{N-1}\right]\end{aligned}$$

where the first term is $NE\left[\min_{j\neq i} \widetilde{C}_j(\lambda) \middle| X = x;\lambda\right]$; and the second term is

$$\begin{aligned}& N \int_0^{\bar{c}} s\widetilde{G}(s \mid x;\lambda)(N-1)\widetilde{G}(s \mid x;\lambda)^{N-2} g(s \mid x;\lambda) ds \\ =\ & (N-1) \int_0^{\bar{c}} sd\left[1 - \widetilde{G}^N(s \mid x;\lambda)\right] = (N-1)E\left[\min_i \widetilde{C}_i(\lambda) \middle| X = x;\lambda\right].\end{aligned}$$

Because $\widetilde{C}_i(\lambda) \equiv C_i - \delta(x;\lambda)$, we can write the right-hand side of (B.2) as $\sigma(x) - \delta(x;\lambda)$, where

$$\sigma(x) \equiv NE[C^{(1:N-1)}|X = x] - (N-1)E[C^{(1:N)}|X = x],$$

with $C^{(m:n)}$ being the $m$-th smallest out of $n$ independent draws from $F_{C|X}$. Given the contractor belief $\lambda$, the buyer's optimization problem is:

$$\alpha(\tilde{x};\lambda) \equiv \arg\max_{x\in\mathcal{X}} \{\pi(x) - \sigma(x) + \mu(x, \tilde{x}) + \delta(x;\lambda)\},$$

where $\pi, \sigma$ and $\mu$ are belief-free in that they only depend on model primitives that are common knowledge to both the buyer and contractors. We maintain that $E(\min_{i=1,\dots,N} C_i | X = x)$ is differentiable in $x$ over $\mathcal{X}$ for all $N$.

Suppose $\alpha^* : \mathcal{X} \to \mathcal{X}$ is the buyer's strategy in a strictly monotone pure-strategy PBE, and is differentiable. By the restriction of consistent beliefs on the equilibrium path of initial announced design, $\lambda^*(.|x) = F_{X^*|\tilde{X}=\alpha^{*-1}(x)}(.)$ for all $x$ on the support of $\alpha^*(\widetilde{X})$. For $\alpha^*(.)$ to be the buyer's equilibrium strategy, it must be the case that for all $\tilde{x} \in \mathcal{X}$,

$$\pi'(\alpha^*(\tilde{x})) + \mu_1(\alpha^*(\tilde{x}), \tilde{x}) = \sigma'(\alpha^*(\tilde{x})) - \delta'(\alpha^*(\tilde{x}); \lambda^*), \tag{B.2}$$

where $\mu_1(z, \tilde{x}) \equiv \partial\mu(x, \tilde{x})/\partial x|_{x=z}$. Let

$$\Lambda(x, \tilde{x}) \equiv E[s_+(x, X^*) \mid \widetilde{X} = \tilde{x}] = \int_{\omega(x)} [\pi(x^*) - \pi(x) - a(x, x^*)]\, dF(x^*|\tilde{x}),$$

where $\omega(x) \equiv \{x^* : s(x, x^*) > 0\}$ as in the text. Note that the functional form of $\Lambda$ depends on the common knowledge primitive $F_{X^*|\tilde{X}=\tilde{x}}$ but not the belief of contractors. By construction, $\mu(x, \tilde{x}) = (1-\gamma)\Lambda(x, \tilde{x})$ and $\delta(x; \lambda^*) = \gamma\Lambda(x, \alpha^{*-1}(x))$. Thus

$$\delta'(\alpha^*(\tilde{x}); \lambda^*) = \gamma\Lambda_1(\alpha^*(\tilde{x}), \tilde{x}) + \frac{\gamma\Lambda_2(\alpha^*(\tilde{x}), \tilde{x})}{\alpha^{*\prime}(\tilde{x})}, \tag{B.3}$$

where $\Lambda_1$ (and $\Lambda_2$) denote partial derivatives of $\Lambda$ with respect to its first (and second) argument respectively. Note that the function form of $\sigma$ and $\Lambda$ are determined by model primitives $F_{C|X}$ and $F_{X^*|\tilde{X}}$ respectively, and do not depend on the belief of contractors. Substitute (B.3) into (B.2) to get

$$\alpha^{*\prime}(\tilde{x}) = \Psi(\alpha^*(\tilde{x}), \tilde{x})$$

where $\Psi : \mathcal{X} \times \widetilde{\mathcal{X}} \to \mathbb{R}$ is defined as

$$\Psi(x, \tilde{x}) \equiv \frac{\gamma\Lambda_2(x, \tilde{x})}{\sigma'(x) - \pi'(x) - \Lambda_1(x, \tilde{x})}. \tag{B.4}$$

We say a contractor's conditional belief $\lambda$ is *ultra-pessimistic at* $x \in \mathcal{X}$ if it assigns no probability mass to new designs to the event that the new designs yield positive net surplus. That is,

$$\int 1\{x^* \in \omega(x)\} d\lambda(x^*|x) = 0$$

where $\omega(x) \equiv \{x^* : s(x, x^*) > 0\}$ as in the text. In other words, a contractor with an ultra-pessimistic belief at $x$ thinks $\Pr\{s(X, X^*) \leq 0 | X = x\} = 1$, which implies $\delta(x; \lambda) = 0$ for all $x \in \mathcal{X}$. Therefore, a buyer's objective function when contractors hold "ultra-pessimistic" beliefs is:

$$H^o(x, \tilde{x}) \equiv \pi(x) + (1-\gamma)\Lambda(x, \tilde{x}) - \sigma(x),$$

which is the buyer's objective function when contractors hold an "ultra-pessimistic" belief that $\Pr\{s(X, X^*) \leq 0 \mid X = x\} = 1$ for all $x \in \mathcal{X}$. This is because $\delta(x; \lambda) = 0$ if the belief $\lambda(.|x)$ is ultra-pessimistic at $x$.

**Assumption B1.** (Smoothness and Concavity) $\pi$, $\sigma$ and $\Lambda$ are differentiable and bounded over their domains. For each $\tilde{x} \in \widetilde{\mathcal{X}}$, $H^o$ is strictly concave in $x$ over $\mathcal{X}$.

It follows from Assumption B1 and the Theorem of Maximum that

$$\mathcal{X}_o \equiv \left\{ x : x = \arg\max_{z \in \mathcal{X}} H^o(z, \tilde{x}) \text{ for some } \tilde{x} \in \mathcal{X} \right\}$$

is convex and compact in $\mathcal{X}$. Let $x_h^o$ (and $x_l^o$) denote the supremum (and infimum) of $\mathcal{X}_o$; $x_h$ (and $x_l$) denote the supremum (and infimum) of $\mathcal{X}$; and $\tilde{x}_h$ (and $\tilde{x}_l$) denote the supremum (and infimum) of $\widetilde{\mathcal{X}}$.

**Assumption B2.** (a) $\Psi$ is continuous over $\mathcal{X} \times \widetilde{\mathcal{X}}$; and there exists $L \in \mathbb{R}_{++}$ such that for all $(\tilde{x}, x, x')$ in $\widetilde{\mathcal{X}} \times \mathcal{X} \times \mathcal{X}$, $|\Psi(x', \tilde{x}) - \Psi(x, \tilde{x})| \leq L\,|x' - x|$. (b) $M(\tilde{x}_h - \tilde{x}_l) \leq x_h - x_l^o$, where $M \equiv \sup_{(x,\tilde{x})\mathcal{X}\times\widetilde{\mathcal{X}}} \Psi(x, \tilde{x}) < \infty$.

Define the following ordinary differential equation with an initial condition:

$$\alpha'(\tilde{x}) = \Psi(\alpha(\tilde{x}), \tilde{x}) \text{ and } \alpha(\tilde{x}_l) = x_l^o. \tag{B.5}$$

It follows from Assumption B2 and Picard's Local Existence Theorem that there exists a solution to (B.5) over $\widetilde{\mathcal{X}} \equiv [\tilde{x}_l, \tilde{x}_h]$. The range for the solution is a subset of $[x_l^o, x_h]$ under Assumption B2.

**Assumption B3.** (Monotonicity) (a) For any pair $x' > x$ on $\mathcal{X}$, $F_{C|X=x'}$ first-order stochastically dominates $F_{C|X=x}$. (b) For each $x \in \mathcal{X}$, $\Lambda(x, \tilde{x})$ is increasing in $\tilde{x}$. (c) For each $\tilde{x} \in \widetilde{\mathcal{X}}$, $\pi(x) + \Lambda(x, \tilde{x})$ is non-increasing in $x$.

**Lemma B.1** *Under Assumption B1, B2 and B3, there exists a strictly monotone solution $\alpha^* : \widetilde{\mathcal{X}} \to \mathcal{X}$ to (B.5).*

*Proof of lemma B.1.* We first show that if the solution exists it must be strictly monotone over its domain. By construction,

$$\sigma(x) \equiv N \int_0^{\bar{c}} [1 - F_{C|X=x}(s)]^{N-1} ds - (N-1) \int_0^{\bar{c}} [1 - F_{C|X=x}(s)]^N ds.$$

Therefore

$$\sigma'(x) = -N(N-1) \int_0^{\bar{c}} \frac{\partial F_{C|X}(s|x)}{\partial x} [1 - F_{C|X}(s)]^{N-2} F_{C|X}(s) ds > 0,$$

where the last line is due to Assumption B3-(a) (that $F_{C|X}$ is stochastically increasing in $x$). Together with Assumption B3-(c), this implies the denominator in the definition of $\Psi$ in (B.4) is positive for all $x, \tilde{x}$. Assumption B3-(b) then implies the numerator in (B.4) is also positive. Thus the solution to (B.5), if exists, must be increasing over $\widetilde{\mathcal{X}}$. By the Picard's Existence Theorem, there exists a solution $\alpha^*$ to (B.5) over $\tilde{\mathcal{X}} \equiv [\tilde{x}_l, \tilde{x}_h]$ under Assumption B2. $\square$

**Assumption B4.** (Support Conditions) (a) For any $(x, \tilde{x}) \in \mathcal{X} \times \tilde{\mathcal{X}}$, the integral $\int 1\{x^* \in \omega(x)\} dF_{X^*|\tilde{X}=\tilde{x}}(x^*) > 0$. (b) For some $\alpha^*$ that solves (B.5), $\mathcal{X}_o \subseteq \mathcal{X}_e \equiv \{x : x = \alpha^*(\tilde{x})$ for some $\tilde{x} \in \widetilde{\mathcal{X}}\}$.

We now define a symmetric monotone pure-strategy Perfect Bayesian Equilibrium as follows:

$$\begin{aligned} &\alpha^* \text{ solves (B.5); } \beta^* \text{ is defined in (B.1) with beliefs } \lambda^*; \\ &\lambda^*(.|x) = F_{X^*|\tilde{X}=\alpha^{*-1}(x)}(.) \text{ for } x \in \mathcal{X}_e \text{; and } \lambda^*(.|x) \text{ is ultra-pessimistic for } x \notin \mathcal{X}_e \end{aligned} \quad (B.6)$$

**Proposition B.1**. *Under Assumptions B1, B2, B3 and B4, the profile of strategies and beliefs in (B.6) exists, and is a symmetric monotone pure-strategy Perfect Bayesian equilibrium.*

*Proof of Proposition B.1.* Existence of the profile $(\alpha^*, \beta^*, \lambda^*)$ in (B.6) follows from Lemma B.1. It only remains to show that such a profile $(\alpha^*, \beta^*, \lambda^*)$ indeed forms a symmetric monotone psPBE. That $\beta^*$ is the contractors' best response given their beliefs $\lambda^*$ is shown earlier in the text of this appendix. That $\alpha^*$ is a solution to (B.5) ensures that the buyer can not make profitable (local) deviation over the equilibrium path of $X$. Also by definition, $\lambda^*$ is consistent on the equilibrium support of $\alpha^*(\tilde{X})$.

It remains to check that the off-equilibrium beliefs in (B.6) guarantee that for any $\tilde{x}$ there is no profitable deviation of the buyer from $\alpha^*(\tilde{x})$ to some $x' \notin \mathcal{X}_e$. To see this, first note that under Assumption B4, $x' \notin \mathcal{X}_e$ implies $x' \notin \mathcal{X}_o$. Thus there exists $x^o \in \mathcal{X}_o$ so that $H^o(x^o, \tilde{x}) \geq H^o(x', \tilde{x})$. By construction,

$$H^o(z, \tilde{x}) \leq \pi(z) + \mu(z, \tilde{x}) - \sigma(z) + \delta(z; \lambda^*),$$

where the inequality holds with equality for $z \notin \mathcal{X}_e$ (because $\delta(z; \lambda^*) = 0$ for such $z$ due to ultra-pessimistic beliefs), and the inequality holds strictly for $z \in \mathcal{X}_e$ due to Assumption B4-(a). Because $x^o \in \mathcal{X}_o$ implies $x^o \in \mathcal{X}_e$ under Assumption B4-(b),

$$\begin{aligned} & \pi(x^o) + \mu(x^o, \tilde{x}) - \sigma(x^o) + \delta(x^o; \lambda^*) \\ > \; & H^o(x^o, \tilde{x}) \geq H^o(x', \tilde{x}) \\ = \; & \pi(x') + \mu(x', \tilde{x}) - \sigma(x') + \delta(x'; \lambda^*). \end{aligned}$$

This implies $x'$ could not be the buyer's optimal choice of initial design given the belief $\lambda^*$. $\square$

**Discussion of Assumption B4-(b)**. We conclude this part of the appendix with primitive conditions that are sufficient for the second support condition in Assumption B4. By construction and the monotonicity of the solution to (B.5), $\alpha^*(\tilde{x}_l) = x_l^o = \inf \mathcal{X}_e = \inf \mathcal{X}_o$, and

$$x_h^e - x_l^e = x_h^e - x_l^o = \int_{\tilde{x}_l}^{\tilde{x}_h} \Psi(\alpha^*(\tilde{x}), \tilde{x}) d\tilde{x}.$$

Define $\alpha_o(\tilde{x}) \equiv \arg\max_{x\in\mathcal{X}} H^o(x,\tilde{x})$ for all $\tilde{x} \in \widetilde{\mathcal{X}}$. Suppose the solution is in the interior of $\mathcal{X}$ for all $\tilde{x}$. By the Implicit Function Theorem,

$$\frac{d}{d\tilde{x}}\alpha_o(\tilde{x}) = -\frac{(1-\gamma)\Lambda_{12}(\alpha_o(\tilde{x}),\tilde{x})}{H^o_{11}(\alpha_o(\tilde{x}),\tilde{x})}$$

for all $\tilde{x}$ such that $\arg\max_{x\in\mathcal{X}} H^o(x,\tilde{x})$ is in the interior of $\mathcal{X}$, where $H^o_{11}$ denotes the second-order derivative with respect to its first argument and $\Lambda_{12}$ denotes the cross-derivatives. Assume $\Lambda_{12}$ is positive over its domain. Then $x^o_h - x^o_l$ is bounded above by $\int_{\tilde{x}_l}^{\tilde{x}_h} \alpha'_o(t)dt$. Therefore for Assumption B4-(b) to hold, it suffices to have

$$-\frac{(1-\gamma)\Lambda_{12}(x,\tilde{x})}{H^o_{11}(x,\tilde{x})} \leq \frac{(1-\gamma)\Lambda_2(x,\tilde{x})}{\sigma'(x)-\pi'(x)-\Lambda_1(x,\tilde{x})}$$

for all $(x,\tilde{x}) \in \mathcal{X}\times\tilde{\mathcal{X}}$. For example, the inequality in the display above holds when $\pi$ and $a$ are close to being linear in $x$. In this case, $\Lambda_{12}$ and therefore the left-hand side will be positive and close to zero while the right-hand side is positive and not close to zero in general.[23]

# C Proofs in Section 3

Let $\mathcal{X}_e \equiv \{x : x = \alpha^*(\tilde{x}) \text{ for some } \tilde{x} \in [0,1]\}$ denote the equilibrium support of the initial design, which is convex and compact under assumptions of our model by the theorem of maximum.

**Lemma C1.** *(a) The inverse of a contractor's equilibrium bidding strategy in (2) is*

$$\beta^{*-1}(b,x) = \tilde{\beta}^{-1}(b,x) + \delta(x;\lambda^*) \tag{C.1}$$

*for all $x$ and $b$ on the support of bids in symmetric monotone psPBE. (b) For all $x$ on the equilibrium support of initial designs, $\tilde{\beta}^{-1}(.,x)$ is identified from the distribution of the auction price (the winning bid) $V$.*

**Proof of Lemma C1.** By changing variables between contractor costs $C_i$ and quoted prices in the procurement auctions $P_i$, we can write the inverse of contractors' equilibrium bidding strategy as:

$$\tilde{\beta}^{-1}(p,x) = p - \frac{1}{N-1}\frac{1-F_{P|X=x}(p)}{f_{P|X=x}(p)} \text{ for all } x \in \mathcal{X}_e, \tag{C.2}$$

where the right-hand side is identified from the distribution of quoted prices conditional on the initial design.

[23] To see this, note that by construction,

$$\Lambda_1(x,\tilde{x}) \equiv \frac{\partial}{\partial x}\int_{\omega(x)} [\pi(x^*)-\pi(x)-a(x,x^*)]\, dF(x^*|\tilde{x}) = \int_{\omega(x)} [-\pi'(x)-a_1(x,x^*)]\, dF(x^*|\tilde{x}).$$

In fact, $\beta^{-1}(.,x)$ is identified for all $x \in \mathcal{X}_e$ even if the data only reports the contract prices determined in the procurement auctions. By definition, the contract price $V$ is the minimum of prices quoted by participating contractors $(P_i : i = 1, 2, ..., N)$. Because contractor costs are i.i.d. conditional on $X$, their bids in a symmetric monotone psPBE are i.i.d. given $X$. Thus

$$1 - F_{V|X} = \left(1 - F_{P|X}\right)^N \Rightarrow F_{P|X} = 1 - \left(1 - F_{V|X}\right)^{\frac{1}{N}}.$$

Substituting this into (C.2) and using $f_{V|X=x}(b) = \frac{\partial}{\partial t} F_{V|X=x}(t)|_{t=b}$, we get

$$\tilde{\beta}^{-1}(p, x) = p - \frac{N}{N-1} \frac{1 - F_{V|X=x}(p)}{f_{V|X=x}(p)} \text{ for all } x \in \mathcal{X}_e,$$

where the right-hand side is identified using the distribution of auction prices conditional on each $x \in \mathcal{X}_e$. □

For each $x \in \mathcal{X}$, define $\omega(x) \equiv \{t \in \mathcal{X} : s(x, t) > 0\}$. That is, $\omega(x)$ is the set of new designs that could be adopted to replace the initial design $x$.

**Assumption C1.** *For each $x \in \mathcal{X}$, $\omega(x)$ is a non-degenerate interval $(\omega_l(x), \omega_h(x))$. Both $\omega_l$ and $\omega_h$ are differentiable in $x$ over $\mathcal{X}$.*[24]

Under this assumption, $\mu(x, \tilde{x})$ is continuous in both arguments and is differentiable in $x$ for each $\tilde{x}$. To see how this condition can be satisfied, suppose for any $x$, the net incremental surplus $s(x, x^*)$ is monotone (or concave or convex) in $x^*$. The implicit function theorem then implies that this condition holds if $s$ is continuously differentiable in both of its arguments.

**Theorem 1** *Under Assumption C1, the marginal effect of contract design on social surplus $\pi'(x)$ is identified for all $x \in \mathcal{X}_e$.*

**Proof of Theorem 1.** Rewrite (6) as:

$$\begin{aligned} & \varphi'(\alpha^*(\tilde{x}); \beta^*) - \pi'(\alpha^*(\tilde{x})) \\ = \ & -\frac{\partial}{\partial x}\left[(1-\gamma)\pi(x)p(x,\tilde{x})\right]_{x=\alpha^*(\tilde{x})} + \frac{\partial}{\partial x}\left[\int_{\omega(x)} (1-\gamma)[\pi(t) - a(x,t)]dF_{X^*|\tilde{x}}(t)\right]_{x=\alpha^*(\tilde{x})} \end{aligned} \quad \text{(C.3)}$$

for any $\tilde{x}$ with an interior solution, where $p(x, \tilde{x}) \equiv \Pr(s(X, X^*) > 0 | X = x, \widetilde{X} = \tilde{x})$ Recall for any $x \in \mathcal{X}_e$ and $t \in \omega(x)$, $y(x, t) = (1 - \gamma)a(x, t) + \gamma[\pi(t) - \pi(x)]$. Hence the second term on the right-hand side is

$$-\frac{\partial}{\partial x}\left[\pi(x)p(x,\tilde{x})\right]_{x=\alpha^*(\tilde{x})} + \frac{\partial}{\partial x}\left[\int_{\omega(x)} \left\{\pi(t) - y(x,t)\right\} dF_{X^*|\tilde{x}}(t)\right]_{x=\alpha^*(\tilde{x})}, \quad \text{(C.4)}$$

[24]Our main identification result in Theorem 1 holds when $\omega(x) \subseteq \mathcal{X}$ is non-convex, i.e., partitioned into disjoint intervals such as $(-\infty, \omega_1(x))$, $(\omega_2(x), \omega_3(x))$, $(\omega_4(x), +\infty)$.

and (C.3), under Assumption C1, can be written as

$$\begin{aligned}&\varphi'(\alpha^*(\tilde{x});\beta^*) - \pi'(\alpha^*(\tilde{x}))\\ = \;& -\pi'(\alpha^*(\tilde{x}))p(\alpha^*(\tilde{x}),\tilde{x}) - \pi(\alpha^*(\tilde{x}))p_1(\alpha^*(\tilde{x}),\tilde{x}) + \frac{\partial}{\partial x}\left[\int_{\omega_l(x)}^{\omega_h(x)} \{\pi(t) - y(x,t)\}\, dF_{X^*|\tilde{x}}(t)\right]_{x=\alpha^*(\tilde{x})} \quad \text{(C.5)}\end{aligned}$$

for all $\tilde{x}$, where $p_1(z,\tilde{x}) \equiv \frac{\partial}{\partial x}p(x,\tilde{x})|_{x=z}$.

In what follows, let $z$ be shorthand for $\alpha^*(\tilde{x})$ at each $\tilde{x}$. Then by Assumption C1,

$$p_1(z,\tilde{x}) = f_{X^*|\tilde{x}}(\omega_h(z))\omega_h'(z) - f_{X^*|\tilde{x}}(\omega_l(z))\omega_l'(z).$$

Applying the Leibniz Rule to the last term on the right-hand side of (C.5), we can write (C.5) as:

$$\begin{aligned}\varphi'(z;\beta^*) &= [1 - p(z,\tilde{x})]\pi'(z) + \pi(z)[f_{X^*|\tilde{x}}(\omega_l(z))\omega_l'(z) - f_{X^*|\tilde{x}}(\omega_h(z))\omega_h'(z)]\\ &+ [\pi(\omega_h(z)) - y(z,\omega_h(z))]\, f_{X^*|\tilde{x}}(\omega_h(z))\omega_h'(z) - [\pi(\omega_l(z)) - y(z,\omega_l(z))]\, f_{X^*|\tilde{x}}(\omega_l(z))\omega_l'(z)\\ &- \int_{\omega_l(z)}^{\omega_h(z)} y_1(z,t) f_{X^*|\tilde{x}}(t)dt \qquad \text{(C.6)}\end{aligned}$$

for any $z \in \mathcal{X}_e$. Next, note that for all $(x, x^*)$,

$$\begin{aligned}\pi(x^*) - y(x,x^*) &= \pi(x^*) - \gamma[\pi(x^*) - \pi(x)] - (1-\gamma)a(x,x^*)\\ &= \pi(x) + (1-\gamma)[\pi(x^*) - \pi(x) - a(x,x^*)]\\ &= \pi(x) + (1-\gamma)s(x,x^*).\end{aligned}$$

Because $s(x,x^*) = 0$ at $x^* = \omega_l(x)$ or $\omega_h(x)$, we have $\pi(x^*) - y(z,x^*) = \pi(z) + (1-\gamma)\cdot 0$ for all $z \in \mathcal{X}_e$ and $x^* = \omega_l(z)$ or $\omega_h(z)$. Substitute these into (C.6) and cancel out several terms. Then for all $\tilde{x} \in \widetilde{\mathcal{X}}$,

$$\pi'(\alpha^*(\tilde{x})) = [1 - p(\alpha^*(\tilde{x}),\tilde{x})]^{-1}\left(\varphi'(\alpha^*(\tilde{x});\beta^*) + \int_{\omega_l(\alpha^*(\tilde{x}))}^{\omega_h(\alpha^*(\tilde{x}))} y_1(\alpha^*(\tilde{x}),t) f_{X^*|\tilde{X}=\tilde{x}}(t)dt\right). \quad \text{(C.7)}$$

The r.h.s. of (C.7) consists of quantities that are directly identifiable in the data. First, because the distribution of $\tilde{X}$ is normalized to a standard uniform distribution, the monotonicity of the buyer strategy implies that $\alpha^*(\tilde{x})$ equals the $(100\times\tilde{x})$-th percentile of the initial design announced in the data. For any $\tilde{x} \in \widetilde{\mathcal{X}}$ and $t \in \omega(\alpha^*(\tilde{x})) \equiv (\omega_l(\alpha^*(\tilde{x}))$, $\omega_h(\alpha^*(\tilde{x})))$,

$$\begin{aligned}&f_{X^*|\tilde{X}=\tilde{x}}(t) = f_{X^*|X=\alpha^*(\tilde{x})}(t)\\ =\;& f_{X^*|X^*\in\omega(\alpha^*(\tilde{x})),X=\alpha^*(\tilde{x})}(t) \times \Pr(X^* \in \omega(\alpha^*(\tilde{x}))|X = \alpha^*(\tilde{x}))\\ =\;& f_{X^*|D=1,X=\alpha^*(\tilde{x})}(t) \times \Pr(D = 1|X = \alpha^*(\tilde{x}))\end{aligned}$$

where the first equality is due to monotonicity of $\alpha^*$ and the second and third equalities hold because "$D = 1$ and $X = x$" if and only if "$X^* \in \omega(x)$ and $X = x$" for all

$x \in \mathcal{X}_e$. Second, note $\varphi'(\alpha^*(\tilde{x});\beta^*) \equiv \frac{\partial}{\partial x}\varphi(x;\beta^*)|_{x=\alpha^*(\tilde{x})}$ by definition and is identified for all $\tilde{x} \in (0,1)$. Third, $y_1(\alpha^*(\tilde{x}),t) \equiv \frac{\partial}{\partial x}y(x,t)|_{x=\alpha^*(\tilde{x})}$ is identified for all $\tilde{x} \in (0,1)$ and $t$ such that $t \in \omega(\alpha^*(\tilde{x}))$. Finally,

$$p(\alpha^*(\tilde{x}),\tilde{x}) \equiv \Pr\{D=1 \mid X=\alpha^*(\tilde{x}),\tilde{X}=\tilde{x}\} = \Pr\{D=1 \mid X=\alpha^*(\tilde{x})\} \equiv p^*(\alpha^*(\tilde{x})),$$

where the equality is due to the monotonicity of $\alpha^*$. This implies the right-hand is identifiable from the data, because $\alpha^*(\tilde{x})$ equals the $100 \times \tilde{x}$-th percentile of $X$ given our normalization of the marginal distribution of $\tilde{X}$. It then follows that $\pi'(\cdot)$ is identified over $\mathcal{X}_e$. Using a location normalization $\pi(\underline{x}) = \pi_0$ where $\underline{x}$ is the infimum of $\mathcal{X}_e$, we obtain a solution for $\pi$:

$$\pi(x) = \pi_0 + \int_{\underline{x}}^{x} \left\{ [1-p^*(z)]^{-1} \left( \varphi'(z;\beta^*) + \int_{\omega(z)} y_1(z,t) f_{X^*|X=z}(t)dt \right) \right\} dz$$

for all $x \in \mathcal{X}_e$. $\square$

**Corollary 1** *(Theorem 1) Suppose that Assumption C1, conditions A1 and A2 hold and $\pi$ is differentiable. Then (a) $a(x,x^*)$ is identified for all $(x,x^*)$ with $x \in \mathcal{X}_e$ and $s(x,x^*) > 0$, and $\pi(x)$ is identified for all $x \in \mathcal{X}_s$; and (b) the holdup $\delta(x;\lambda^*)$ and the cost distribution $F_{C|X=x}$ are identified for all $x \in \mathcal{X}_e$.*

**Proof of Corollary 1.** Part (a). That $\gamma$ is identified under conditions (A0) and (A1) is shown in the text of Section 3. The social surplus $\pi(x)$ is identified for all $x \in \mathcal{X}_e$ while $a(x,t)$ is identified for all $x \in \mathcal{X}_e$ and $t \in \omega(x) \cap \mathcal{X}_e$. Next, consider any $x \in \mathcal{X}_e$ and $t \in \omega(x)$ (but $t$ may not belong to $\mathcal{X}_e$). Let $y_j$ denote the partial derivative of $y(.,.)$ with respect to its $j$-th argument. It follows from (1) that

$$\begin{aligned} y_1(x,t) &= -\gamma\pi'(x) + (1-\gamma)a_1(x,t); \\ y_2(x,t) &= \gamma\pi'(t) + (1-\gamma)a_2(x,t). \end{aligned}$$

Adding the two equations and using the fact that $a_1(x,t) + a_2(x,t) = 0$ under (A2), we get

$$\pi'(t) = \pi'(x) + [y_1(x,t) + y_2(x,t)]/\gamma \text{ for all } t \in \omega(x)$$

where the right-hand side is identified. By the smoothness of $s$ in $x^*$ given any $x$, the set $\mathcal{X}_e \cup \omega(x)$ is convex for all $x \in \mathcal{X}_e$. Therefore, under the location normalization $\pi(\underline{x}) = \pi_0$, $\pi(t)$ is identified for all $t \in \mathcal{X}_e \cup \omega(x)$. Thus, $a(x,t)$ is identified as

$$a(x,t) = \frac{y(x,t) - \gamma\pi(t) + \gamma\pi(x)}{1-\gamma}$$

for such a pair of $(x,t)$ with $x \in \mathcal{X}_e$ and $t \in \omega(x)$.

Part (b). By definition, the holdup on the buyer conditional on any $x \in \mathcal{X}_e$ is

$$\delta(x;\lambda^*) = \gamma \int_{\omega(x)} [\pi(t) - \pi(x) - a(x,t)]\, dF_{X^*|X=x}(t), \tag{C.8}$$

where we have used the fact that $F_{X^*|\tilde{X}=\alpha^{*-1}(x)}(.) = F_{X^*|X=x}(.)$ for all $x \in \mathcal{X}_e$. Part (a) showed that the integrand on the right-hand side of (C.8) is identified for all $x \in \mathcal{X}_e$ and $t \in \omega(x)$. Besides, for any $x \in \mathcal{X}_e$, the density $f_{X^*|X=x}(t)$ is identified at any $t \in \omega(x)$ as

$$f_{X^*|X=x}(t) = f_{X^*|D=1,X=x}(t) \times \Pr\{D = 1|X = x\}.$$

Thus $\delta(x; \lambda^*)$ is identified for all $x \in \mathcal{X}_e$. It then follows from Lemma C1 that the inverse of bidding strategy $\beta^*(., x)$, and therefore the cost distribution $F_{C|X=x}$ is identified for all $x \in \mathcal{X}_e$. □

# D Buyer surplus in cost-plus and fixed-price contracts

We now explain how to use estimates from Section 6 to calculate the buyer surplus under cost-plus and fixed-price contracts. Recall the expected surplus for buyers under a cost-plus contract is

$$E[\pi(X_j^*, Job_j)|w_j] - E(C_{j,i}^* + \zeta_{j,i}|w_j), \tag{D.1}$$

where $i$ denotes the winner, and $w_j \equiv (auti_j, afri_j, adis_j, nbid_j, job_j)$.

By construction, ex ante costs from contractor $i$ is

$$E(C_{j,i}^*|\tilde{z}_{j,i}) = E(C_{j,i}|\tilde{z}_{j,i}) + E[a(X_j, X_j^*, \tilde{z}_{j,i})|\tilde{z}_{j,i}]. \tag{D.2}$$

That is, the actual costs for implementing the new design $X^*$ is decomposed into the sum of costs based on the initial design $C_{j,i}$ and the incremental costs $a(X_j, X_j^*, \tilde{z}_{j,i})$. We assume $X_j^*$ and the initial design in equilibrium $X_j = \alpha^*(\tilde{X}_j)$ are independent from the contractor-specific information once conditional on the project-level characteristic $job_j$ in $\tilde{z}_{j,i}$.

Our first step is to estimate the expected profit margin quoted by the winner in equilibrium. We begin by estimating the ex ante actual costs by the mean of $\hat{c}_{j,i} + \hat{a}_{j,i}$ conditional on $\tilde{z}_{j,i}$, where $\hat{c}_{j,i}$ are estimated costs under the initial design in Section 4 and $\hat{a}_{j,i} \equiv a(x_j, x_j^*, \tilde{z}_{j,i}; \hat{\theta})$ are estimates for contractor-specific incremental costs in Section 4. For contracts with no transfers in the data, $\hat{a}_{j,i} = 0$ by construction. In what follows, we discretize the support of $\tilde{z}_{j,i}$ into disjoint bins, and estimate ex ante actual costs in (D.2) by the sample average of $\hat{c}_{j,i} + \hat{a}_{j,i}$ conditional on discretized values of $\tilde{z}_{j,i}$. Denote the estimates by $\hat{v}_{j,i}$. In each auction $j$, we find contractors $i$ and $i'$ who have the two lowest estimates for ex ante actual costs, and calculate the winner's quote for profit margin in equilibrium as $\hat{v}_{j,i'} - \hat{v}_{j,i}$.

We then estimate the second term in (D.1) by the average of $\hat{v}_{j,i'}$ across contracts conditional on discretized values of $w_j$ (because $C_{j,i}^* + \zeta_{j,i} = C_{j,i'}^*$ in equilibrium). To estimate the first term in (D.1), we calculate $\hat{\pi}_j^* \equiv \pi(x_j^*, job_j; \hat{\theta})$ for auctions with transfers; and $\hat{\pi}_j^* \equiv \pi(x_j, job_j; \hat{\theta})$ otherwise. We then calculate the average of $\hat{\pi}_j^*$ conditional on (discretized) values of $w_j$. The sum of these two conditional averages are our estimates for the buyer surplus in (D.1).

Next, we turn to the estimation of buyer surplus in equilibrium under fixed-price contracts, which is equal to

$$E[\pi(\alpha^*(\tilde{X}_j), Job_j) + \mu(\alpha^*(\tilde{X}_j), \tilde{X}_j, W_j) - \varphi(\alpha^*(\tilde{X}_j), W_j; \beta^*)|W_j = w_j],$$

where $\mu$ and $\varphi$ are defined in Section 2. The first term $E[\pi(\alpha^*(\tilde{X}_j), Job_j)|w_j]$ is estimated by the average of $\hat{\pi}_j \equiv \pi(x_j, job_j; \hat{\theta})$ conditional on (discretized) values of $w_j$. To estimate $E[\mu(\alpha^*(\tilde{X}_j), \tilde{X}_j, W_j)|w_j]$, we first calculate $\hat{m}_j \equiv \widehat{\Delta}_{j,i}\hat{q}_j[1 - \gamma(w_j; \hat{\varrho})]/\gamma(w_j; \hat{\varrho})$ for each contract $j$, with $i$ being the winning contractor, and then take the average of $\hat{m}_{j,i}$ across contracts with $w_j$.[25] The last term $E[\varphi(\alpha^*(\tilde{X}_j), W_j; \beta^*)|w_j]$ is estimated by a kernel regression of auction prices given $W_j = w_j$. Alternatively if components in $w_j$ are discrete, we can estimate it by a simple sample average.

[25] We use $\widehat{\Delta}_{j,i} \times \hat{q}_j$ as an estimator for $\delta(.; \lambda^*)$, and use the fact that $y - a = \gamma s$ in the Nash Bargaining solution.

# E Figures and Tables

Table 1: Summary Statistics[a]

| Variable | Mean | Std. Dev. | 5-th pctile | median | 95-th pctile |
|---|---|---|---|---|---|
| **Contract (Project)-level** | | | | | |
| number of bidders | 4.521 | 2.267 | 2 | 4 | 9 |
| engineering estimate (million dollars) | 2.813 | 6.681 | .227 | .947 | 10.212 |
| actual project size (million dollars) | 2.667 | 5.978 | .256 | 1.044 | 9.811 |
| job | 0.379 | .485 | 0 | 0 | 1 |
| winning bid (million dollars) | 2.701 | 6.604 | .195 | .885 | 9.423 |
| transfer (million dollars) | .258 | 1.1 | 0 | .039 | 1.09 |
| fringe[b] | .484 | .5 | 0 | 0 | 1 |
| distance (miles)[b] | 98.174 | 145.743 | 5 | 46.47 | 376 |
| utilization[b] | .125 | .217 | 0 | .005 | .637 |
| avg. fringe[c] | .564 | .293 | 0 | .6 | 1 |
| avg. utilization[c] | .121 | .127 | 0 | .084 | .377 |
| avg. distance[c] | 108.342 | 99.301 | 17.785 | 82.052 | 279 |
| **Contractor (Bidder)-level** | | | | | |
| bids (million dollars) | 3.055 | 17.185 | .202 | .912 | 10 |
| fringe | .620 | .485 | 0 | 1 | 1 |
| distance (miles) | 103.574 | 144.633 | 7.2 | 55.555 | 374 |
| utilization | .112 | .225 | 0 | 0 | .7 |
| min. dis. among rivals (miles) | 43.346 | 57.868 | 2.88 | 25.4 | 149 |
| min. uti. among rivals | .017 | .075 | 0 | 0 | .11 |

[a] There are 1306 auctions and 5908 bids in the sample.
[b] These rows report the characteristics of the auction winner.
[c] The average is taken for all bidders within an auction.

Table 2: Regression Results of Transfer[a]

| | (1) | (2) | (3) |
|---|---|---|---|
| spec. change ($x^* - x$) | 0.0681 | 0.122* | -0.0436 |
| | (0.0662) | (0.0698) | (0.0805) |
| avg. fringe | -0.211** | -0.206** | -0.104 |
| | (0.0989) | (0.101) | (0.102) |
| number of bidders | -0.0135 | -0.0158 | -0.0150 |
| | (0.0130) | (0.0130) | (0.0129) |
| job | 0.0387 | 0.0295 | 0.00889 |
| | (0.0549) | (0.0550) | (0.0547) |
| spec. change ∗ avg. fringe | 0.655*** | 0.648*** | 0.790*** |
| | (0.0959) | (0.0960) | (0.103) |
| spec. change ∗ number of bidders | -0.143*** | -0.144*** | -0.144*** |
| | (0.0158) | (0.0158) | (0.0158) |
| spec. change ∗ job | -0.0541 | -0.0826* | -0.132*** |
| | (0.0456) | (0.0471) | (0.0482) |
| spec. change ∗ spec. change | 0.00236*** | 0.00279*** | 0.00225*** |
| | (0.000646) | (0.000671) | (0.000682) |
| avg. utilization | | -0.238 | -0.240 |
| | | (0.217) | (0.215) |
| spec. change ∗ avg. utilization | | -0.419** | -0.305* |
| | | (0.173) | (0.174) |
| avg. distance | | | 0.00103*** |
| | | | (0.000290) |
| spec. change ∗ avg. distance | | | 0.000456*** |
| | | | (0.000132) |
| constant | 0.355*** | 0.389*** | 0.219** |
| | (0.0714) | (0.0819) | (0.0916) |
| $N$ | 1224 | 1224 | 1224 |
| $R^2$ | 0.334 | 0.338 | 0.351 |
| adj. $R^2$ | 0.330 | 0.332 | 0.344 |

[a] Standard errors in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. We only use observations of contracts that are modified with negotiated transfers.

Table 3: Regression Results of Specification Change[a]

| | (1) | (2) | (3) |
|---|---|---|---|
| number of bidders | -0.0479* | -0.0485* | -0.0485* |
| | (0.0281) | (0.0281) | (0.0281) |
| avg. fringe | 0.519** | 0.503** | 0.498** |
| | (0.215) | (0.219) | (0.223) |
| job | 0.0835 | 0.0815 | 0.0818 |
| | (0.120) | (0.120) | (0.120) |
| avg. utilization | | -0.186 | -0.185 |
| | | (0.468) | (0.469) |
| avg. distance | | | -0.0000769 |
| | | | (0.000634) |
| constant | -0.264* | -0.228 | -0.218 |
| | (0.155) | (0.179) | (0.200) |
| $R^2$ | 0.006 | 0.006 | 0.006 |
| adj. $R^2$ | 0.003 | 0.002 | 0.002 |
| $p$-value (overall signifiance) | 0.08 | 0.14 | 0.22 |

[a] Standard errors in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. The regression uses the observations with contract revisions and observed transfers.

Table 4: Regression results of expected payment[a]

| | (1) | (2) | (3) |
|---|---|---|---|
| engineering estimate | 1.105*** | 1.080*** | 1.029*** |
| | (0.0160) | (0.0165) | (0.0170) |
| engineering estimate ∗ engineering estimate | 0.0000175 | 0.000107 | -0.000228* |
| | (0.000133) | (0.000133) | (0.000135) |
| avg. fringe | -0.0465 | -0.0574 | -0.108 |
| | (0.117) | (0.118) | (0.115) |
| number of bidders | 0.0151 | 0.0126 | 0.0121 |
| | (0.0146) | (0.0145) | (0.0140) |
| job | -0.137** | -0.145** | -0.121** |
| | (0.0620) | (0.0614) | (0.0596) |
| engineering estimate ∗ avg. fringe | 0.128*** | 0.120*** | 0.150*** |
| | (0.0198) | (0.0197) | (0.0194) |
| engineering estimate ∗ number of bidders | -0.0490*** | -0.0484*** | -0.0487*** |
| | (0.00314) | (0.00312) | (0.00302) |
| engineering estimate ∗ job | 0.0622*** | 0.0648*** | 0.0539*** |
| | (0.00842) | (0.00835) | (0.00818) |
| avg. utilization | | -0.631** | -0.613** |
| | | (0.246) | (0.239) |
| engineering estimate ∗ avg. utilization | | 0.150*** | 0.142*** |
| | | (0.0283) | (0.0274) |
| avg. distance | | | -0.0000780 |
| | | | (0.000302) |
| engineering estimate ∗ avg. distance | | | 0.000299*** |
| | | | (0.0000347) |
| constant | -0.0460 | 0.0583 | 0.104 |
| | (0.0872) | (0.0972) | (0.104) |
| $N$ | 1306 | 1306 | 1306 |
| $R^2$ | 0.977 | 0.978 | 0.979 |
| adj. $R^2$ | 0.977 | 0.978 | 0.979 |

[a] The units of expected payment and engineering estimate are million dollars. Standard errors in parentheses, * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 5: Estimates of Average Marginal Effects on Bargaining Power[a]

| | | (1) | (2) | (3) |
|---|---|---|---|---|
| avg. bargaining power | job=1 | 0.911*** | 0.887*** | 0.689** |
| | | (0.185) | (0.192) | (0.323) |
| | job= 0 | 0.903*** | 0.953*** | 0.793** |
| | | (0.214) | (0.254) | (0.323) |
| avg. marginal effects | number of bidders | 0.060** | 0.074 | 0.051 |
| | | (0.032) | (0.047) | (0.071) |
| | avg. fringe | -0.077 | -0.086 | -0.281 |
| | | (0.178) | (0.238) | (0.471) |
| | avg. utilization | | -0.178 | 0.440** |
| | | | (0.214) | (0.189) |
| | avg. distance | | | 0.001 |
| | | | | (0.002) |

[a] Bootstrap standard errors in parentheses are calculated using 200 bootstrap samples: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. Specifications (1), (2), and (3) are for $w \equiv [job, nbid, afri]$, $w \equiv [job, nbid, afri, auti]$ and $w \equiv [job, nbid, afri, auti, adis]$, respectively.

Table 6: Estimates of Surplus and Cost Functions[a]

| | | (1) | (2) | (3) |
|---|---|---|---|---|
| Surplus function $\pi(\cdot)$ | $X$ | -0.239** | -0.233** | -0.262*** |
| | | (0.083) | (0.082) | (0.083) |
| | $X *$ job | -0.118 | -0.134 | -0.115 |
| | | (0.088) | (0.085) | (0.114) |
| | $X^2$ | 0.006*** | 0.006*** | 0.006*** |
| | | (0.002) | (0.002) | (0.002) |
| Cost function $a(\cdot)$ | $(X^* - X)$ | 0.031 | 0.037 | 0.008 |
| | | (0.048) | (0.051) | (0.061) |
| | $(X^* - X) *$ job | -0.104 | -0.120* | -0.101 |
| | | (0.065) | (0.071) | (0.083) |
| | $(X^* - X) * (X^* - X)$ | -0.010** | -0.010** | -0.010** |
| | | (0.005) | (0.005) | (0.004) |
| | constant | 1.566*** | 1.566*** | 1.566*** |
| | | (0.057) | (0.057) | (0.057) |

[a] Bootstrap standard errors in parentheses are calculated using 200 bootstrap samples: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. Columns (1), (2), and (3) report estimates based on the first-stage estimates from specifications (1), (2), and (3) in Table 4, respectively.

Table 7: Logit Regression of Probability of Incompleteness[a]

| | (1) | (2) | (3) |
|---|---|---|---|
| constant | 0.997*** | 1.179*** | 1.161*** |
| | (0.249) | (0.273) | (0.275) |
| engineering estimate | 2.149*** | 1.798*** | 1.846*** |
| | (0.381) | (0.410) | (0.417) |
| job | 0.226 | -0.403 | -0.391 |
| | (0.256) | (0.512) | (0.512) |
| engineering estimate ∗ job | | 1.309 | 1.287 |
| | | (0.968) | (0.967) |
| engineering estimate ∗ engineering estimate | -0.0198*** | | -0.0202*** |
| | (0.00529) | | (0.00675) |
| observations | 1306 | 1306 | 1306 |
| log-likelihood | -246.72 | -245.81 | -245.70 |
| pseudo $R^2$ | 0.157 | 0.160 | 0.161 |

[a] Standard errors in parentheses, * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 8: Estimates of Markup Ratio (unit: %)[a]

| | | 10-th pctile | 25-th pctile | median | 75-th pctile | 90-th pctile |
|---|---|---|---|---|---|---|
| job | 1 | .017 | .031 | .058 | .131 | .236 |
| | 0 | .020 | .036 | .065 | .129 | .244 |
| winners | Yes | .038 | .063 | .122 | .221 | .397 |
| | No | .017 | .028 | .054 | .105 | .190 |
| fringe | Yes | .017 | .028 | .054 | .112 | .210 |
| | No | .024 | .045 | .078 | .154 | .291 |
| utilization[b] | 0 | .017 | .028 | .054 | .109 | .205 |
| | $(0, 0.3)$ | .025 | .047 | .082 | .161 | .289 |
| | $[0.3, 1]$ | .021 | .040 | .067 | .147 | .292 |
| distance[b] | $[0.1, 11.9]$ | .022 | .037 | .065 | .142 | .278 |
| | $[12, 247.9]$ | .018 | .033 | .062 | .128 | .233 |
| | $> 247.9$ | .020 | .036 | .063 | .132 | .284 |
| number of bidders[c] | 2 | .153 | .181 | .232 | .343 | .523 |
| | $3 \leq n \leq 5$ | .038 | .051 | .078 | .137 | .251 |
| | $6 \leq n \leq 8$ | .017 | .022 | .035 | .073 | .152 |
| | $8 < n$ | .010 | .014 | .019 | .039 | .081 |
| overall | | .019 | .034 | .062 | .129 | .241 |

[a] The markup ratio is defined as a contractor's markup $p_{j,i} + \widehat{\Delta}_{j,i}\hat{q}_j - \hat{c}_{j,i}$ over her initial cost $\hat{c}_{j,i}$.
[b] The intervals of distance are defined by their lower, middle and upper terciles. For all the positive utilization rates, we divide them into $(0, 0.3)$ and $[0.3, 1]$ such that the observations in the two intervals equal.
[c] $n$ is the number of bidders.

Table 9: Estimates of Markup Ratios Assuming away Incompleteness (unit: %)[a]

| | | 10-th pctile | 25-th pctile | median | 75-th pctile | 90-th pctile |
|---|---|---|---|---|---|---|
| job | 1 | .022 | .038 | .071 | .162 | .301 |
| | 0 | .024 | .045 | .077 | .161 | .306 |
| winning bidders | Yes | .050 | .082 | .149 | .277 | .512 |
| | No | .022 | .036 | .064 | .129 | .240 |
| fringe | Yes | .022 | .036 | .066 | .137 | .271 |
| | No | .029 | .054 | .092 | .189 | .359 |
| utilization[b] | 0 | .020 | .036 | .065 | .134 | .262 |
| | $(0, 0.3)$ | .032 | .058 | .098 | .194 | .365 |
| | $[0.3, 1]$ | .026 | .042 | .083 | .175 | .363 |
| distance[b] | $[0.1, 11.9]$ | .025 | .046 | .074 | .168 | .334 |
| | $[12, 247.9]$ | .023 | .042 | .076 | .161 | .299 |
| | $> 247.9$ | .024 | .042 | .073 | .156 | .326 |
| number of bidders[c] | 2 | .188 | .199 | .269 | .406 | .670 |
| | $3 \leq n \leq 5$ | .047 | .061 | .094 | .170 | .311 |
| | $6 \leq n \leq 8$ | .022 | .027 | .045 | .095 | .211 |
| | $8 < n$ | .014 | .017 | .027 | .058 | .106 |
| overall | | .023 | .043 | .075 | .161 | .305 |

[a] The markup ratio assuming away incompleteness is defined as a contractor's markup $\hat{\varpi}_{j,i} \equiv \left(\sum_{k=1,k\neq i}^{n_j} f_{P|Z}(p_{j,i}|z_{j,k})/[1 - F_{P|Z}(p_{j,i}|z_{j,k})]\right)^{-1}$ over her initial cost $p_{j,i} - \hat{\varpi}_{j,i}$.

[b] The intervals of distance are defined by their lower, middle and upper terciles. For all the positive utilization rates, we divide them into $(0, 0.3)$ and $[0.3, 1]$ such that the observations in the two intervals equal.

Table 10: Estimates of Average Marginal Effects on the Holdup Ratio[a]

| | $n=2$ | | | $5 \geq n \geq 3$ | | | $8 \geq n \geq 6$ | | | $n > 8$ | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Variable | (1) | (2) | (3) | (1) | (2) | (3) | (1) | (2) | (3) | (1) | (2) | (3) |
| avg. fringe | 0.062*** | 0.053*** | .057*** | 0.144*** | 0.135*** | 0.134*** | 0.135*** | 0.129*** | 0.0821*** | 0.359*** | 0.417*** | 0.383*** |
| | (0.0001) | (0.011) | (0.0007) | (0.001) | (0.001) | (0.001) | (0.001) | (0.0007) | (0.0006) | (0.009) | (0.0007) | (0.014) |
| job | -0.010*** | -0.011*** | -0.010*** | 0.033*** | 0.031*** | 0.033*** | 0.032*** | 0.031*** | 0.033*** | -0.010*** | -0.015 | 0.011 |
| | (0.0001) | (0.0004) | (0.002) | (0.006) | (0.005) | (0.006) | (0.0004) | (0.001) | (0.0006) | (0.0003) | (0.011) | (0.013) |
| avg. utilization | | -0.126*** | -0.124*** | | -0.095*** | -0.097*** | | -0.045*** | -0.020*** | | -0.251*** | -0.183*** |
| | | (0.0008) | (0.001) | | (0.001) | (0.001) | | (0.005) | (0.002) | | (0.065) | (0.074) |
| avg. distance | | | -1.13e-4*** | | | -0.045*** | | | -3.39e-4*** | | | 4.39e-4 |
| | | | (1.10e-5) | | | (0.005) | | | (2.37e-6) | | | (7.00e-6) |

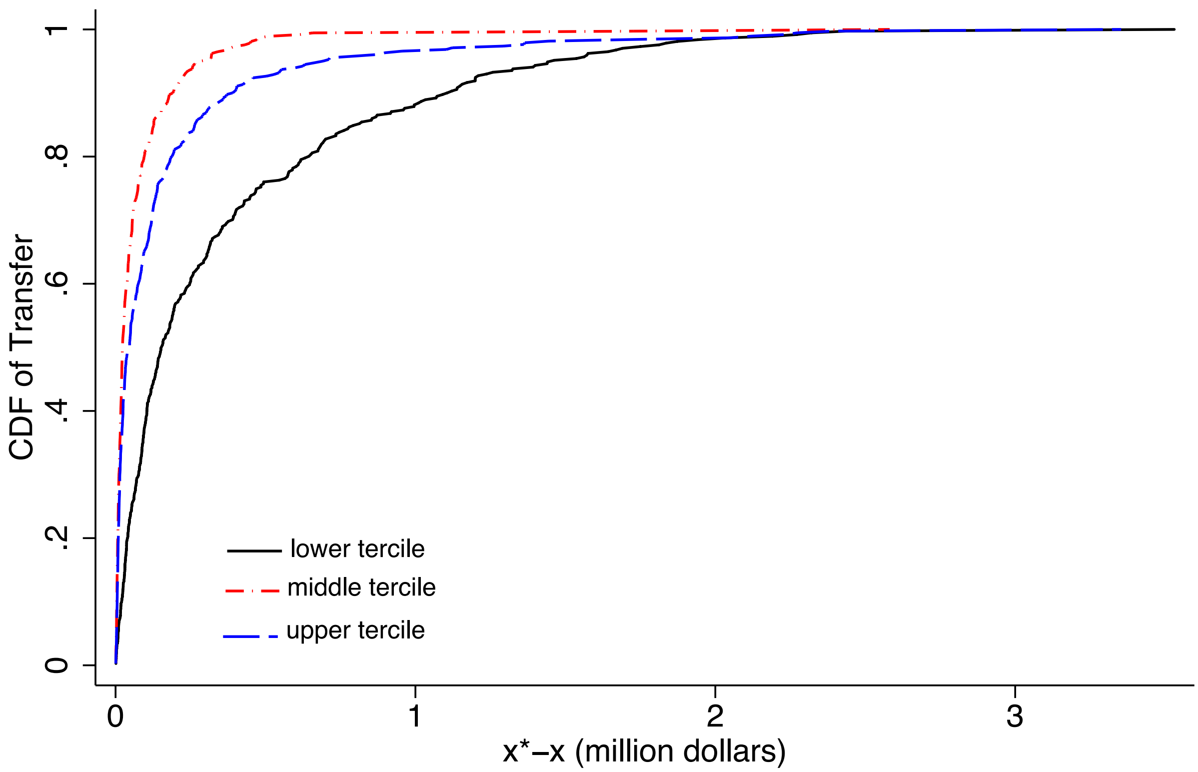
[a] The holdup ratio is defined as the ratio of holdup over the engineering estimate in each contract. Standard errors in parentheses, * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. Specification (1): ratio=$\beta_0$+$\beta_1$avg. fringe+$\beta_2$job +$\beta_3$avg. fringe×job +$\epsilon$. Specification (2): ratio=$\beta_0$+$\beta_1$avg. fringe+$\beta_2$avg. utilization+$\beta_3$job +$\beta_4$avg. fringe×job +$\beta_5$avg. utilization×job+$\epsilon$. Specification (3): ratio=$\beta_0$+$\beta_1$avg. fringe+$\beta_2$avg. utilization+$\beta_3$avg. distance+ $\beta_4$job +$\beta_5$avg. fringe×job +$\beta_6$avg. utilization×job+$\beta_7$avg. distance×job+$\epsilon$.

Table 11: Estimates of Average Marginal Effects on Surplus Difference (Cost-plus Minus Fixed-price, in Million Dollars)[a]

| | $n = 2$ | | | $5 \geq n \geq 3$ | | | $8 \geq n \geq 6$ | | | $n > 8$ | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Variable | (1) | (2) | (3) | (1) | (2) | (3) | (1) | (2) | (3) | (1) | (2) | (3) |
| engineering estimate | 1.029*** | 1.028*** | 1.036*** | 0.980*** | 0.979*** | 0.976*** | 0.878*** | 0.876*** | 0.868*** | 0.745*** | 0.741*** | 0.745*** |
| | (0.007) | (0.007) | (0.006) | (0.002) | (0.002) | (0.002) | (0.002) | (0.002) | (0.002) | (0.017) | (0.019) | (0.015) |
| avg. fringe | 0.919*** | 0.943*** | 0.871*** | 0.454*** | 0.533*** | 0.549*** | 0.397*** | 0.484*** | 0.050*** | 0.451 | 0.072 | 0.028 |
| | (0.518) | (0.062) | (0.046) | (0.013) | (0.014) | (0.015) | (0.015) | (0.017) | (0.003) | (0.146) | (0.102) | (0.093) |
| job | -0.061*** | -0.059*** | -0.079*** | -0.234*** | -0.228*** | -0.237*** | -0.240*** | -0.247*** | -0.234*** | -0.333*** | -0.407*** | -0.376*** |
| | (0.017) | (0.016) | (0.013) | (0.011) | (0.011) | (0.009) | (0.008) | (0.010) | (0.009) | (0.038) | (0.004) | (0.030) |
| avg. utilization | | 0.147*** | 0.102** | | 0.720*** | 0.727*** | | 0.124*** | 0.137*** | | 1.776 | 2.001*** |
| | | (0.049) | (0.043) | | (0.011) | (0.009) | | (0.030) | (0.001) | | (0.157) | (0.078) |
| avg. distance | | | 2.78e-4*** | | | 5.01e-4*** | | | -4.00e-4*** | | | -3.36e-4*** |
| | | | (6.75e-5) | | | (3.986e-5) | | | (9.06e-5) | | | (6.76e-5) |

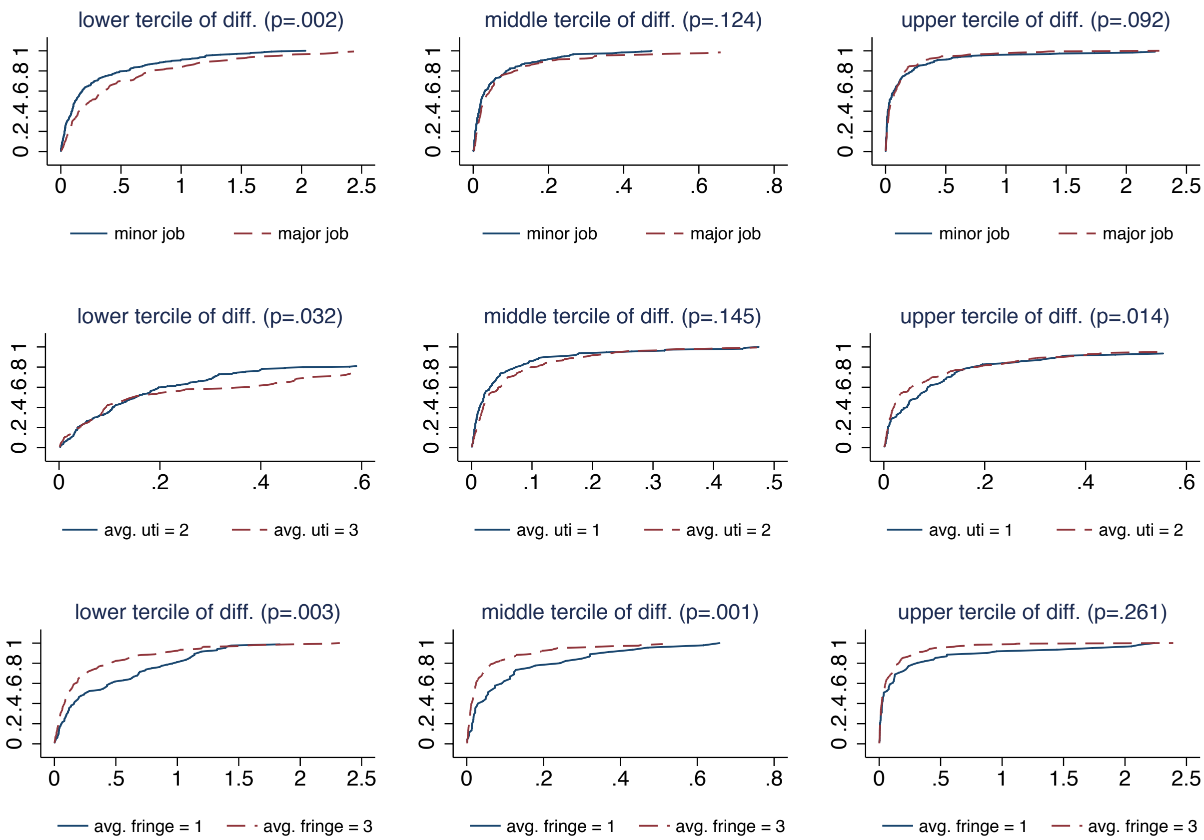
[a] Standard errors in parentheses, * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. Specification (1): diff=$\beta_0$+$\beta_1$engineering estimate+$\beta_2$avg. fringe+$\beta_3$job +$\beta_4$avg. fringe×engineering estimate+ $\beta_5$job×engineering estimate+$\beta_6$engineering estimate×engineering estimate+$\epsilon$. Specification (2): diff=$\beta_0$+$\beta_1$engineering estimate+$\beta_2$avg. fringe+$\beta_3$avg. utilization+$\beta_4$job +$\beta_5$avg. fringe×engineering estimate+$\beta_6$avg. utilization×engineering estimate+ $\beta_7$job×engineering estimate+$\beta_8$engineering estimate×engineering estimate+$\epsilon$. Specification (3): diff=$\beta_0$+$\beta_1$engineering estimate+$\beta_2$avg. fringe+$\beta_3$avg. utilization+$\beta_4$avg. distance+$\beta_5$job +$\beta_6$avg. fringe×eng.+$\beta_7$avg. utilization×engineering estimate+$\beta_8$avg. distance×engineering estimate+ $\beta_9$job×engineering estimate+$\beta_{10}$engineering estimate×engineering estimate+$\epsilon$.

Figure 1: CDF of Transfer Conditional on Specification Difference



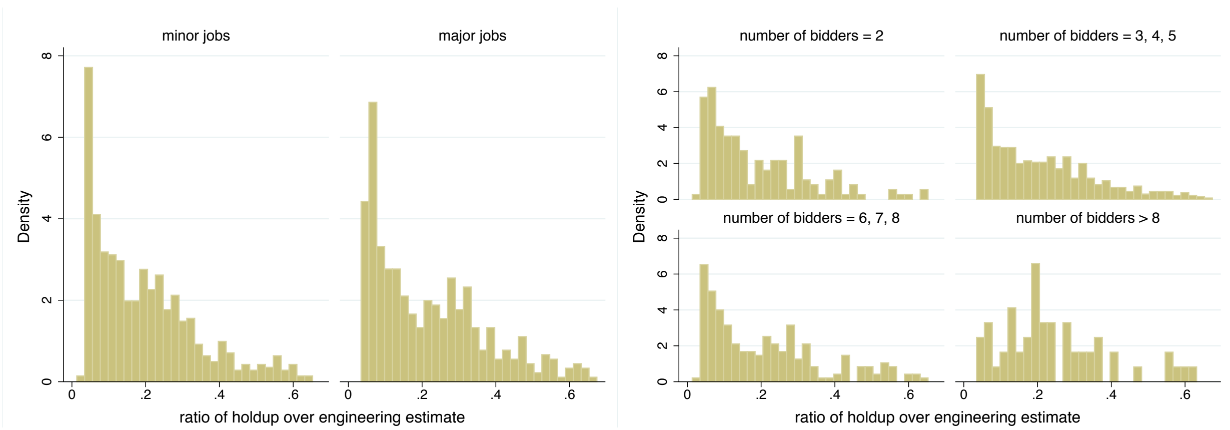

Note: This figure plots the empirical distributions of transfers conditional on the lower, middle and upper terciles of the difference between the initial specification $x$ and the final specification adopted $x^*$. For each pair of transfer distributions, the Kolmogorov-Smirnov test rejects the null of identical distribution at the 1% significance level.

Figure 2: CDF of Transfer Conditional on Contract-level Characteristics




Note: In all the subfigures, the $x$-axis is the specification change $x^* - x$ (million dollars). The subfigures in the left, middle and right columns are conditional on the lower, middle and upper terciles of the specification change $x^* - x$, respectively. $avg.uti = 1, 2$, and 3 correspond to the lower, middle and upper terciles of average utilization. $avg.fringe$ is similarly defined. In the caption of each subfigure, we provide the $p$-value for the Kolmogorov-Smirnov test in the parenthesis.

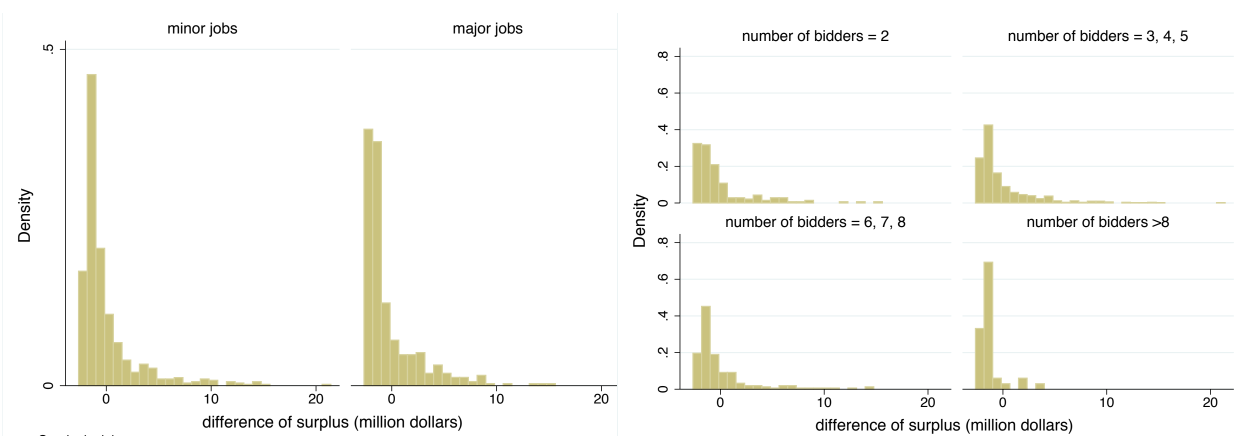
Figure 3: Histogram of the Holdup Ratio




(a) the ratio: by job type (b) the ratio: by number of bidders

Note: The holdup ratio is defined as the ratio of holdup over the engineering estimate in each contract.

Figure 4: Histogram of the Surplus Difference




(a) the surplus difference: by job type (b) the surplus difference: by number of bidders

Note: The surplus difference is defined as the surplus under cost-plus contract subtracts that of the fixed-price contract.